Stochastic Electromagnetic Fields

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Outline

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Introduction

- In the case of EMI problems we have to deal with stochastic electromagnetic fields.
- If a stochastic electromagnetic field originates from a sufficiently large number of statistically independent processes, the field amplitudes will exhibit a Gaussian probability distribution.
- A Gaussian process can be described completely by its mean value and its second order moments.
- We investigate the propagation of the cross-spectral density of stochastic electromagnetic fields.

• Johannes A. Russer and Peter Russer. “Modeling of Noisy EM Field Propagation Using Correlation Information”. In: *IEEE Transactions on Microwave Theory and Techniques* 63.1 (Jan 2015), 76–89

Outline

Network Oriented Noise Modeling

- Numerical values of noise amplitudes cannot be specified for stochastic signals.
- For numerical modeling of noisy circuits one has to deal with energy and power spectra.
- Stationary stochastic signals with Gaussian amplitude probability distribution can be completely described by their *auto-* and *cross correlation spectra*.

- Literature:
• W. B. Davenport and W. L. Root. *An Introduction to the Theory of Random Signals and Noise*. New York: McGraw-Hill, 1958
• H. A. Haus and R. W. Adler. *Circuit Theory of Linear Noisy Networks*. New York. John Wiley, 1959
• H. Hillbrand and P. Russer. "An efficient method for computer aided noise analysis of linear amplifier networks". In: *IEEE Trans. Circuits and Systems* 23.4 (Apr. 1976), pp. 235–238
• Peter Russer and Stefan Müller. "Noise analysis of linear microwave circuits". In: *International Journal of Numerical Modelling, Electronic Networks, Devices and Fields* 3 (1990), pp. 287–316

Network Oriented Noise Modeling

- The spectrum of a stochastic signal does not exist.
- We can, however take a time-windowed sample $s_T(t)$ of a signal $s(t)$ defined by

$$s_T(t) = \begin{cases} s(t) & \text{for} \quad |t| \leqslant T \\ 0 & \text{for} \quad |t| > T \end{cases} . \tag{1}$$

- From this time-windowed signal we can compute the spectrum $S_T(\omega)$

$$S_T(\omega) = \int_{-\infty}^{\infty} s_T(t) \mathrm{e}^{-\jmath\omega t}\, dt , \tag{2a}$$

$$s_T(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_T(\omega) \mathrm{e}^{\jmath\omega t}\, d\omega . \tag{2b}$$

Correlation Functions and Correlation Spectra

- For stationary stochastic signals $s_i(t)$ and $s_j(t)$ we define the *correlation function*

$$c_{ij}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} s_{iT}(t) s_{jT}(t-\tau)\, dt\,. \tag{3}$$

- For $i = j$, the function $c_{ii}(\tau)$ is called the *autocorrelation function*; $c_{ij}(\tau)$ with $i \neq j$ is called the *cross correlation function*.
- The Fourier transform $C_{ij}(\omega)$ of $c_{ij}(\tau)$ is the *correlation spectrum*:

$$C_{ij}(\omega) = \int_{-\infty}^{+\infty} c_{ij}(\tau) e^{-\jmath\omega\tau}\, d\tau\ , \tag{4a}$$

$$c_{ij}(\tau) = \int_{-\infty}^{+\infty} C_{ij}(\omega) e^{\jmath\omega\tau}\, d\omega\ , \tag{4b}$$

where $C_{ii}(\omega)$ is an *autocorrelation spectrum* and $C_{ij}(\omega)$ with $i \neq j$ is a *cross correlation spectrum*.

Network Oriented Noise Modeling

- We can also write the correlation spectra as

$$C_{ij}(\omega) = \lim_{T\to\infty} \frac{1}{2T} \langle\langle S_{iT}(\omega) S^*_{jT}(\omega) \rangle\rangle \,, \tag{5}$$

  where the brackets $\langle\langle \ldots \rangle\rangle$ denote the forming of the *ensemble average*. The ensemble average has to be formed before the limiting process $T \to \infty$ since the amplitude spectrum does not exist for $T \to \infty$.
- The autocorrelation spectrum $C_{ii}(\omega)$ describes the *spectral energy density* of the signal $s_i(t)$.
- To compute the spectral energy densities of linear superpositions of signals we need also their cross correlation spectra.

Correlation Matrices

- We can summarize the correlation spectra of a number of n signals in the *correlation matrix*

$$\boldsymbol{C}(\omega) = \begin{pmatrix} C_{11}(\omega) & C_{12}(\omega) & \cdots & C_{1n}(\omega) \\ C_{21}(\omega) & C_{22}(\omega) & \cdots & C_{2n}(\omega) \\ \vdots & \vdots & & \\ C_{n1}(\omega) & C_{n2}(\omega) & \cdots & C_{nn}(\omega) \end{pmatrix}. \tag{6}$$

- The correlation matrix is *Hermitian*.
- Summarizing the spectra of the time windowed signals $S_{1T}(\omega) \ldots S_{nT}(\omega)$ in the vector $\boldsymbol{S}_T(\omega)$ we can represent the correlation matrix in the compact form

$$\boldsymbol{C}(\omega) = \lim_{T\to\infty} \frac{1}{2T} \langle\!\langle \boldsymbol{S}_T(\omega)\, \boldsymbol{S}_T^{\dagger}(\omega) \rangle\!\rangle . \tag{7}$$

A General Rule for the Derivation of Network Equations for Correlation Matrices

- The equations describing linear networks have the form

$$\boldsymbol{S}_T^{'}(\omega) = \boldsymbol{M}(\omega)\ \boldsymbol{S}_T(\omega)\,. \tag{8}$$

- This yields to the relation between the correlation matrices

$$\boldsymbol{C}'(\omega) = \boldsymbol{M}(\omega)\ \boldsymbol{C}(\omega)\ \boldsymbol{M}^{\dagger}(\omega)\,. \tag{9}$$

• H. Hillbrand and P. Russer. "An efficient method for computer aided noise analysis of linear amplifier networks". In: *IEEE Trans. Circuits and Systems* 23.4 (Apr. 1976), pp. 235–238
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Outline

Stochastic Electromagnetic Fields

Stochastic electric and magnetic fields can be described by the dyadic

$$\underline{\underline{\Gamma}}_E(\boldsymbol{x},\boldsymbol{x}',\omega) = \lim_{T\to\infty} \frac{1}{2T} \langle\!\langle \boldsymbol{E}_T(\boldsymbol{x},\omega)\boldsymbol{E}_T^{\dagger}(\boldsymbol{x}',\omega)\rangle\!\rangle\,, \tag{10a}$$

$$\underline{\underline{\Gamma}}_H(\boldsymbol{x},\boldsymbol{x}',\omega) = \lim_{T\to\infty} \frac{1}{2T} \langle\!\langle \boldsymbol{H}_T(\boldsymbol{x},\omega)\boldsymbol{H}_T^{\dagger}(\boldsymbol{x}',\omega)\rangle\!\rangle\,, \tag{10b}$$

$\langle\!\langle \ldots \rangle\!\rangle$: Ensemble average.
$E_T(\boldsymbol{x},\omega)$, $H_T(\boldsymbol{x},\omega)$: Fourier transforms of the time–windowed field vectors.

