The characterisation and propagation of stochastic fields from printed circuit boards

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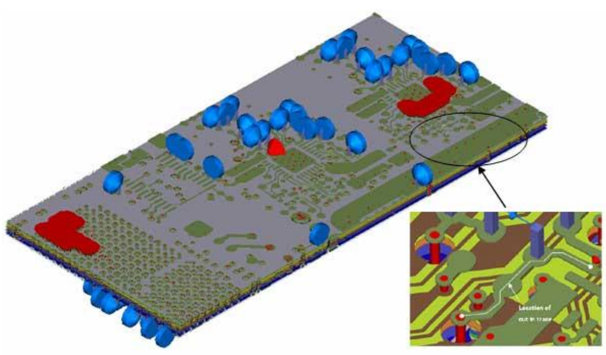
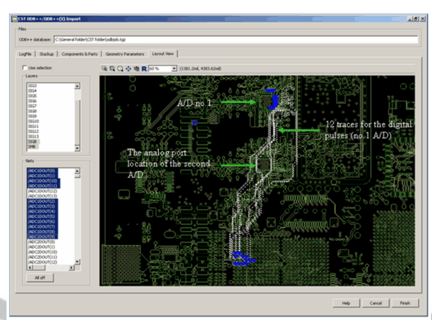
Direct modelling or Equivalent methods

3D EM simulation of mixed analog / digital PCB





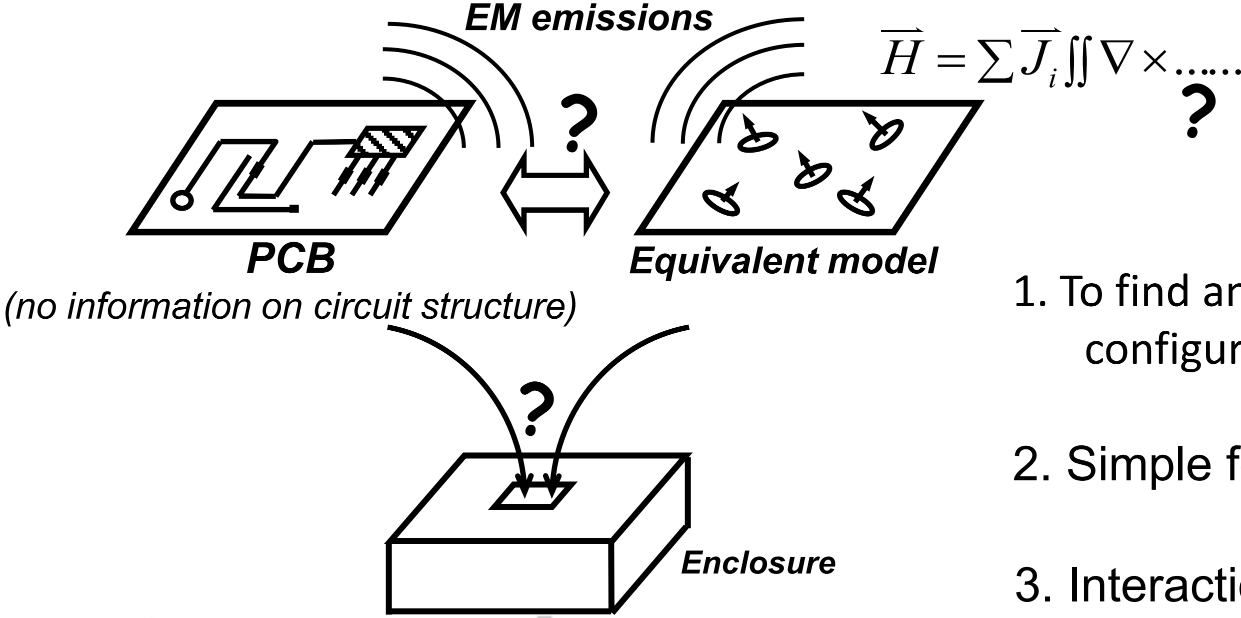
modeling time	running time	memory required
1 week	10 h	3 GB

Difficulties

- unrealistic computational resources and time due to increasingly complex circuit structure
- unknown characteristics of the circuit
- confidential reasons

Equivalent modelling

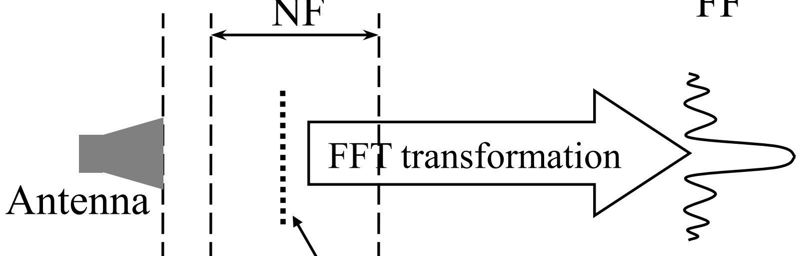
- not modeling the complete complexity of PCBs
- representing the radiations by equivalent sources
- fast and computationally low-cost
- general for radiators at printed board level



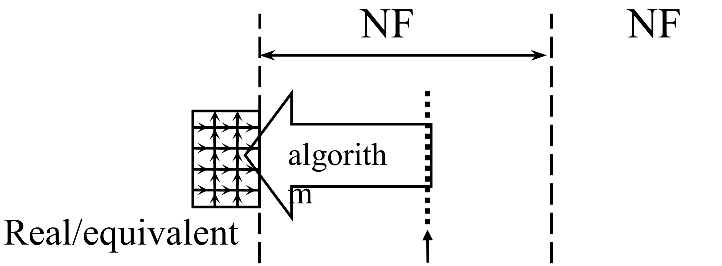
1. To find an efficient equivalent configuration to represent the PCB
2. Simple formulation
3. Interactions with packages

