IC 1407 ACCREDIT

STOCHASTIC SIGNAL PROCESSING

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Motivation of statistical SP

- Electromagnetic field (EMF) emitted by the printed circuit board (PCB) contains a stochastic component sensing by the near-field scanning probe and registering by the real-time digital oscilloscope
- Statistical signal processing can be used for the characterization of the received stochastic EMF and for the localization and identification of the radiating sources
- The measured stochastic signals contain the superposition of different radiating sources located in the environment of the DUT:
  - ✓ emissions from transmission lines transferring the data sequences between distinct blocks of the DUT;
  - ✓ thermal noise and interference from surrounding radiations

Classification of random processes

Random process

Time sequence of random variables

$$\mathcal{S}(t) = \{S_{t_i}, i = [1, n]\}$$

S_{t_i} is a random variable at $t = t_i$

The set of random signals

$$\mathcal{S}(t) = \{S(t, \omega) : \omega \in \Omega , t \in \mathbb{R}\}$$

ω is an elementary event

Ω is a sample space

$\mathbb{R}$ is a set of real numbers

- **The stochastic phenomena can be mathematically characterized by the model of random process constructed from the measured random signals**
- **The random variable is fully described by the probability density function for each time $t = t_i$**
- **Random signal is a time domain realization of the stochastic phenomena for each event $\omega \in \Omega$**

Cumulative Distribution Function

- The ***cumulative distribution function*** of the stochastic process is:

$$F_S(s,t) = P[S(t,\omega) \le s] = \int_\Omega \mathbf{I}_{\{\omega:S(t,\omega)\le s\}}\,\mathrm{d}P(\omega) = E\left\{\mathbf{I}_{\{\omega:S(t,\omega)\le s\}}\right\}$$

- ✓ indicator function $\mathbf{I}_{\{\omega:S(t,\omega)\le s\}} = \begin{cases} 1, \omega: S(t,\omega) \le s \\ 0, \omega: S(t,\omega) > s \end{cases}$
- ✓ $E\{\cdot\}$ is the operator of ***statistical expectation or ensemble averaging***[1] using Lebesgue integral

- The expected value of the stochastic process is the ***statistical mean***

$$m_S(t) = E\{S(t,\omega)\} = \int_\Omega S(t,\omega)\,\mathrm{d}P(\omega) = \int_{\mathbb{R}} s\,\mathrm{d}F_S(s,t)$$

[1] A. Napolitano, *Generalizations of Cyclostationary Signal Processing: Spectral Analysis and Applications*. John Wiley & Sons Ltd - IEEE Press, 2012.

Second-Order Characterization

- The ***second-order joint cumulative distribution function*** of the stochastic process is:

$$F_{\mathcal{S}}(s_1, s_2; t_1, t_2) = P[S(t_1, \omega) \leq s_1, S(t_2, \omega) \leq s_2] = E\left\{\mathbf{I}_{\{\omega: S(t_1,\omega)\leq s_1\}}\mathbf{I}_{\{\omega: S(t_2,\omega)\leq s_2\}}\right\}$$

- The ***autocorrelation function*** of the stochastic process is:

$$\mathcal{R}_{\mathcal{S}}(t, \tau) = E\{S(t, \omega)S(t+\tau, \omega)\} = \int_{\mathbb{R}^2} s_1 s_2 \,\mathrm{d}F_{\mathcal{S}}(s_1, s_2; t, t+\tau)$$

- The ***autocovariance*** of the stochastic process is its autocorrelation function of the zero-mean process:

$$\mathcal{C}_{\mathcal{S}}(t, \tau) = E\{[S(t, \omega) - m_{\mathcal{S}}(t)][S(t+\tau, \omega) - m_{\mathcal{S}}(t+\tau)]\}$$

Spectral Characterization

- The second order stochastic process is ***harmonizable*** and can be characterized in the frequency domain if its autocorrelation function can be expressed by the Fourier-Stieltjes integral:

$$E\{S(t_1,\omega)S(t_2,\omega)\} = \int_{\mathbb{R}^2} e^{j2\pi(f_1t_1+f_2t_2)}\,\mathrm{d}\mathcal{G}_{\mathcal{S}}(f_1,f_2)$$

- ✓ where $\mathcal{G}_{\mathcal{S}}(f_1,f_2)$ is a ***spectral correlation function*** with bounded variation. It is also known that a stochastic process is harmonizable if and only if its covariance function is harmonizable