• J. A. Russer and P. Russer. “Stochastic electromagnetic fields”. In: *German Microwave Conference (GeMIC)*. Mar. 2011, pp. 1–4. ISBN: 978-1-4244-9225-1
• Johannes A. Russer and Peter Russer. “Modeling of Noisy EM Field Propagation Using Correlation Information”. In: *IEEE Transactions on Microwave Theory and Techniques* 63.1 (Jan 2015), 76–89

Vectorial Stochastic Fields

- Consider a current density vector $\boldsymbol{J}(\boldsymbol{x}, \omega)$ describing the source of the electromagnetic field. The electric field excited from $\boldsymbol{J}(\boldsymbol{x}, \omega)$ is given by

$$\boldsymbol{E}(\boldsymbol{x}, \omega) = \int_V \boldsymbol{G}(\boldsymbol{x}, \boldsymbol{x}', \omega) \boldsymbol{J}(\boldsymbol{x}', \omega) d^3 x' , \tag{11}$$

  where $\boldsymbol{G}(\boldsymbol{x}, \boldsymbol{x}', \omega)$ is the total *Green's dyadic* .
- The integration is extended over the whole volume V where $\boldsymbol{J}(\boldsymbol{x}, \omega)$ is nonvanishing.
  - J. Van Bladel. *Electromagnetic Fields*. 2nd. New York: John Wiley & Sons, 2007

Field Computation with Green's Function

$$\boldsymbol{E}(\boldsymbol{x},\omega) = \int_V \boldsymbol{G}_{EJ}(\boldsymbol{x}-\boldsymbol{x}',\omega)\boldsymbol{J}(\boldsymbol{x}',\omega)d^3x'$$

$$\boldsymbol{H}(\boldsymbol{x},\omega) = \int_V \boldsymbol{G}_{HJ}(\boldsymbol{x}-\boldsymbol{x}',\omega)\boldsymbol{J}(\boldsymbol{x}',\omega)d^3x'$$

$$\Gamma_J(\boldsymbol{x}_a,\boldsymbol{x}_b,\omega) = \lim_{T\to\infty}\frac{1}{2T}\langle\langle \boldsymbol{J}_T(\boldsymbol{x}_a,\omega)\boldsymbol{J}_T^\dagger(\boldsymbol{x}_b,\omega)\rangle\rangle$$

$$\Gamma_E(\boldsymbol{x}_a,\boldsymbol{x}_b,\omega) = \lim_{T\to\infty}\frac{1}{2T}\langle\langle \boldsymbol{E}_T(\boldsymbol{x}_a,\omega)\boldsymbol{E}_T^\dagger(\boldsymbol{x}_b,\omega)\rangle\rangle$$

$$\Gamma_H(\boldsymbol{x}_a,\boldsymbol{x}_b,\omega) = \lim_{T\to\infty}\frac{1}{2T}\langle\langle \boldsymbol{H}_T(\boldsymbol{x}_a,\omega)\boldsymbol{H}_T^\dagger(\boldsymbol{x}_b,\omega)\rangle\rangle$$

$$\Gamma_E(\boldsymbol{x}_a,\boldsymbol{x}_b,\omega) = \iint_V \boldsymbol{G}_{EJ}(\boldsymbol{x}_a-\boldsymbol{x}'_a)\,\Gamma_J(\boldsymbol{x}'_a,\boldsymbol{x}'_b,\omega)\,\boldsymbol{G}^\dagger_{EJ}(\boldsymbol{x}_b-\boldsymbol{x}'_b)d^3x'_a d^3x'_b$$

$$\Gamma_H(\boldsymbol{x}_a,\boldsymbol{x}_b,\omega) = \iint_V \boldsymbol{G}_{HJ}(\boldsymbol{x}_a-\boldsymbol{x}'_a)\,\Gamma_J(\boldsymbol{x}'_a,\boldsymbol{x}'_b,\omega)\,\boldsymbol{G}^\dagger_{HJ}(\boldsymbol{x}_b-\boldsymbol{x}'_b)d^3x'_a d^3x'_b$$

- J. A. Russer and P. Russer. "Stochastic electromagnetic fields". In: *German Microwave Conference (GeMIC)*. Mar. 2011, pp. 1–4. ISBN: 978-1-4244-9225-1
- Johannes A. Russer and Peter Russer. "Modeling of Noisy EM Field Propagation Using Correlation Information". In: *IEEE Transactions on Microwave Theory and Techniques* 63.1 (Jan 2015), 76–89

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Uniqueness Theorem and Equivalence Principle

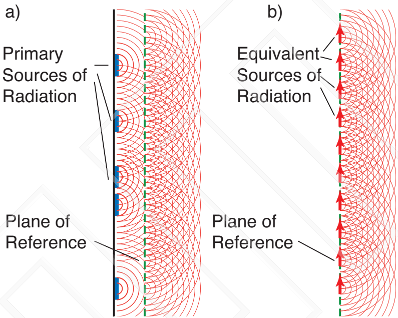
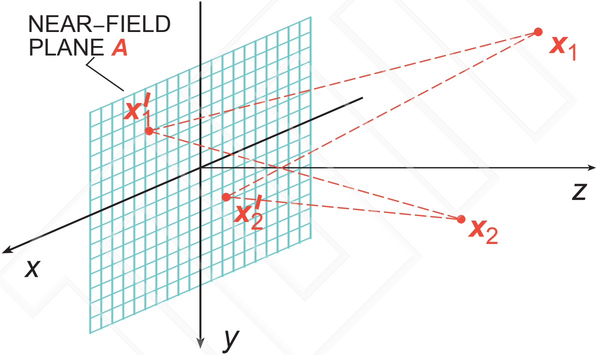


Figure: Uniqueness theorem and equivalence principle: (a) Reference plane with the source-free region to the right in which the EM field is completely determined by the tangential electric or magnetic field in the reference plane, originating from the primary sources of radiation left from the reference plane. (b) Equivalent sources positioned in the reference plane at excite the same field as the primary sources on the right side of the reference plane.

Near-Field Scanning

Consider an area A with an impressed stochastic stationary Gaussian electric polarization $P_e(\boldsymbol{x}', t')$.



We can compute correlation dyadics $\underline{\underline{\Gamma}}_E(\boldsymbol{x}_1, \boldsymbol{x}_2, \omega)$ and $\underline{\underline{\Gamma}}_H(\boldsymbol{x}_1, \boldsymbol{x}_2, \omega)$ for all pairs of points $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ if the correlation dyadic $\underline{\underline{\Gamma}}_{Pe}(\boldsymbol{x}'_1, \boldsymbol{x}'_2, \omega)$ is known for all pairs of points $\boldsymbol{x}'_1$ and $\boldsymbol{x}'_2$.

Near-Field Scanning

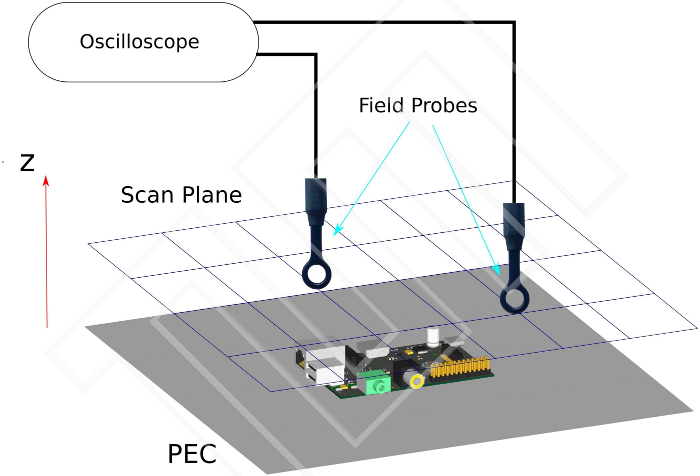
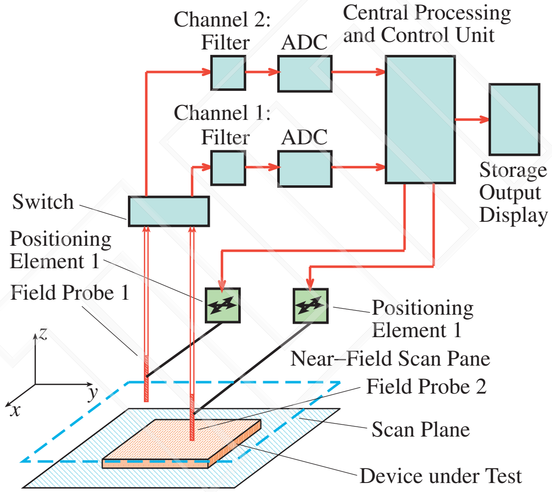


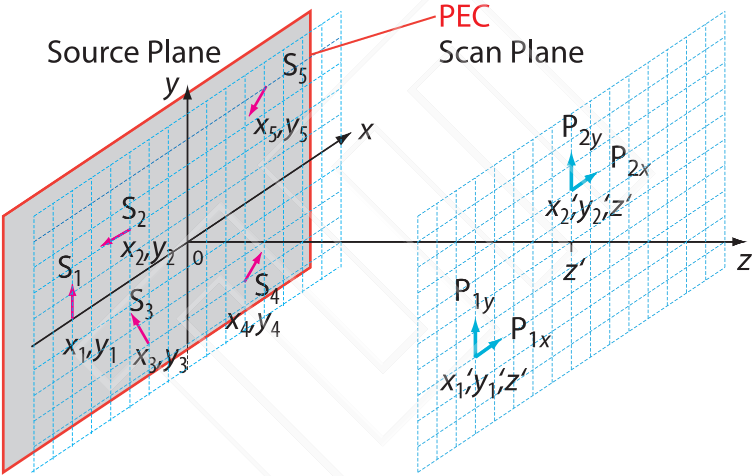
Figure: Device under test with two field probes, connected to a multi–channel digital oscilloscope, in the scanning plane.