Near Field Scanning

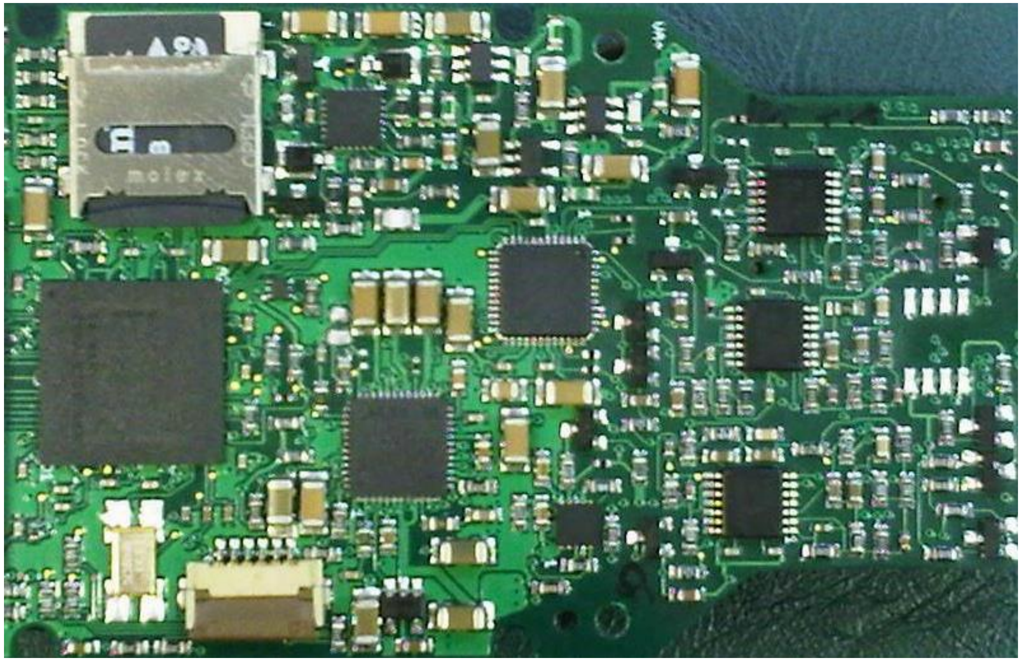
Popular technique for providing EM fields closely surrounding DUTs



NF – FF transformation



Source model from NF



Mixed signal PCBs radiate across a broad frequency range with a range of correlated and uncorrelated signals

Correlation spectrum

$$\Gamma_H(x_1,x_2,\omega)=\int_{-\infty}^{\infty} c_h(x_1,x_2,\tau)e^{-j\omega\tau}d\tau=\lim_{T\to\infty}\frac{1}{2T}\left\langle \boldsymbol{H}_T(x_1,\omega)\boldsymbol{H}_T^*(x_2,\omega)\right\rangle$$

The spectral magnetic energy density is then

$$W_H(x,\omega)=\frac{\mu}{2}\Gamma_H(x,x,\omega)$$

Correlation function

$$\boldsymbol{h}_T(x,t) = \begin{cases} \boldsymbol{h}(x,t) & for\ -T < t < T \\ 0 & for\ |t| \geq T \end{cases}$$

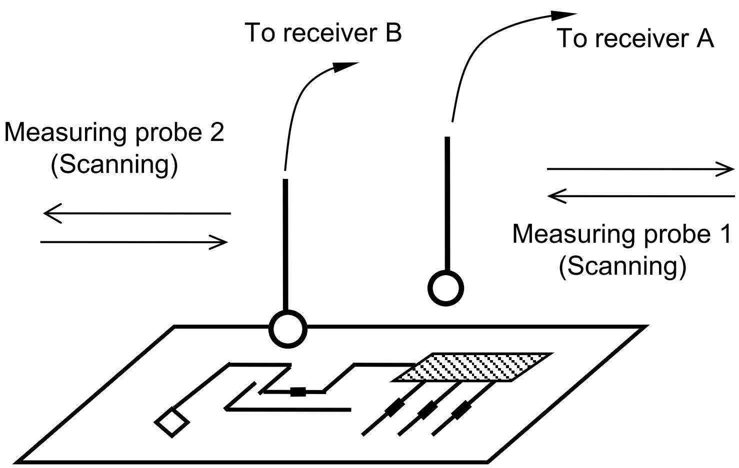
The correlation function:

$$c_h(x_1,x_2,\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-\infty}^{\infty} \boldsymbol{h}_T(x_1,t)\,\boldsymbol{h}(x_2,t-\tau)\,dt$$

Obtained from the FT of the correlation function or the FT of the windowed magnetic field $\mathbf{H}_T(x,\omega)$ through an ensemble average,

Measurement

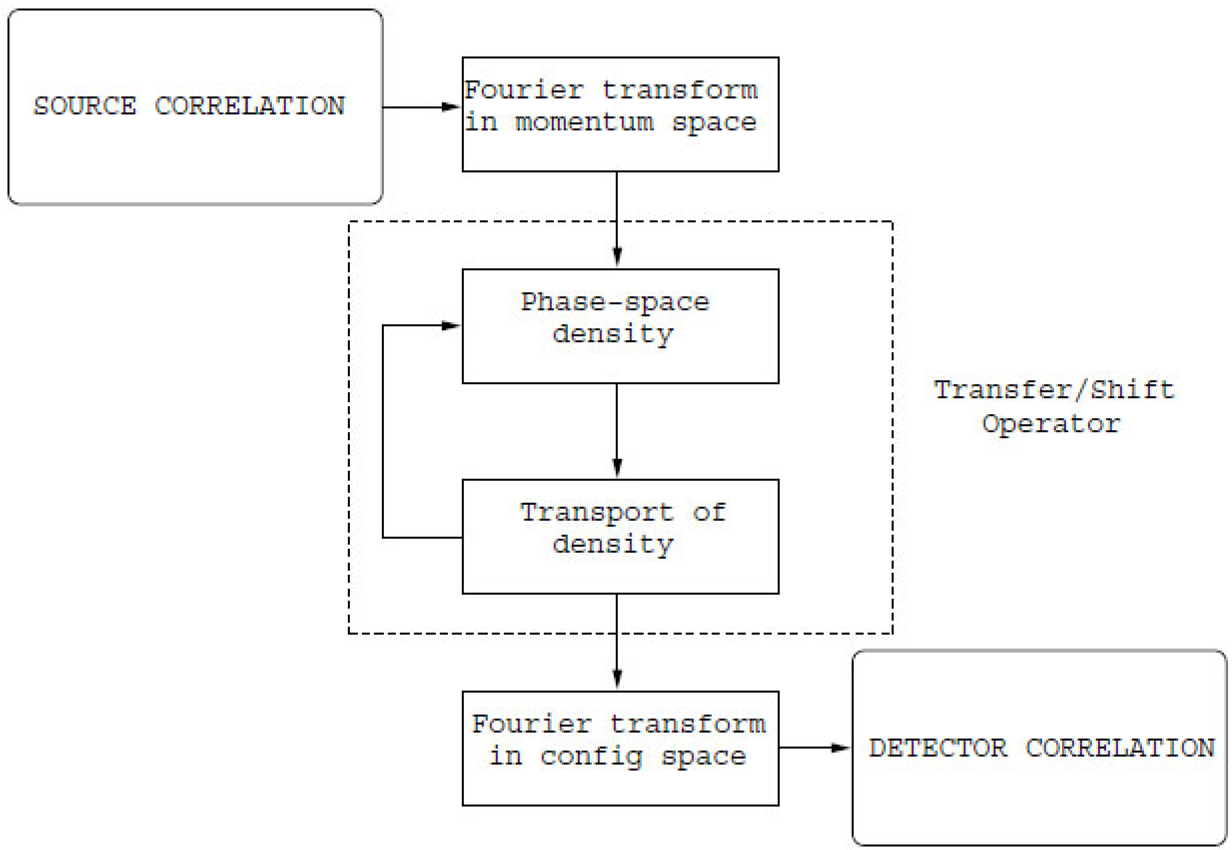
Experimentally we obtain $\Gamma_H(\boldsymbol{r},\boldsymbol{r},\omega)$ using the two probe arrangement below,