- The Fourier transform of the realization $S(t,\omega)$ of the harmonizable stochastic process can be expressed as:

$$\hat{S}(f,\omega) = \int_{\mathbb{R}} S(t,\omega)e^{-j2\pi ft}\,\mathrm{d}t$$

- ✓ where $\hat{S}(f,\omega)$ can contain Dirac delta functions

Time-Frequency Relation

- The ***spectral correlation function*** (Loève bifrequency spectrum) of the harmonizable stochastic process is defined as:

$$\mathcal{G}_{\mathcal{S}}(f_1, f_2) = E\left\{\hat{S}(f_1, \omega)\hat{S}(f_2, \omega)\right\}$$

- The relation between the autocorrelation function and the spectral correlation function is defined by two-dimensional Fourier transform:

$$E\{S(t_1)S(t_2)\} = \int_{\mathbb{R}^2} \mathcal{G}_{\mathcal{S}}(f_1, f_2) e^{j2\pi(f_1 t_1 + f_2 t_2)} \,\mathrm{d}f_1 \mathrm{d}f_2$$

$$\mathcal{G}_{\mathcal{S}}(f_1, f_2) = \int_{\mathbb{R}^2} E\{S(t_1)S(t_2)\} e^{-j2\pi(f_1 t_1 + f_2 t_2)} \,\mathrm{d}t_1 \mathrm{d}t_2$$

Time-Variant Spectrum

- The ***time-variant spectrum*** of the stochastic process is the Fourier transform of the autocorrelation function with respect to the lag parameter $\tau = t_2 - t_1$:

$$\mathcal{V}_{\mathcal{S}}(t,f) = \int_{\mathbb{R}} \mathcal{R}_{\mathcal{S}}(t,\tau) e^{-j2\pi f\tau}\,\mathrm{d}\tau$$

- By introducing the variables $t_1 = t + \tau/2$ and $t_2 = t - \tau/2$ it can be obtained a time-frequency representation in terms of ***Wigner-Ville spectrum*** for stochastic processes:

$$\mathcal{W}_{\mathcal{S}}(t,f) = \int_{\mathbb{R}} E\{S(t+\tau/2)S(t-\tau/2)\} e^{-j2\pi f\tau}\,\mathrm{d}\tau = \int_{\mathbb{R}} E\{\hat{S}(f+\nu/2)\hat{S}(f-\nu/2)\} e^{-j2\pi\nu t}\,\mathrm{d}\nu$$

Characteristics of RP

- Distribution of the probability over RVs of the random process
- Ensemble averaging of the random process by using expectation operator
- Statistical mean and 2D-autocorrelation function in time domain
- Spectral correlation function in bi-frequency domain
- Time-variant and Wigner-Ville spectra in time-frequency domain
- Probabilistic approach is more theoretical then practical

Stationary process

- Second-order ***wide-sense stationary process* (WSS)** can be characterized by an autocorrelation function (ACF) and a power spectral density linked by the Wiener-Khinchin relation

$$\mathcal{R}_{\mathcal{S}}(t,\tau) = R_{\mathcal{S}}(\tau) = \int_{\mathbb{R}} V_{\mathcal{S}}(f) e^{j2\pi f\tau}\,\mathrm{d}f$$

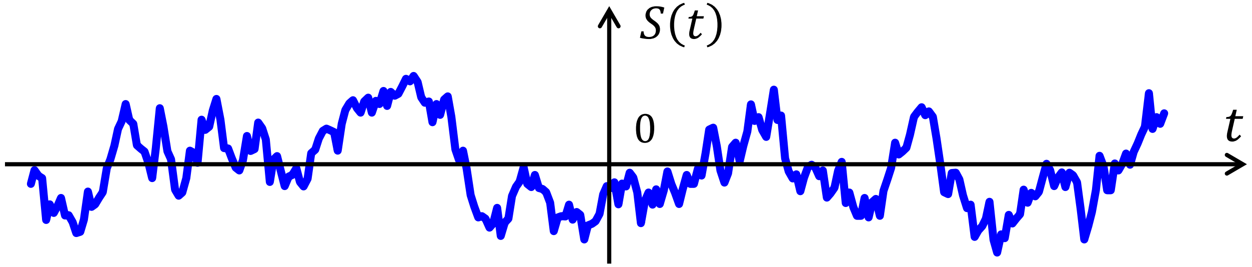
$$\mathcal{V}_{\mathcal{S}}(t,f) = V_{\mathcal{S}}(f) = \int_{\mathbb{R}} R_{\mathcal{S}}(\tau) e^{-j2\pi f\tau}\,\mathrm{d}\tau$$