Near-Field Scanning System



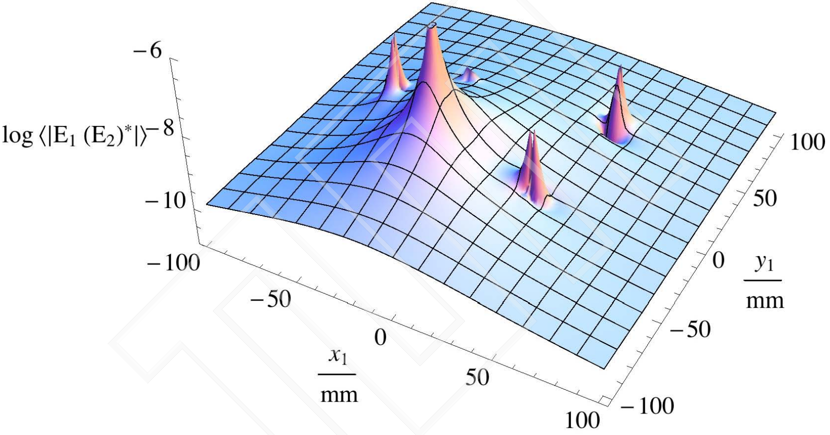
Outline

Stochastic EM Field Propagation



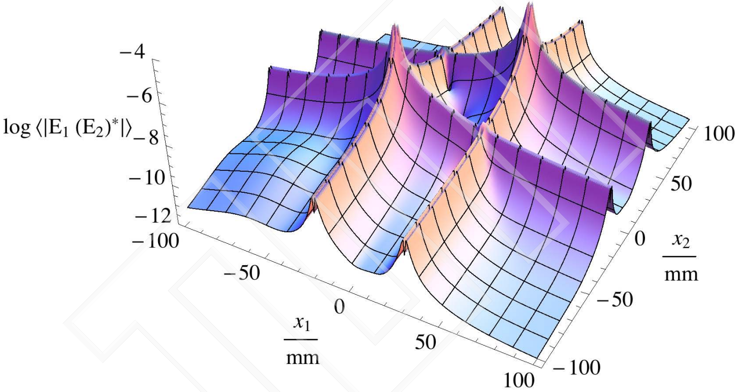
Planar array of $N = 5$ stochastic sources sampled at a sampling plane parallel to the source plane.

Stochastic Near-Field Propagation



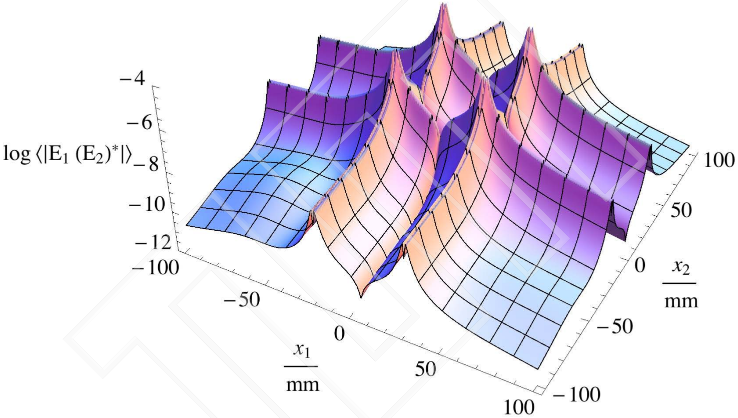
3D logarithmic plot of the magnitude of the electric field correlation density $\mathbf{\Gamma}_E(\boldsymbol{x}_1{}', \boldsymbol{x}_2{}', \omega)$ for $x_1', y_1' \in \{-100\ \mathrm{mm}, 100\ \mathrm{mm}\}$, $z' = 1$ mm, fixed reference point $\boldsymbol{x}_2' = [-20, -30, 1]^T\mathrm{mm}$, and $\beta = 0.1\ \mathrm{mm}^{-1}$.

Near-Field Propagation – Uncorrelated Sources



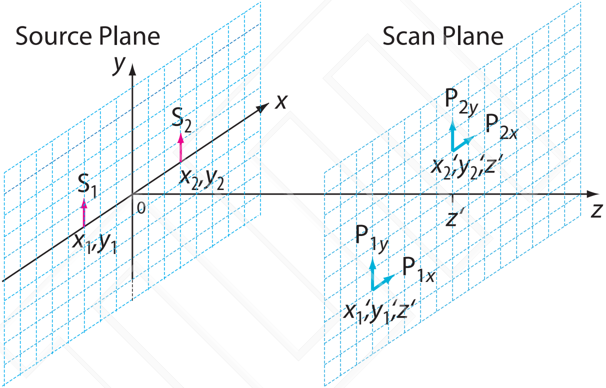
3D logarithmic plot of the magnitude of the electric field correlation density $\boldsymbol{\Gamma}_E(\boldsymbol{x}'_1, \boldsymbol{x}'_2, \omega)$ for $x_1{}'$ and $x_2{}'$ are varied independently, $z' = 1$ mm, and $\beta = 0.1\ \mathrm{mm}^{-1}$.

Near-Field Propagation – Correlated Sources



3D logarithmic plot of the magnitude of the electric field correlation density $\boldsymbol{\Gamma}_E(\boldsymbol{x}_1{}', \boldsymbol{x}_2{}', \omega)$ for fully correlated in-phase sources of equal amplitude and for x_1 and x_2 are varied independently, $z' = 1$ mm, and $\beta = 0.1\ \mathrm{mm}^{-1}$.

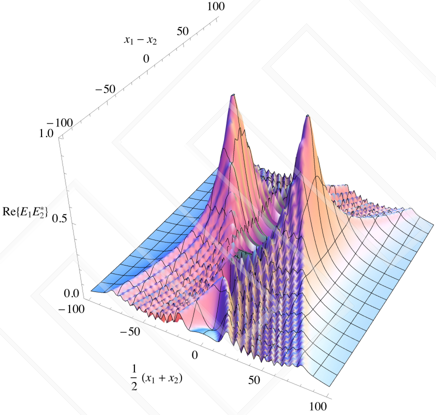
Evolution of the Correlation - Two Stochastic Sources



Two stochastic sources sampled at a scan plane

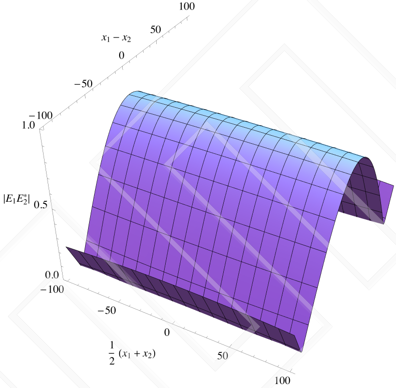
- Johannes A. Russer et al. “Evolution of Transverse Correlation in Stochastic Electromagnetic Fields”. In: *Proceeding of: IEEE International Microwave Symposium, IMS*. Phoenix, Arizona, USA, May 17–22 2015

Electric Field Correlation Density



3D linear plot of the magnitude of the electric field correlation density $\underline{\underline{\Gamma}}_E(\boldsymbol{x}_1, \boldsymbol{x}_2, \omega)$ for $y_1' = y_2' = 0$ and $x_1', x_2' \in \{-100\ \mathrm{mm}, 100\ \mathrm{mm}\}$, $z' = 10$ mm.