Then by inverting:

$$C_M(\boldsymbol{r},\omega) = \xi(\boldsymbol{r},\omega)^{-1}\,\Gamma_H(\boldsymbol{r},\boldsymbol{r},\omega)\,\xi(\boldsymbol{r},\omega)^{*-1}$$

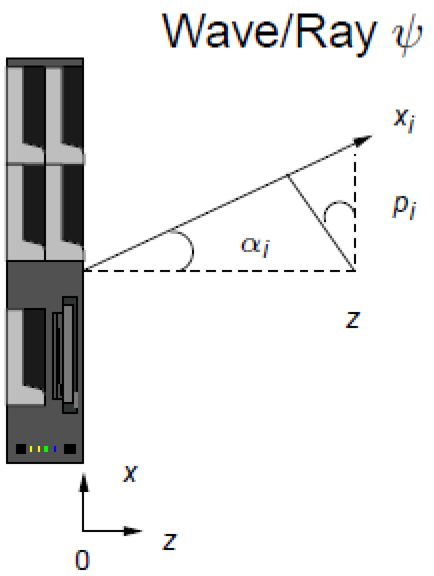
Procedure


SOURCE CORRELATION
Fourier transform
in momentum space
Phase-space
density
Transport of
density
Transfer/Shift
Operator
Fourier transform
in config space
DETECTOR CORRELATION

Fields in momentum space

- Generalized coordinates (x_i) live in spatial planes parallel to source surface
- Conjugate variables (momenta p_i) are direction cosines of the wave vector

$$\hat{p}_i = \sin\alpha_i$$



$$T(p_i) = \sqrt{1 - p_i^2} = \cos\alpha_i$$

Huygens principle

$$\psi(p,z) = \hat{G}_0\nu(p,z=0) + \hat{G}_1\psi(p,z=0)$$

$$\hat{G}_{0,1}(p,z) = \int G_{0,1} e^{-i\,k\,p\,x'}\,dx'$$

with $\nu(p,z=0) = \partial\psi/\partial z|_{z=0}$

[Creagh *et al.*, Journal of Physics A, 2013]

Correlation in momentum space

$$\psi(p,z) = \hat{G}_0 \nu(p) - \hat{G}_1 \psi(p)$$

$$\psi(p,z) = \underbrace{\frac{i}{2k\sqrt{1-p^2}} e^{ikz\sqrt{1-p^2}}}_{\hat{G}_0} \nu(p, z=0) + \underbrace{\frac{1}{2} e^{ikz\sqrt{1-p^2}}}_{\hat{G}_1} \psi(p, z=0)$$

Solution of Helmholtz Equation

$$\psi(p,z) = e^{ikzT(p)} \psi(p, z=0)$$

with $T(p) = \sqrt{1-p^2}$ “kinetic” operator for propagating waves

Fields carry fluctuations

$$\hat{\Gamma}_z(p_1,p_2) = \langle \psi(p_1,z)\, \psi^*(p_2,z) \rangle = e^{ikz(T(p_1) - T^*(p_2))}\, \hat{\Gamma}_0(p_1,p_2)$$

$\hat{\Gamma}_0$ measured or inferred.

Wigner distribution function

$$W_z(x,p) = \int e^{ikxq}\,\rho\left(p+\frac{q}{2}, p-\frac{q}{2}\right)\,dq$$

$$\rho(p_1,p_2) = \langle \psi(p_1)\,\psi^*(p_2)\rangle$$

- $\hat{\Gamma}_z(p_1,p_2)$ is just the single-particle density matrix $\rho(p_1,p_2)$
- $\langle\cdot\rangle$ ensemble average: necessary in absence of information
- WDF of waves in phase-space $\psi(p)$
- Enough to predict the flow of energy

Properties of WDF

Phase-space configures as $(x, p) \in \mathbb{R}^n \times \mathbb{R}^n$ [1]. For $n = 1$ we get

$$W_z(x,p) = \int e^{ikxq}\, \hat{\Gamma}_z\left(p + \frac{q}{2}, p - \frac{q}{2}\right) dq$$

$$\hat{\Gamma}_z\left(p + \frac{q}{2}, p - \frac{q}{2}\right) = \int e^{ikx'q}\, W_z\left(x', p\right) dx'$$

Symmetric in x and p!

$$W_z(x,p) = \int e^{-ikps}\, \Gamma_z\left(x + \frac{s}{2}, x - \frac{s}{2}\right) ds$$

Needs following change of variables in phase-space/configuration correlation [2] [3]

$$p_1 = p + \tfrac{q}{2},\; p_2 = p - \tfrac{q}{2}$$
$$x_1 = x + \tfrac{s}{2},\; x_2 = x - \tfrac{s}{2} \rightarrow s = x_1 - x_2,\; x = \tfrac{x_1 + x_2}{2}$$

[1] E. Wigner, Phys. Rev., 40, 749759, 1932 [2] N. Marcuvitz, Proc. IEEE, 79-10, 1991 [3] R. G. Littlejohn, Phys. Rep., 138, 193-291, 1986

Transport of WDF for waves

Given source correlation ($z = 0$), detector correlation ($z \neq 0$) involves kinetic operator

$$W_z\left(x,p\right) = \int e^{ikxq}\, \underbrace{e^{ikz\left(\sqrt{1-\left(p+\frac{q}{2}\right)^2}-\sqrt{1-\left(p-\frac{q}{2}\right)^2}^{\,*}\right)}\, \hat{\Gamma}_0\left(p+\frac{q}{2},p-\frac{q}{2}\right)}_{\hat{\Gamma}_z\left(p+\frac{q}{2},p-\frac{q}{2}\right)}\, dq$$

But

$$\hat{\Gamma}_0\left(p+\frac{q}{2},p-\frac{q}{2}\right) = \int e^{ikx'q}\, W_0\left(x',p\right)\, dx'$$

allows

Exact

$$W_z\left(x,p\right) = \int\int \hat{\mathcal{G}}\left(x-x',z;p,p'\right)\, W_0\left(x',p'\right)\, dx'\, dp'$$

$\hat{\mathcal{G}}$ transports Wigner distribution functions in phase space.