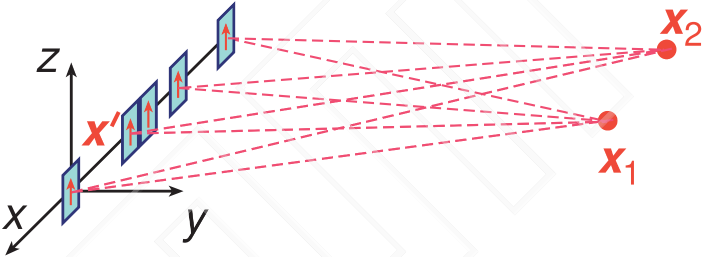
Electric Field Correlation Density



3D linear plot of the magnitude of the electric field correlation density $\underline{\underline{\Gamma}}_E(\boldsymbol{x}_1, \boldsymbol{x}_2, \omega)$ for $y_1' = y_2' = 0$ and $x_1', x_2' \in \{-100 \text{ mm}, 100 \text{ mm}\}$, $z' = 1$ m

For 1m distance the correlation depends only on the transverse position difference of the scan coordinates and not on their mean position.

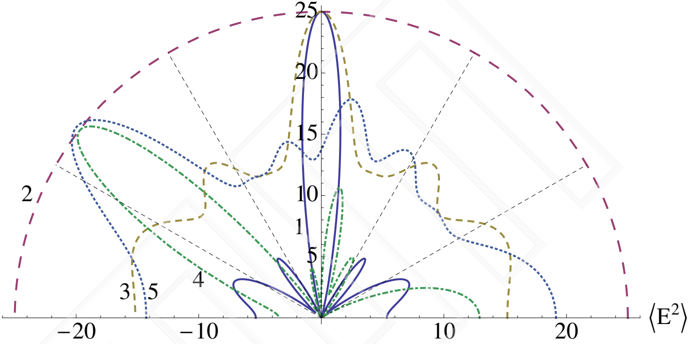
Correlation in the Far-Field
Non-Uniform Linear Array of Hertzian Dipoles



Non-uniform linear array of Hertzian dipoles positioned at
$x' = 0, -2.5\lambda, -3.25\lambda, -4.5\lambda, -6.5\lambda$.

• Johannes A. Russer and Peter Russer. "Modeling of Noisy EM Field Propagation Using Correlation Information". In: *IEEE Transactions on Microwave Theory and Techniques* 63.1 (Jan 2015), 76–89

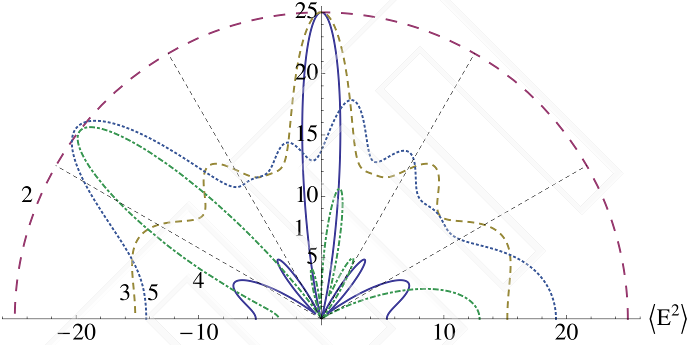
Far-Field Characteristics (1)



Angular distribution of $\langle |E_z(\varphi)|^2 \rangle$ far-field electric field spectral energy density.

$$C_1^I = i_0^2 \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}, \quad C_2^I = 5 i_0^2 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \begin{aligned} C_3^I &= 0.5 C_1^I + 0.5 C_2^I, \\ C_5^I &= 2.5 C_2^I + 0.5 C_3^I. \end{aligned}$$

Far-Field Characteristics (2)



Angular distribution of $\langle |E_z(\varphi)|^2 \rangle$ far-field electric field spectral energy density.

$$C_4^I = i_0^2 \begin{bmatrix} 1 & e^{0.2\pi\jmath} & e^{-0.4\pi\jmath} & e^{-1.5\pi\jmath} & e^{-1.1\pi\jmath} \\ e^{-0.2\pi\jmath} & 1 & e^{-0.6\pi\jmath} & e^{-1.7\pi\jmath} & e^{-1.3\pi\jmath} \\ e^{0.4\pi\jmath} & e^{0.6\pi\jmath} & 1 & e^{-1.1\pi\jmath} & e^{-0.7\pi\jmath} \\ e^{1.5\pi\jmath} & e^{1.7\pi\jmath} & e^{1.1\pi\jmath} & 1 & e^{0.4\pi\jmath} \\ e^{1.1\pi\jmath} & e^{1.3\pi\jmath} & e^{0.7\pi\jmath} & e^{-0.4\pi\jmath} & 1 \end{bmatrix}, \quad C_5^I = 2.5C_2^I + 0.54C_3^I.$$

Outline

Part I: Numerical Computation of Stochastic Fields

- The numerical computation of stochastic electromagnetic fields can be performed in an efficient way by transforming the field problem to a network problem.
- Like in the case of deterministic electromagnetic fields also in the case of stochastic electromagnetic fields network methods can reduce the computational effort considerably and beyond this can contribute to *compact model* generation.
- Network methods for deterministic fields already have been described in

• Peter Russer. *Electromagnetics, Microwave Circuit and Antenna Design for Communications Engineering*. Second. Boston: Artech House, 2006
• Leopold B. Felsen, Mauro Mongiardo, and Peter Russer. *Electromagnetic Field Computation by Network Methods*. 1st ed. Berlin Heidelberg New York: Springer, 2009. ISBN: 3540939458

Numerical Computation of Stochastic Fields

- In the following we describe the computation of stochastic electromagnetic fields by the *Method of Moments* (MoM).
- The MoM allows to transform a field problem into a network-like problem described by algebraic equations.
  - R. F. Harrington. *Time Harmonic Electromagnetic Fields*. New York: McGraw-Hill, 1961

Numerical Computation of Stochastic Fields

- Let us first apply the MoM to compute the integral expression for deterministic fields. We expand the field functions $\boldsymbol{J}(\boldsymbol{x},\omega)$ and $\boldsymbol{E}(\boldsymbol{x},\omega)$ into basis functions

$$\boldsymbol{J}(\boldsymbol{x},\omega) = \sum_n I_n(\omega)\boldsymbol{u}_n(\boldsymbol{x})\,, \tag{12a}$$

$$\boldsymbol{E}(\boldsymbol{x},\omega) = \sum_n V_n(\omega)\boldsymbol{u}_n(\boldsymbol{x})\,, \tag{12b}$$

  where the $\boldsymbol{u}_n(\boldsymbol{x})$ are *vectorial basis functions* and $I_n(\omega)$ and $V_n(\omega)$ are the expansion coefficients.
- We can consider $I_n(\omega)$ and $V_n(\omega)$ as *generalized voltages* and *currents*, respectively. If use a complete set of basis functions, the series expansions will converge to the exact value.
- However, to facilitate a numerical treatment of the problem we have to truncate the series expansion after a finite number of elements.

Numerical Computation of Stochastic Fields

- With these series expansions we obtain

$$\sum_n V_n(\omega)\boldsymbol{u}_n(\boldsymbol{x}) = \sum_n I_n(\omega) \int_V \boldsymbol{G}(\boldsymbol{x},\boldsymbol{x}',\omega)\boldsymbol{u}_n(\boldsymbol{x}')d^3x' \,. \qquad (13)$$

- Using expansion functions $\boldsymbol{u}_n(\boldsymbol{x})$ with the property

$$\int_V \boldsymbol{u}_m^\dagger(\boldsymbol{x})\boldsymbol{u}_n(\boldsymbol{x})d^3x = \delta_{mn} \,, \qquad (14)$$

where δ_{mn} is the Kronecker delta, and multiplying (13) from the left with $\boldsymbol{u}_m^\dagger(\boldsymbol{x})$ and integrating over V yields

$$V_m(\omega) = \sum_m Z_{mn}(\omega)I_n(\omega) \,. \qquad (15)$$

Numerical Computation of Stochastic Fields

We expand the correlation dyadics $\Gamma_J(\boldsymbol{x}_a, \boldsymbol{x}_b, \omega)$ and $\Gamma_E(\boldsymbol{x}_a, \boldsymbol{x}_b, \omega)$ into basis functions

$$C_{I,mn}(\omega) = \iint\limits_V \boldsymbol{u}_m^\dagger(\boldsymbol{x})\Gamma_J(\boldsymbol{x}, \boldsymbol{x}', \omega)\boldsymbol{u}_n(\boldsymbol{x}')d^3x\, d^3x', \tag{16a}$$

$$C_{V,mn}(\omega) = \iint\limits_V \boldsymbol{u}_m^\dagger(\boldsymbol{x})\Gamma_E(\boldsymbol{x}, \boldsymbol{x}', \omega)\boldsymbol{u}_n(\boldsymbol{x}')d^3x\, d^3x'. \tag{16b}$$