Suggests a scheme for numerical computation ($FFT \leftrightarrow IFFT$)

Linearization

Taylor series expansion of the kinetic operator (small q)

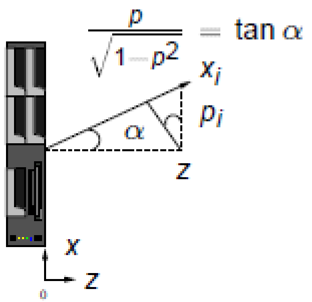
$$\Delta T(p,q) = \sqrt{1-\left(p+\frac{q}{2}\right)^2} - \sqrt{1-\left(p-\frac{q}{2}\right)^2}^{\,*}$$

- Even-order derivatives equate to zero for propagating waves
- Linearization of $\Delta T \approx \frac{p}{\sqrt{1-p^2}}q$ results in a good approximation of $\hat{\mathcal{G}}$

$$\hat{\mathcal{G}}\left(x-x',z;p\right) \approx \int e^{ik\left(x-x'\right)q-ikz\frac{p}{\sqrt{1-p^2}}q}\,dq = \delta\left(x-x'-\frac{p}{\sqrt{1-p^2}}z\right)$$

This yields a Frobenius-Perron equation

$$W_z(x,p) \approx W_0\left(x-z\frac{p}{\sqrt{1-p^2}},p\right)$$



Next (third-) order

Taylor series expansion of the kinetic operator (small q)

$$\Delta T(p,q)=\sqrt{1-\left(p+\frac{q}{2}\right)^2}-\sqrt{1-\left(p-\frac{q}{2}\right)^2}^{\,*}$$

- Even-order derivatives equate to zero for propagating waves
- Up to the third-order $\Delta T \approx \left[\frac{p}{\sqrt{1-p^2}}\right] q + \left[\frac{p}{4(1-p^2)^{3/2}}+\frac{p^3}{4(1-p^2)^{5/2}}\right] q^3$

$$\hat{\mathcal{G}}\left(x-x',z;p\right)\approx\int e^{ik\left(x-x'\right)q-ikz\left[\frac{p}{\sqrt{1-p^2}}\right]q+ikz\left[\frac{p}{4(1-p^2)^{3/2}}+\frac{p^3}{4(1-p^2)^{5/2}}\right]q^3}\,dq$$

$$=-\frac{\mathrm{Ai}\left\{-\frac{\left(\underline{x}-\underline{x}'\right)k^{2/3}}{z^{1/3}}-\frac{(kz)^{2/3}}{\left[g(\underline{p})\right]^{1/3}\sqrt{1-|\underline{p}|^2}}\right\}}{4\pi\,(kz)^{1/3}\left[g\left(\underline{p}\right)\right]^{1/3}},$$

$$g\left(\underline{p}\right)=-\frac{3\underline{p}}{4\left(1-|\underline{p}|^2\right)^{3/2}}-\frac{3\left|\underline{p}\right|^3}{4\left(1-|\underline{p}|^2\right)^{5/2}}.$$

N. Marcuvitz, Proc. IEEE, 79-10, 1991

Frobenius-Perron equation

We take the exact propagator

$$\hat{\mathcal{G}}\left(x-x',z;p,p'\right)=\delta\left(p-p'\right)\int e^{ik\left(x-x'\right)q+ikz\left(\sqrt{1-\left(p+\frac{q}{2}\right)^2}-\sqrt{1-\left(p-\frac{q}{2}\right)^2}^{*}\right)}\,dq$$

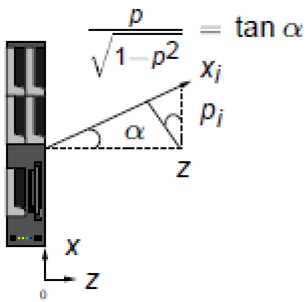
and we go back to the linear approximation. If we restrict to propagating waves $(x,p)\in\mathbb{R}^n\times C_n$, $\hat{\mathcal{G}}$ can be approximated

Approximate

$$W_z(x,p)\approx\int\delta\left(x-x'-\frac{p}{\sqrt{1-p^2}}z\right)W_0\left(x',p\right)dx'$$

$$W_z(x,p)\approx W_0\left(\mathcal{M}_z(x),p\right)$$

$$\mathcal{M}_z(x)=x-z\frac{p}{\sqrt{1-p^2}}$$



Evanescent waves

If we extend the phase-space to $(x, p) \in \mathbb{R}^n \times (\mathbb{R}^n / C_n)$

$$\Delta T(p, q) = i\sqrt{\left(p + \frac{q}{2}\right)^2 - 1} + i\sqrt{\left(p - \frac{q}{2}\right)^2 - 1}$$

- Taylor expand ΔT around $q = 0$
- Odd-order derivatives null for evanescent waves
- Second-order approximation is

$$\Delta T(p, q) \approx 2\sqrt{p^2 - 1} - \frac{p^2}{2(p^2 - 1)^{\frac{3}{2}}} \frac{q^2}{2} + \frac{1}{2\sqrt{p^2 - 1}} \frac{q^2}{2}$$

- Zero-order approximation yields

$$W_z(x, p) \approx e^{-2kz\sqrt{p^2 - 1}} W_0(x, p)$$

Back to correlation function

Propagation law

$$\Gamma_z(x,s) = \int\int e^{ikp'(s-s')}\left[\int \hat{\mathcal{G}}\left(x-x',z;p'\right)\Gamma_0\left(x',s'\right)dx'\right]ds'\,dp'$$

Linear displacement approximation

$$\hat{\mathcal{G}}\left(x-x',z;p'\right) \approx \delta\left(x-x'-\frac{p'}{\sqrt{1-p'^2}}z\right)$$

$$\Gamma_z(x,s) \approx \int\int e^{ikp'(s-s')}\,\hat{\Gamma}_0\left(x-\frac{p'}{\sqrt{1-p^2}}z, s'\right)ds'\,dp'$$

Connection with Zernike's theorem

Quasi-homogeneous source $\Gamma_0(x,s) = I_0(x)\,\mu_0(s)$ in far-field

$$\Gamma_z^{ZT}(x,s) \propto \frac{\mu_0\left(\frac{x}{\lambda z}\right) e^{\frac{i2\pi xs}{\lambda z}}}{\lambda^2 z^2} I_0\left(\frac{s}{\lambda z}\right)$$

Paraxial regime, $p << 1$

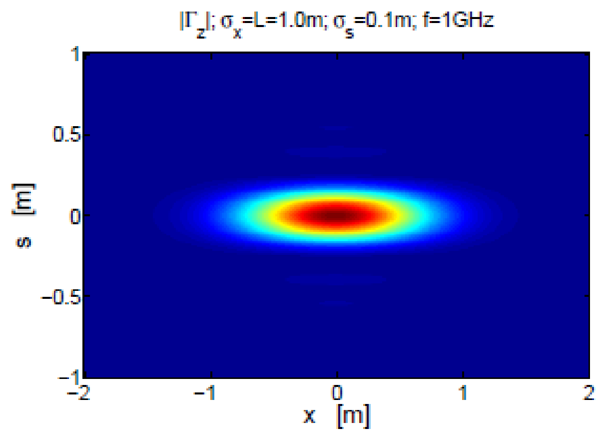
$$\Gamma_z(x,s) \approx \int\int e^{ikp'\left(s-s'\right)}\,\Gamma_0\left(x-p'z, s'\right)\,ds'\,dp'$$