These matrix elements can be summarized in the matrices

$$\boldsymbol{C}_I(\omega) = \begin{bmatrix} C_{I,11}(\omega) & \dots & C_{I,1N}(\omega) \\ \vdots & \ddots & \vdots \\ C_{I,N1}(\omega) & \dots & C_{I,NN}(\omega) \end{bmatrix}, \tag{17a}$$

$$\boldsymbol{C}_V(\omega) = \begin{bmatrix} C_{V,11}(\omega) & \dots & C_{V,1N}(\omega) \\ \vdots & \ddots & \vdots \\ C_{V,N1}(\omega) & \dots & C_{V,NN}(\omega) \end{bmatrix}. \tag{17b}$$

Numerical Computation of Stochastic Fields

The matrix elements $Z_{mn}(\omega)$ are given by

$$Z_{mn}(\omega) = \iint_V \boldsymbol{u}_m^\dagger(\boldsymbol{x}) \boldsymbol{G}(\boldsymbol{x}, \boldsymbol{x}', \omega) \boldsymbol{u}_n(\boldsymbol{x}') \, d^3x \, d^3x' \, . \tag{18}$$

For a chosen dimension N of the series expansions (12a) and (12b) we introduce the *generalized current and voltage vectors*

$$\boldsymbol{I}(\omega) = [I_1(\omega) \dots I_N(\omega)]^T \, , \tag{19}$$

$$\boldsymbol{V}(\omega) = [V_1(\omega) \dots V_N(\omega)]^T \, , \tag{20}$$

and the *impedance matrix*

$$\boldsymbol{Z}(\omega) = \begin{bmatrix} Z_{11}(\omega) & \dots & Z_{1N}(\omega) \\ \vdots & \ddots & \vdots \\ Z_{N1}(\omega) & \dots & Z_{NN}(\omega) \end{bmatrix} . \tag{21}$$

Numerical Computation of Stochastic Fields

We can write

$$V_m(\omega) = \sum_m Z_{mn}(\omega) I_n(\omega)\,,$$

in matrix form as

$$\boldsymbol{V}(\omega) = \boldsymbol{Z}(\omega)\boldsymbol{I}(\omega)\,. \tag{22}$$

as

$$\boldsymbol{C}_V(\omega) = \boldsymbol{Z}(\omega)\ \boldsymbol{C}_I(\omega)\ \boldsymbol{Z}^\dagger(\omega) \tag{23}$$

with

$$\boldsymbol{C}_I(\omega) = \lim_{T\to\infty} \frac{1}{2T}\langle \boldsymbol{I}_T(\omega)\boldsymbol{I}_T(\omega)^\dagger\rangle\,, \tag{24}$$

$$\boldsymbol{C}_V(\omega) = \lim_{T\to\infty} \frac{1}{2T}\langle \boldsymbol{V}_T(\omega)\boldsymbol{V}_T(\omega)^\dagger\rangle\,. \tag{25}$$

Using the MoM we have reduced the field problem to a network problem.

Part II: Digital Signal Processing - Time-Discrete Signals

The discrete-time signal function $s[n]$ is related to the continuous-time signal function $s(t)$ of length T_0 by

$$s[n] = s(n\Delta t) \quad \text{with } T_0 = N\Delta t\,. \tag{26}$$

Δt: sampling interval.
For a real-valued discrete time sequence $x_j[n]$, applied to a linear time-invariant (LTI) system with the impulse response $h_{ij}[n]$, the output sequence $y_i[n]$ is

$$y_i[n] = h_{ij}[n] * x_j[n] \equiv \sum_{m=-\infty}^{\infty} h_{ij}[n-m]x_j[m]\,, \tag{27}$$

where the symbol $*$ denotes the convolution operation.

Discrete-Time Correlation Functions

The discrete-time correlation function $c_{ij}[n, n+m]$ of two real-valued discrete time sequences $x_i[n]$ and $x_j[n]$ is defined as

$$c_{ij}^{x}[n, n+m] = \langle x_i[n] x_j[n+m] \rangle. \tag{28}$$

If $s_i[n]$ and $s_j[n]$ are stationary ergodic processes, $c_{ij}[n, n+m]$ is independent from n and the ensemble average is identical with the time average:

$$c_{ij}^{x[m]} = \langle x_i[n] x_j[n+m] \rangle = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x_1[n] x_2[n+m] \, .$$

Discrete-Time Correlation Functions

Hence, for the response we obtain

$$\langle y_p[n] y_q[n+m] \rangle = \tag{29}$$

$$= \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} h_{pr}[k] \langle x_r[n-k] x_s[n+m-l] \rangle h_{qs}[l] \,,$$

$$c_{pq}^{y}[m] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} h_{pr}[k] c_{rs}^{x}[m+k-l] h_{qs}[l] \,. \tag{30}$$

Correlation Transfer Function

We define the *correlation transfer function*

$$k^{y[l]}_{pq,rs} = \sum_{k=-\infty}^{\infty} h_{pr}[k] h_{qs}[l+k] \,. \tag{31}$$

We can write

$$c^{y}_{pq}[m] = \sum_{l=-\infty}^{\infty} k_{pq,rs}[l] c^{x}_{rs}[m-l] \,. \tag{32}$$

• Johannes A. Russer, Andreas Cangellaris, and Peter Russer. "Correlation Transmission Line Matrix (CTLM) modeling of stochastic electromagnetic fields". In: *2016 IEEE MTT-S International Microwave Symposium (IMS)*. San Francisco, May 2016, pp. 1–4. DOI: 10.1109/MWSYM.2016.7540091

TLM - A Discrete Scheme of Electromagnetism

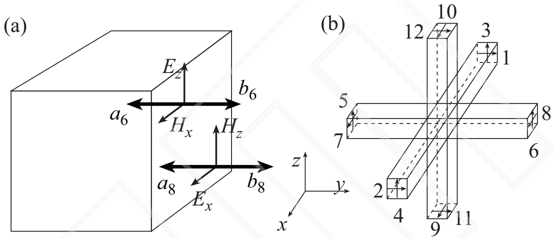


Figure: Schematic of the TLM cell: a) Space cell with samples of the tangential electric and magnetic field values and wave pulse amplitudes, b) TLM node.

In the TLM–method, the electromagnetic field is modeled by wave pulses propagating on a Cartesian mesh of transmission lines.

- W.J.R. Hoefer. "The Transmission Line Matrix (TLM) Method". In: *Numerical Techniques for Microwave and Millimeter Wave Passive Structures*. Ed. by T. Itoh. New York: J. Wiley., 1989, pp. 496–591

TLM - A Discrete Scheme of Electromagnetism

- In a compact formulation of the TLM scheme we summarize all $12N$ incident wave pulses in the vector $a[k]$ and all $12N$ scattered wave pulses in the vector $b[k]$.
- The argument k enumerates the discrete time step. We can formulate the TLM scheme in the compact Hilbert space notation where the scattering matrix $\boldsymbol{S}$ describes the instantaneous scattering of the wave pulses in the TLM node and $\boldsymbol{\Gamma}$ describes the connection of the TLM nodes with the adjacent TLM nodes.

Discrete TLM Green's Functions

- The discrete Green's function for TLM can be written as $G[n_i, k; n_j, k']$.
- It relates the wave pulses $a[n_i, k']$ incident on boundary port n_i and time k' to the wave pulses $b[n_j, k]$ scattered from boundary port n_j and time k.
- We can write

$$b_i[k] = \sum_{n_j \in B} \sum_{k'=-\infty}^{\infty} G_{i,j}[k-k'] a_j[k'] \,, \tag{33}$$

where B is a set $\{n_1, n_2, \ldots n_N\}$ of N boundary nodes.