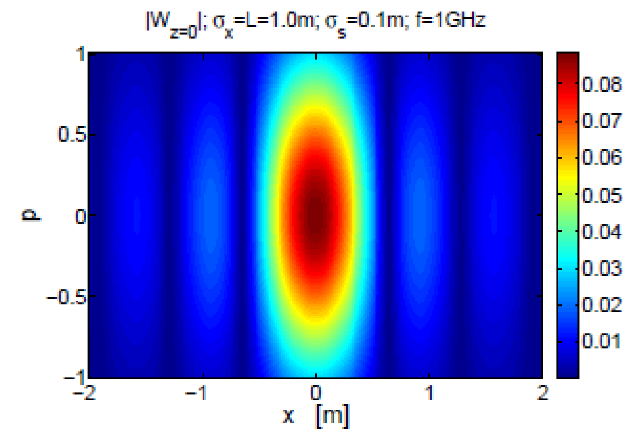
Substitution of variable

Generalization [1]

$$\Gamma_z(x,s) = \mu_0(s) \star \Gamma_z^{ZT}(x,s)$$

Near-field correlation function: Gauss-Schell moded





Source correlation function

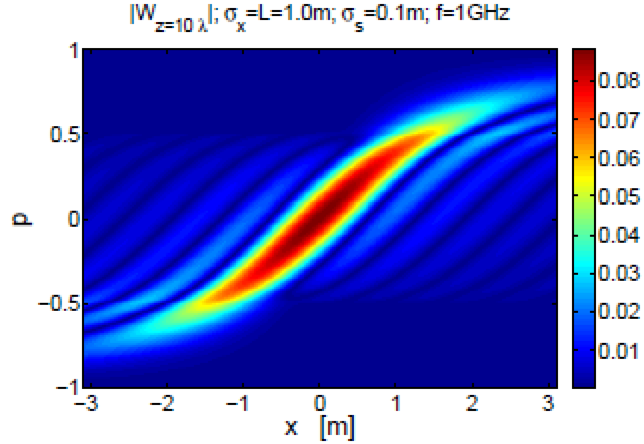
$$\Gamma_0\left(x_1, x_2\right) = I_0 \exp\left[-\frac{s^2}{2\sigma_s^2}\right] \exp\left[-\frac{x^2}{2\sigma_x^2}\right]$$

Source Wigner distribution function

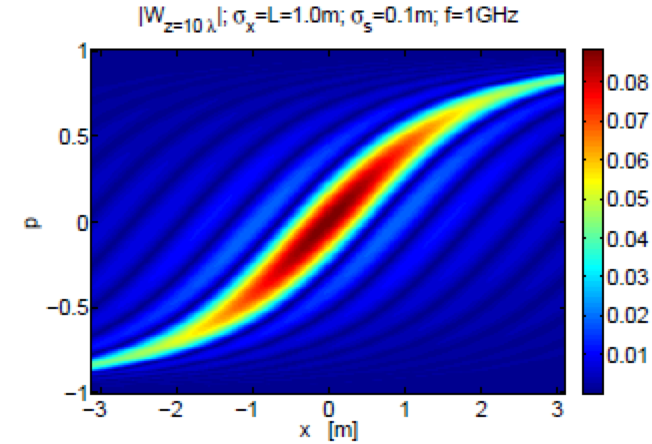
$$W_0\left(x, p\right) = I_0 \exp\left[-\frac{x^2}{2\sigma_x^2}\right] \sqrt{\frac{\pi}{2}}\sigma_s \exp\left(-\frac{k^2 p^2 \sigma_s^2}{2}\right)$$