Discrete TLM Green's Functions

For stationary stochastic electromagnetic fields we can introduce the following auto- and cross correlation functions of the wave amplitudes:

$$c^{a}_{ij}[m] = \langle a_i[n] a_j[n+m] \rangle \,, \tag{34}$$

$$c^{b}_{ij}[m] = \langle b_i[n] b_j[n+m] \rangle \,. \tag{35}$$

Correlation Green's Function

We introduce the *Correlation Green's Function (CGF)* $K_{ij;pq}[k]$ for the TLM wave amplitude correlation functions

$$K_{ij;pq}[k] = \sum_{l=-\infty}^{\infty} G_{i,p}[l] G_{j,q}[l+k] \,. \tag{36}$$

We obtain

$$c^b_{ij}[m] = \sum_{n_r, n_s \in B} \sum_{l=-\infty}^{\infty} K_{ij;rs}[l] c^a_{rs}[m-l] \,, \tag{37}$$

relating the auto- and cross correlation functions $c^b_{ij}[m]$ of the wave amplitudes scattered from the boundary to the auto- and cross correlation functions $c^a_{rs}[m]$ incident to the boundary.

Numerical Example

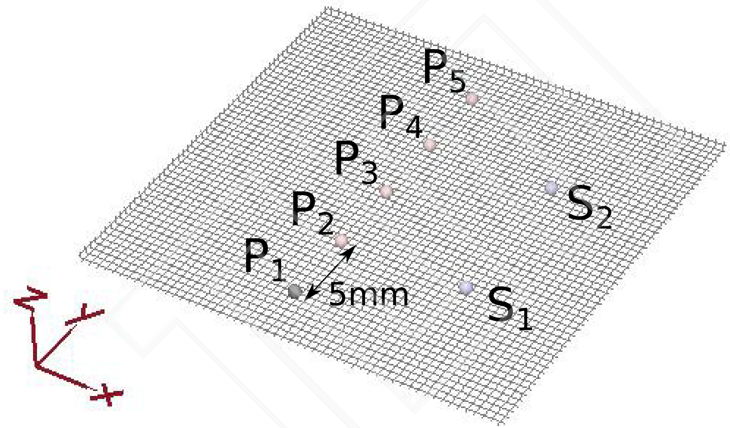


Figure: Arrangement of source and observation points.

Numerical Example

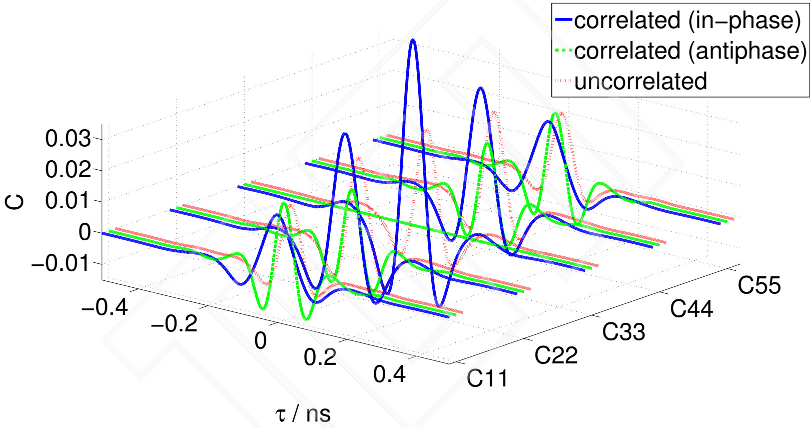


Figure: Time domain autocorrelations of the observation points for a two-source excitation with correlated in-phase, correlated antiphase, and uncorrelated sources.

Outline

Cyclostationary Stochastic Fields

- The signals of high-speed digital electronic circuits can be considered as stochastic signals, since in a global consideration as sources of radiated EMI the information content is irrelevant and is not considered to be predictable.
- The radiated EMI is mainly caused by the transients of the digital circuits.
- Since signals in digital ICs are clocked, the statistical averages have a periodic time dependence with the period of the clock interval.
- Whereas the bit sequences are stochastic the dependence of the radiated EMI pulse contribution on each transient is deterministic.
- We encounter in electronic systems cyclostationary (CS) stochastic EMI.
- A CS random EM field is a special kind of a random process with periodic time dependence of its ensemble averages.

• Professor Peter J. Schreier and Professor Louis L. Scharf. *Statistical Signal Processing of Complex-Valued Data: The Theory of Improper and Noncircular Signals*. 1st ed. Cambridge, New York: Cambridge University Press, 2010

• Johannes A. Russer et al. “Analysis of Cyclostationary Stochastic Electromagnetic Fields”. In: *International Conference on Electromagnetics in Advanced Applications (ICEAA), 2015*. IEEE, Sept. 2015, pp. 1452–1455

Cyclostationary Signals

- *Time-domain correlation matrix* (TDCM) of signal vectors $\boldsymbol{s}_i(t)$ and $\boldsymbol{s}_j(t)$:

$$\boldsymbol{c}_{ij}(t,\tau) = \boldsymbol{s}_i(t)\boldsymbol{s}_j^\dagger(t-\tau)\,. \tag{38}$$

- For *nonstationary stochastic processes* the TDCM $\boldsymbol{c}_{ij}(t,\tau)$ depends on the global time t and the time difference τ whereas for *stationary stochastic processes* the TDCM only depends on the time difference τ and we can write $\boldsymbol{c}_{ij}(\tau)$.
- *Cyclostationary stochastic processes* are nonstationary stochastic processes, where the TDCM depends periodically on t with a period T_0.

Cyclostationary Signals

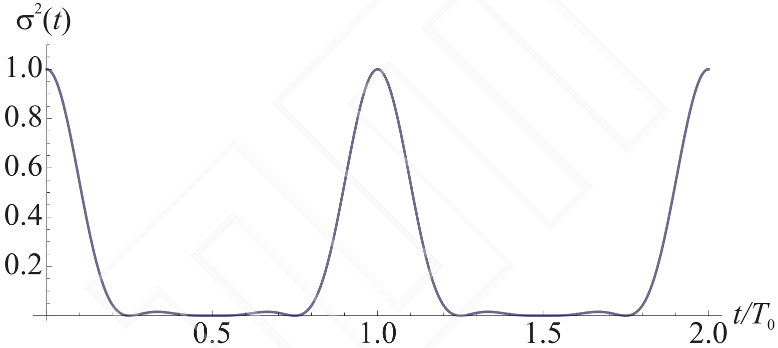
- The signals $s_i(t)$ and $s_j(t)$ are said to be *second-order cyclostationary in the wide sense* if their mean values $\langle\langle s_i(t)\rangle\rangle$ and their correlation functions $\langle\langle s_i(t)s_j(t-\tau)\rangle\rangle$ are periodic with a period T_0, i.e.

$$\langle\langle s_i(t+T_0)\rangle\rangle = \langle\langle s_i(t)\rangle\rangle\,, \tag{39a}$$

$$\langle\langle c_{ij}(t+T_0,\tau)\rangle\rangle = \langle\langle c_{ij}(t,\tau)\rangle\rangle\,. \tag{39b}$$

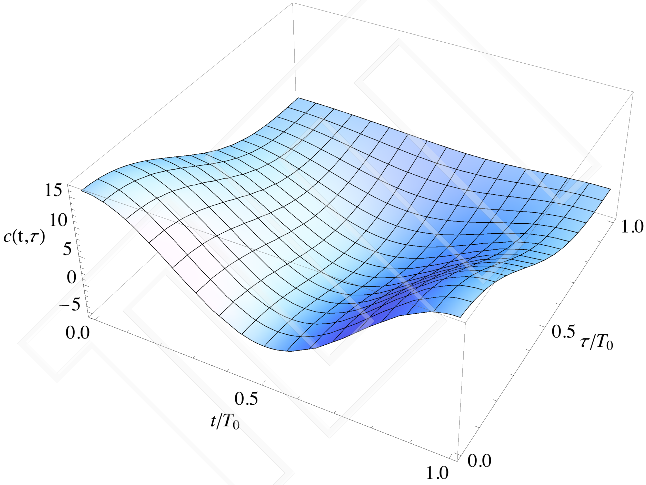
• Johannes A. Russer et al. “Analysis of Cyclostationary Stochastic Electromagnetic Fields”. In: *International Conference on Electromagnetics in Advanced Applications (ICEAA), 2015*. IEEE, Sept. 2015, pp. 1452–1455
• Johannes A. Russer et al. “Near-Field Propagation of Cyclostationary Stochastic Electromagnetic Fields”. In: *International Conference on Electromagnetics in Advanced Applications (ICEAA), 2015*. Torino, Italy, Sept. 2015, pp. 1456–1459. DOI: 10.1109/ICEAA.2015.7297360

Stochastic EM Fields



Time-dependence of the variance $\sigma^2(t)$ of the CS random signal $s_1(t)$.