Propagation of correlation functions

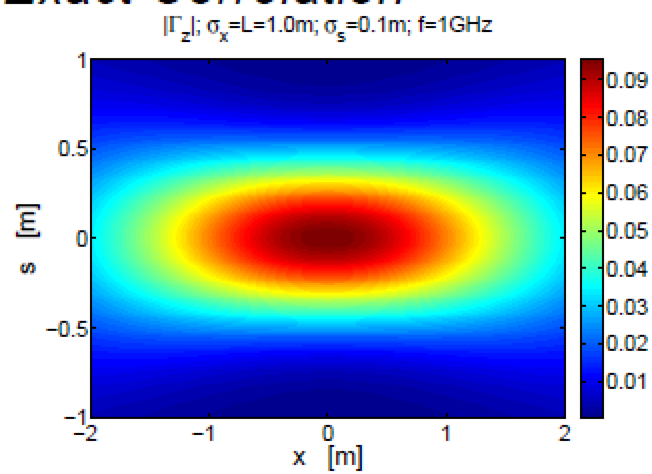
Exact Wigner



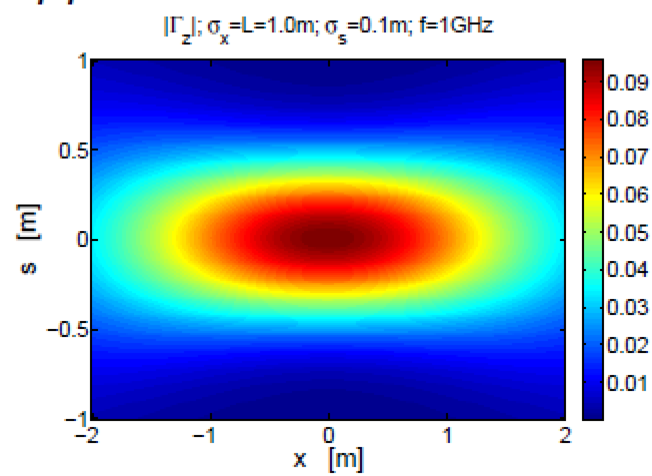
Approximate Wigner



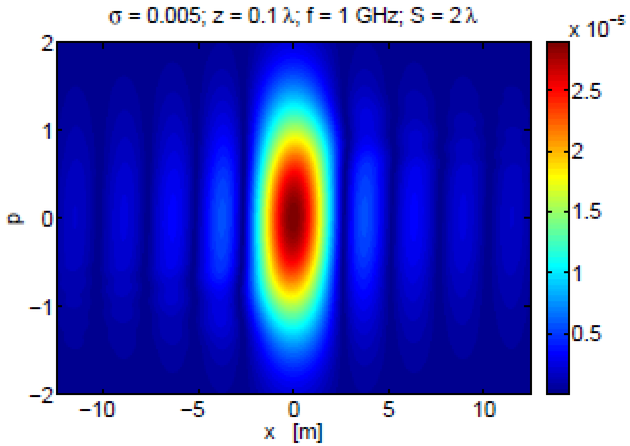
Exact Correlation

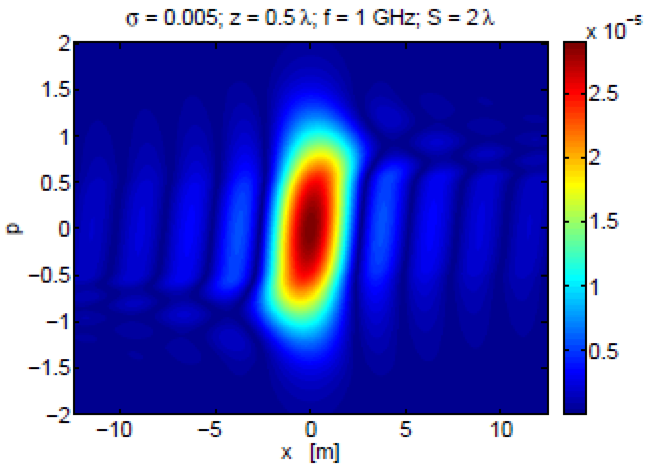


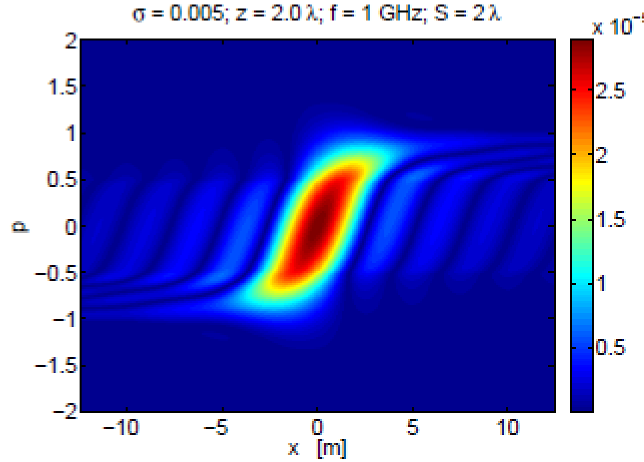
Approximate Correlation



Evanescent component

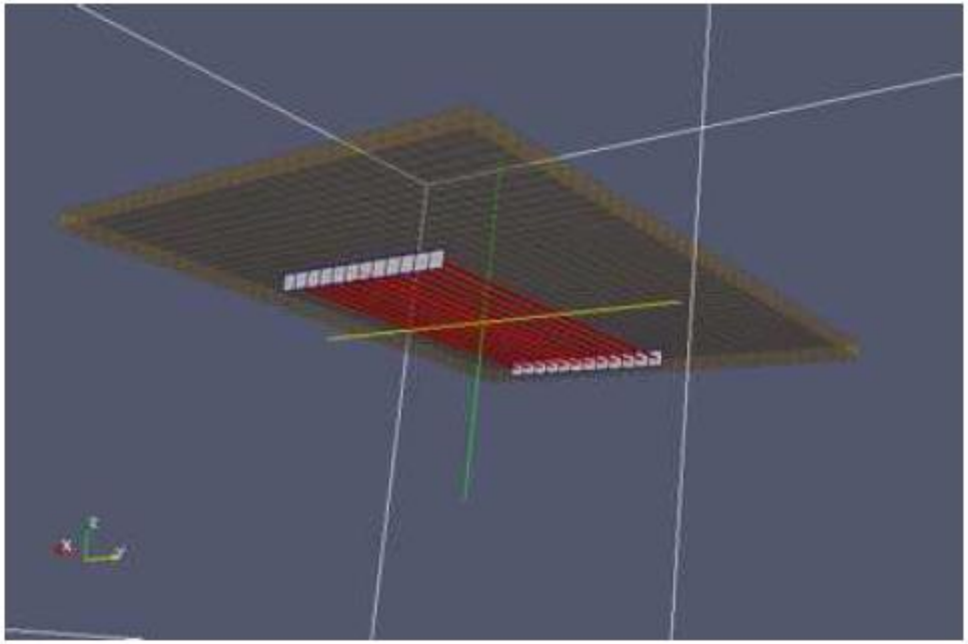