Cyclic Correlation Functions



Autocorrelation function $c_{11}(t, \tau)$ of the CS random signal $s_1(t)$ with period T_0.

Cyclic Correlation Functions

- The time-periodic correlation function can be expanded into a Fourier series as

$$\boldsymbol{c}_{ij}(t,\tau) = \sum_{n=-\infty}^{+\infty} \boldsymbol{c}_{n,ij}(\tau)\mathrm{e}^{\jmath n\omega_0 t} \quad \text{with } \omega_0 = \frac{2\pi}{T_0}\,, \tag{40}$$

- The Fourier coefficients

$$\boldsymbol{c}_{n,ij}(\tau) = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \boldsymbol{c}_{ij}(t,\tau)\mathrm{e}^{-\jmath n\omega_0 t}dt \tag{41}$$

are called the *cyclic correlation functions* and the frequencies $nf_0 = n\omega_0/2\pi$ are called the *cycle frequencies*.

Two-Dimensional Spectral Power Density Matrix

- We introduce the *two-dimensional spectral power density matrix*

$$\boldsymbol{C}(\omega_1, \omega_2) = \boldsymbol{c}(t_1, t_2)\mathrm{e}^{-\jmath(\omega_1 t_1 - \omega_2 t_2)} dt_1 dt_2 \,. \tag{42}$$

- Inserting

$$\boldsymbol{c}(t, \tau) = \sum_{n=-\infty}^{+\infty} \boldsymbol{c}_n(\tau)\mathrm{e}^{\jmath n \omega_0 t}$$

and substituting $t_1 = t$, $t_2 = t - \tau$, $dt_1 = dt$, $t_2 = -d\tau$ yields

$$\boldsymbol{C}(\omega_1, \omega_2) = \sum_{n=-\infty}^{+\infty} \{\boldsymbol{c}_n(\tau)\mathrm{e}^{\jmath\omega_2\tau} d\tau \mathrm{e}^{-\jmath(\omega_1 - \omega_2 - n\omega_0)t} dt\} \,.$$

The Cyclic Correlation Spectrum

- $$C(\omega_1,\omega_2) = \sum_{n=-\infty}^{+\infty} \{ c_n(\tau) e^{\jmath\omega_2\tau} d\tau e^{-\jmath(\omega_1-\omega_2-n\omega_0)t} dt \} .$$

- This equation yields

$$C(\omega_1,\omega_2) = \sum_{n=-\infty}^{+\infty} C_n(\omega_1)\delta(\omega_1-\omega_2-n\omega_0) , \quad (43)$$

- where the *cyclic correlation spectrum* $C_n(\omega_1)$ is given by

$$C_n(\omega_1) = c_n(\tau) e^{\jmath\omega_2\tau} d\tau . \quad (44)$$

The Pulse Response Matrix

- In a linear time-invariant system the output signal vector $\boldsymbol{s}_2(t)$ is related to the input signal vector $\boldsymbol{s}_1(t)$ by the *pulse response matrix* $\boldsymbol{h}(t)$ via

$$\boldsymbol{s}_2(t) = \boldsymbol{h}(t-t')\boldsymbol{s}_1(t')dt' \,. \tag{45}$$

- The relations between the TDCM $\boldsymbol{c}_o(t_1, t_2)$ of the output signal vectors and the TDCM $\boldsymbol{c}_i(t_1, t_2)$ of the input signal vectors is given by

$$\boldsymbol{c}_o(t_1, t_2) = \boldsymbol{h}(t_1 - t_1')\boldsymbol{c}_i(t_1', t_2')\boldsymbol{h}^{\dagger}(t_2 - t_2')dt_1' dt_2' \,. \tag{46}$$

The Frequency Response Matrix

- By Fourier transformation of

$$\boldsymbol{c}_o(t_1, t_2) = \boldsymbol{h}(t_1 - t_1')\boldsymbol{c}_i(t_1', t_2')\boldsymbol{h}^\dagger(t_2 - t_2')dt_1'dt_2' \,.$$

  we obtain the *output spectral power density matrix*

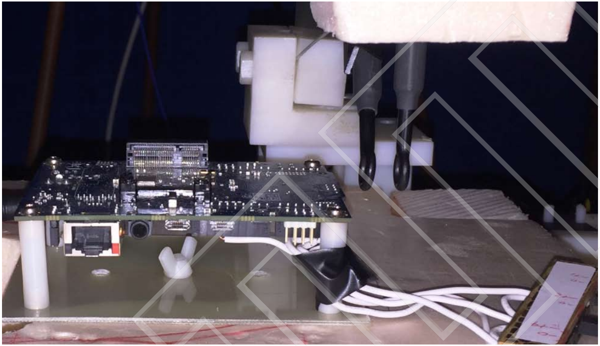
$$\boldsymbol{C}_o(\omega_1, \omega_2) = \boldsymbol{H}(\omega_1)\boldsymbol{C}_i(\omega_1, \omega_2)\boldsymbol{H}^\dagger(\omega_2) \tag{47}$$

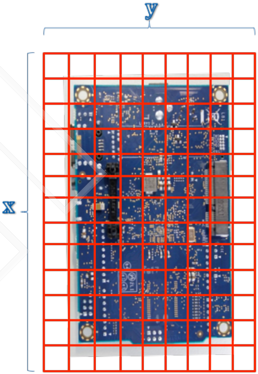
- The *frequency response matrix* is given by

$$\boldsymbol{H}(\omega) = \boldsymbol{h}(t)\mathrm{e}^{-\jmath\omega t}dt \,. \tag{48}$$

Outline

Measurements

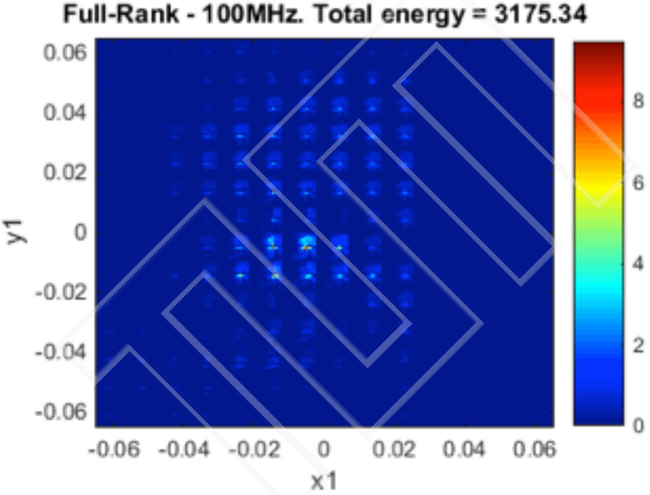




Galileo microcontroller board based on the 400 MHz Intel Quark SoC X1000, a 32-bit Intel Pentium class system on chip and the scanning grid used ($x = 9$ cm, $y = 13$ cm).

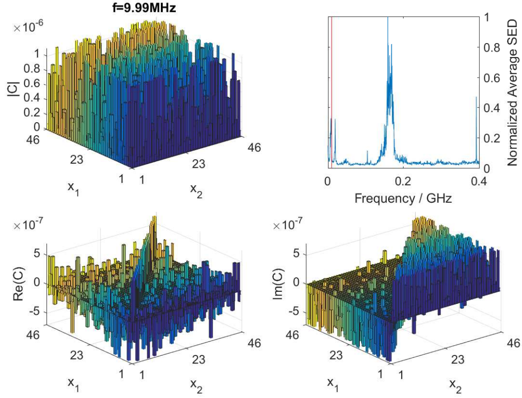
- David W. P. Thomas et al. “Near-Field Scanning of Stochastic Fields Considering Reduction of Complexity”. In: *Proc. International Symposium on Electromagnetic Compatibility, EMC*. Angers, France, Sep 4-8 2017

Measurements



Full cross-correlation measured from the Galileo board for 100 MHz emissions.

Frequency Dependence of the Correlation Matrix



• Johannes A. Russer et al. “Correlation Measurement and Evaluation of Stochastic Electromagnetic Fields”. In: *Proceedings of the International Symposium on Electromagnetic Compatibility, EMC Europe*. Wroclaw, Poland, Sept. 2016

Data Analysis - PCA

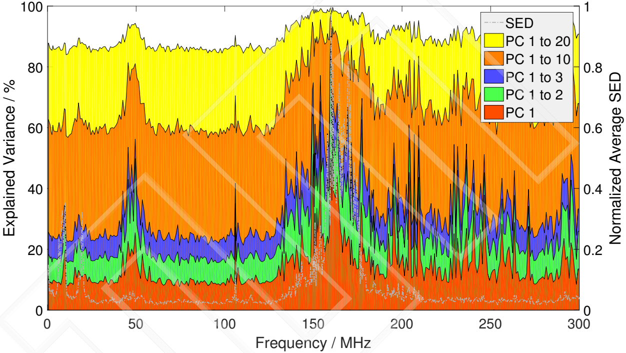


Figure: Cumulative explained variance of the principal component vs. frequency and spectral energy density.

• Johannes A. Russer et al. "Correlation Measurement and Evaluation of Stochastic Electromagnetic Fields". In: *Proceedings of the International Symposium on Electromagnetic Compatibility, EMC Europe*. Wroclaw, Poland, Sept. 2016

Efficient Approximation of Measured Data by PCA

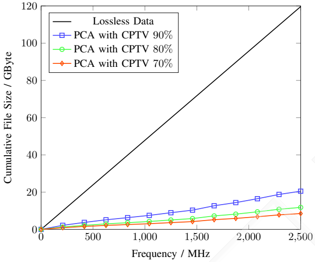


Figure: Cumulative file-size vs. frequency for different CPTV.

- The tangential magnetic field components were measured on a grid of 19×13 points.
- Determining all correlations requires measurement at about 30,000 point pairs.
- By storing only the few most dominant PCs, a significant reduction in required memory can be achieved.
- The required storage could be reduced from $120\,\mathrm{GB}$ to below $20\,\mathrm{GB}$, while retaining 90% of the total variation.

• M. Haider and J. A. Russer. “Principal component analysis for efficient characterization of stochastic electromagnetic fields”. In: *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields, IJNM* (2017)

• Michael Haider et al. “Principal Component Analysis Applied in Modeling of Stochastic Electromagnetic Field Propagation”. In: *Proceedings of the European Microwave Conference (EuMC)*. Nuremberg, Germany, Oct 8-13 2017

Experimental Results

- Initially, a single probe scan above a PCB is performed to establish the location of major sources of radiation.
- Two Langer EMV-Technik RF R50-1 magnetic field probes are connected to the input ports of an Agilent MSO8104A digital time domain oscilloscope.
- A two probe scanning measurement is performed for an Arduino Galileo board which is programmed for data transfer between central processor unit (CPU) and internal memory.

• Johannes A. Russer et al. "Near-Field Correlation Measurement and Evaluation of Stationary and Cyclostationary Stochastic Electromagnetic Fields". In: *Proc. European Microwave Conference (EuMC)*. London, U.K.: IEEE, Oct. 2016, pp. 484–484. DOI: 10.1109/EuMC.2016.7824384

Experimental Results

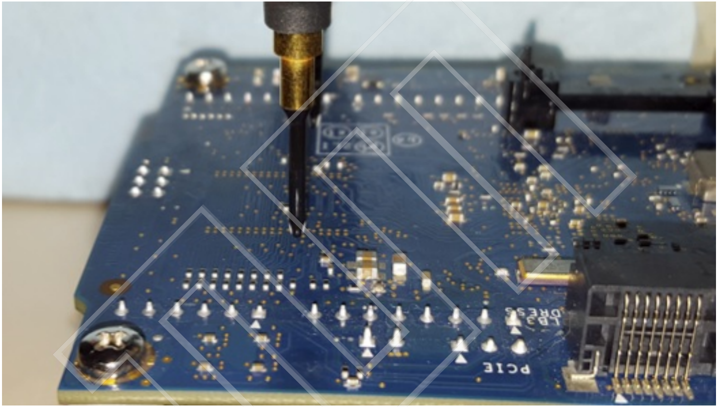


Figure: Measurement setup.

• Johannes A. Russer et al. "Near-Field Correlation Measurement and Evaluation of Stationary and Cyclostationary Stochastic Electromagnetic Fields". In: *Proc. European Microwave Conference (EuMC)*. London, U.K.: IEEE, Oct. 2016, pp. 484–484. DOI: 10.1109/EuMC.2016.7824384

Experimental Results

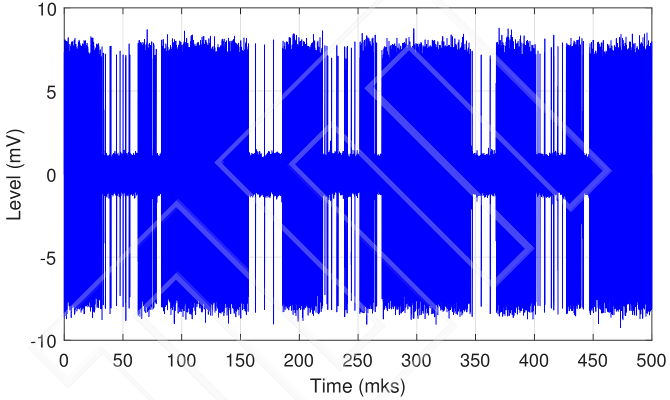


Figure: Realization of the stochastic signal.

• Johannes A. Russer et al. “Near-Field Correlation Measurement and Evaluation of Stationary and Cyclostationary Stochastic Electromagnetic Fields”. In: *Proc. European Microwave Conference (EuMC)*. London, U.K.: IEEE, Oct. 2016, pp. 484–484. DOI: 10.1109/EuMC.2016.7824384

Fourier Series Transform of the Cyclic AC Function

The spectral correlation reveals a coherent clock frequency of 400 MHz and its harmonics (vertical red lines) while cyclic autocorrelation function represents only cyclic periodic properties of the cyclostationary process.

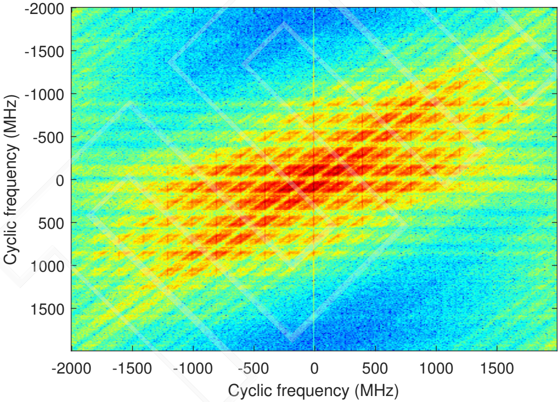


Figure: Spectral Correlation Density Function.

Outline

Conclusion

- We have discussed the propagation of stationary stochastic EM (GSSEM) fields with Gaussian propability distribution in a general context.
- The propagation of GSSEM fields can be computed numerically on the basis of correlation dyadics.
- In the near-field the GSSEM field may be scanned with arbitrary fine resolution. Scanning in every pair of measurement points is required.
- In the far-field only a single reference point is required, but the resolution is wavelength limited.
- The PCA method considerably reduce the computational effort for computing the environmental field from the field sampled by identifying and retaining only the relevant variables without loss of information.
- Second order CS stochastic processes are non-stationary stochastic processes, where the TDCM depends on the global time and the time difference, and the dependence on the global time is periodic.

Acknowledgment

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