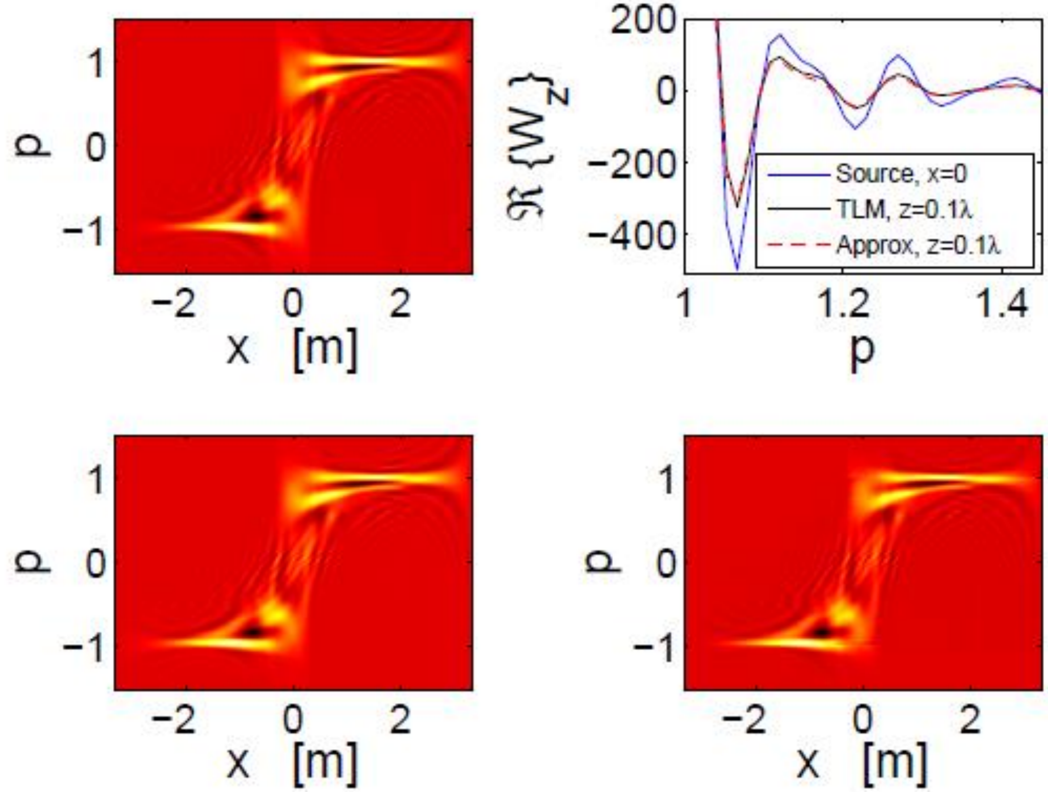


- Significant energy in near-field
- Disappears in far-field

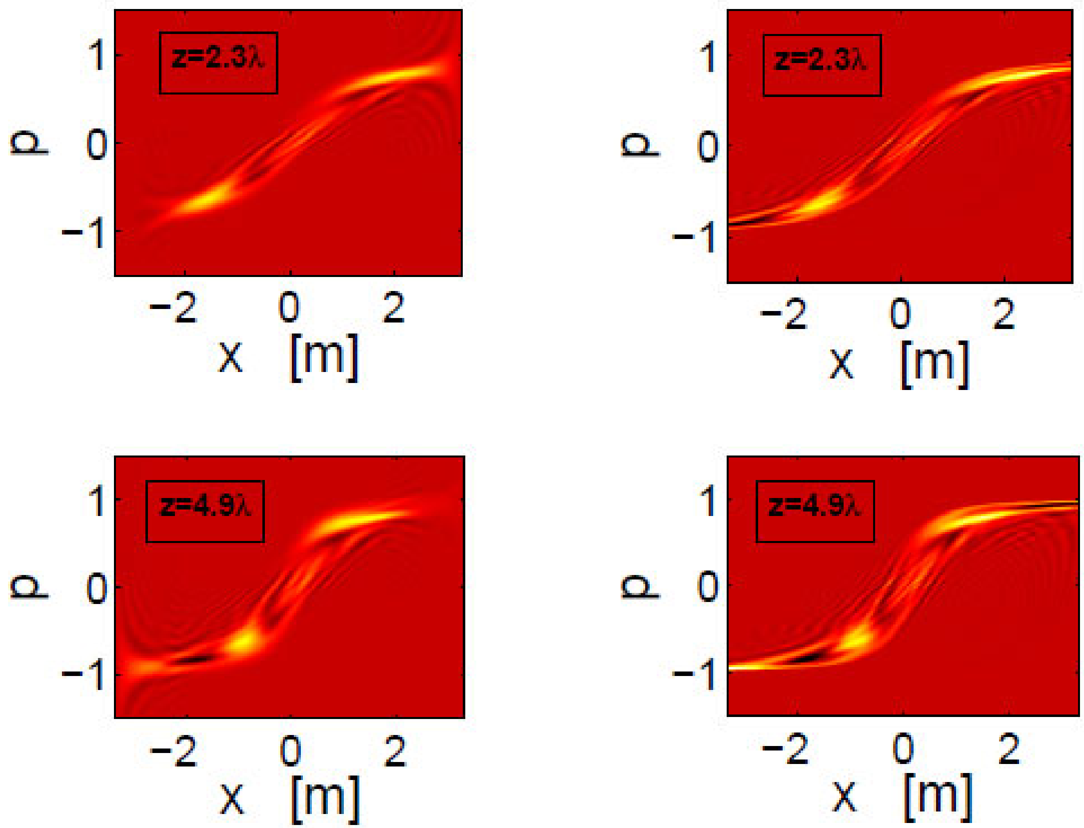
Full wave validation
Cable driven by random voltages



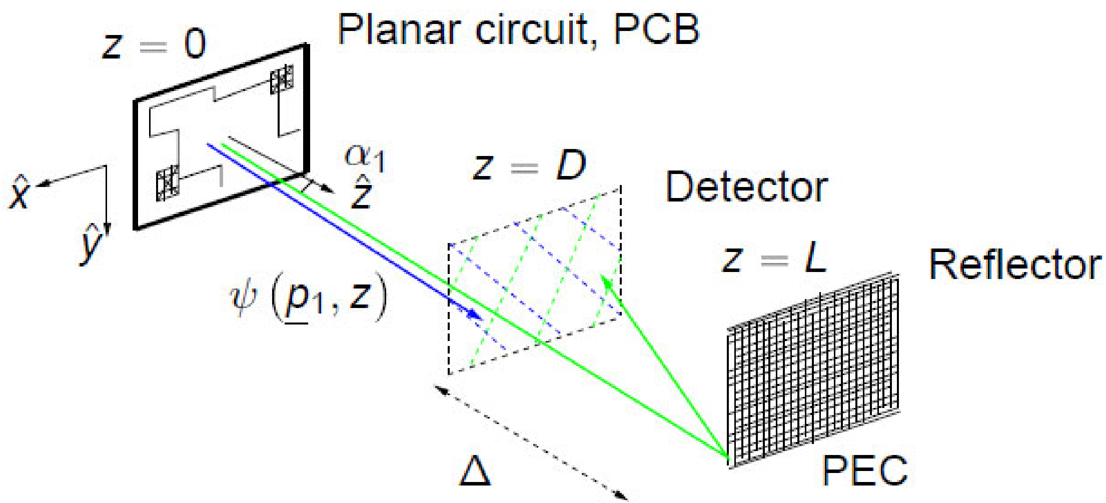
Full wave validation
Near-field distribution



Full wave validation
Far-field distribution



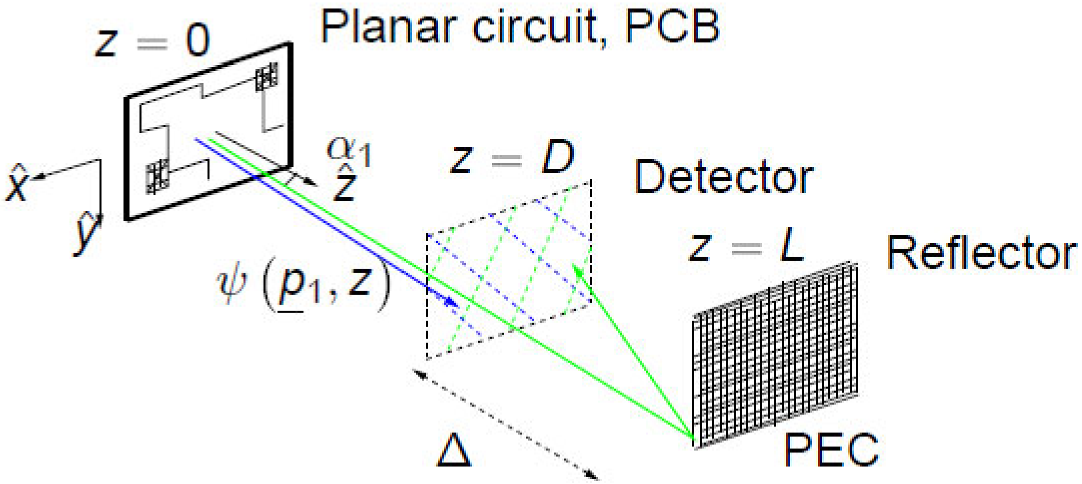
Wigner distribution functions in reflecting environments



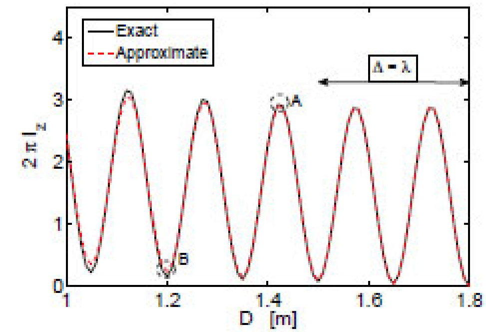
$$\mathcal{W}_D(\underline{x}, \underline{p}) \approx \frac{1}{(2\pi)^2}\Bigg[W_0\left(\underline{x} - \frac{\underline{p}}{\sqrt{1-|\underline{p}|^2}}D, \underline{p}\right)$$

$$+ W_0\left(\underline{x} - \frac{\underline{p}}{\sqrt{1-|\underline{p}|^2}}(2L-D), \underline{p}\right) - 2\cos\left(2k\Delta\hat{T}(\underline{p})\right) W_0\left(\underline{x} - \frac{\underline{p}}{\sqrt{1-|\underline{p}|^2}}L, \underline{p}\right)\Bigg]$$

Wigner distribution functions in reflecting environments

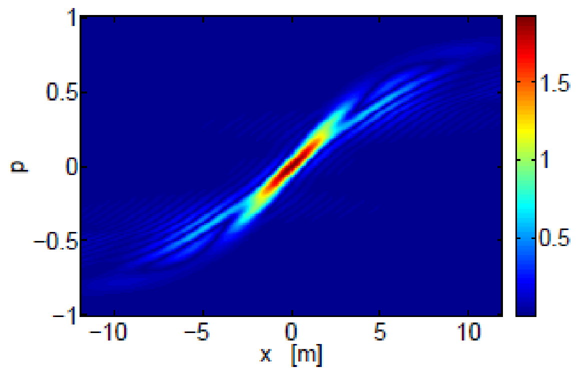


Interference pattern

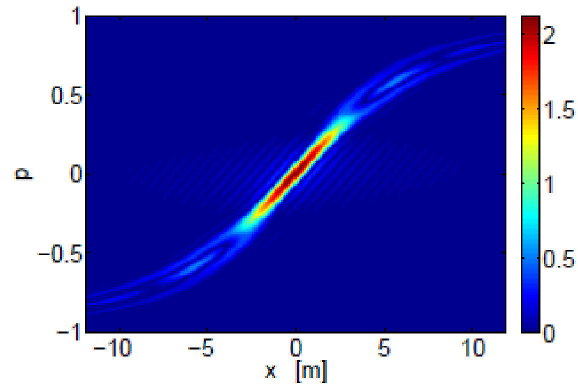


Numerical results in reflecting environments

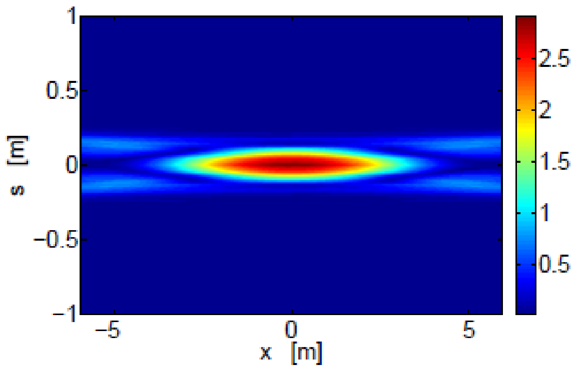
Exact Wigner



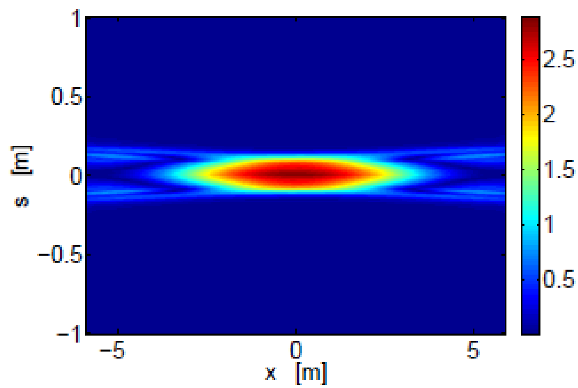
Approximate Wigner



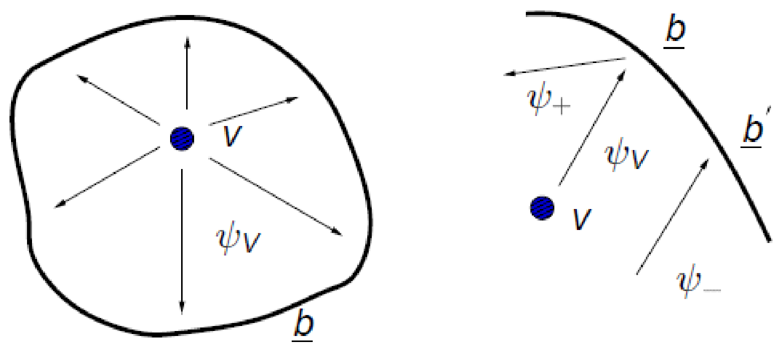
Exact Correlation



Approximate Correlation



Complex source in a Closed-environments



In either configuration or momentum space

Shift operator

$$|\psi_-\rangle = \mathrm{S}\,|\psi_+\rangle + |\psi_V\rangle$$

Reflection operator

$$|\psi_+\rangle = \mathrm{R}\,|\psi_-\rangle$$

Transfer operator defined as

$$|\psi_-\rangle = \mathrm{T}\,|\psi_-\rangle + |\psi_V\rangle$$

$$\boxed{\mathrm{T} = \mathrm{SR}}$$

[Creagh *et al.*, Journal of Physics A, 2013]

Correlation Tensor

Form the product

$$\Gamma = |\psi_-\rangle \langle\psi_-|$$

$$= (\mathrm{I} - \mathrm{T})^{-1} |\psi_v\rangle \langle\psi_v| (\mathrm{I} - \mathrm{T})^{-1,\star}$$

$$= (\mathrm{I} - \mathrm{T})^{-1} \Gamma_0 (\mathrm{I} - \mathrm{T})^{-1,\star}$$

$$= \sum_{n,m=0}^{\infty} \mathrm{T}^n \Gamma_0 \mathrm{T}^{m,\star}$$

Then, write the correlation as

$$\Gamma = K + \sum_{n=1}^{\infty} \left[K\mathrm{T}^n + \mathrm{T}^{n,\star}K\right]$$

with

$$K = \sum_{n=0}^{\infty} \mathrm{T}^n \Gamma_0 \mathrm{T}^{n,\star}$$

Which can be Wigner transformed (approximate propagation through Frobenius-Perron equation)

Conclusion

- Problem of radiation from complex sources
- Evaluation of field to field correlation in phase space
- Propagation conveniently described as transformation of Wigner functions
- Example near homogeneous, partially coherent source
- Transport of Wigner functions as in billiard

Questions?