- Due to the dependency of the ACF for the WSS process only from $\tau = t_2 - t_1$, the spectral correlation function $\mathcal{G}_{\mathcal{S}}(f_1, f_2)$ can be non-zero only for $f_1 = -f_2$

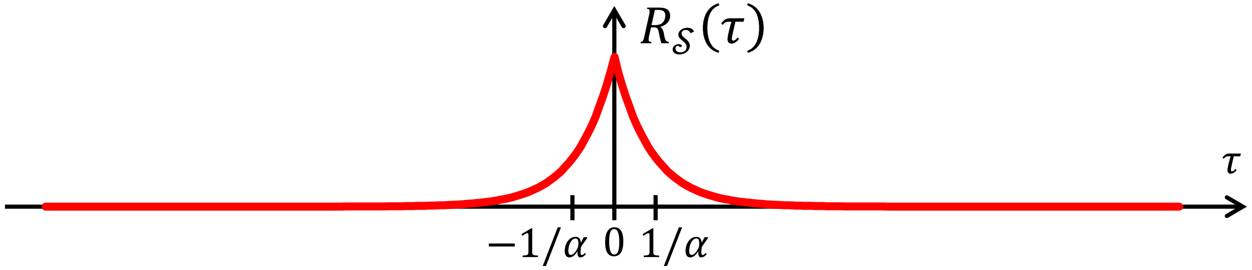
$$E\{S(t_1)S(t_1-\tau)\} = \int_{\mathbb{R}^2} \mathcal{G}_{\mathcal{S}}(f_1,f_2) e^{j2\pi(f_1 t_1 + f_2(t_1-\tau))}\,\mathrm{d}f_1\mathrm{d}f_2 = \int_{\mathbb{R}^2} \mathcal{G}_{\mathcal{S}}(f_1,-f_1) e^{j2\pi f_1\tau}\,\mathrm{d}f_1\mathrm{d}f_2$$

Stationary process

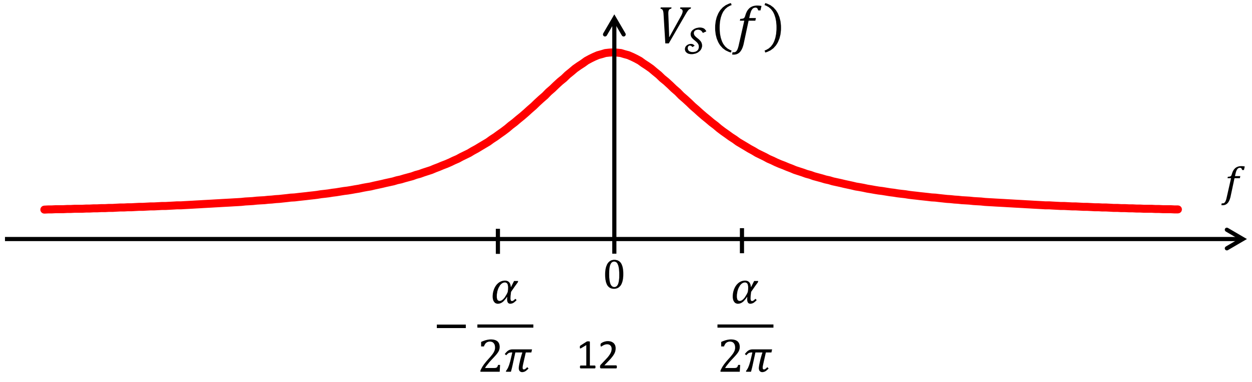
➢ Random process realization



➢ Autocorrelation function



➢ Power spectral density



Discrete-time random process

- ***Discrete time process*** can be analyzed as the sampled continuous time realizations of the stochastic processes:

$$\mathcal{S}[n] = \mathcal{S}(t = n\Delta) = \sum_{k=-\infty}^{\infty} S_k \delta[n-k]$$

- where Δ is a sample interval. The Discrete Time Fourier Transform (DTFT) of the realization $S[n]$ of the harmonizable discrete stochastic process can be expressed as:

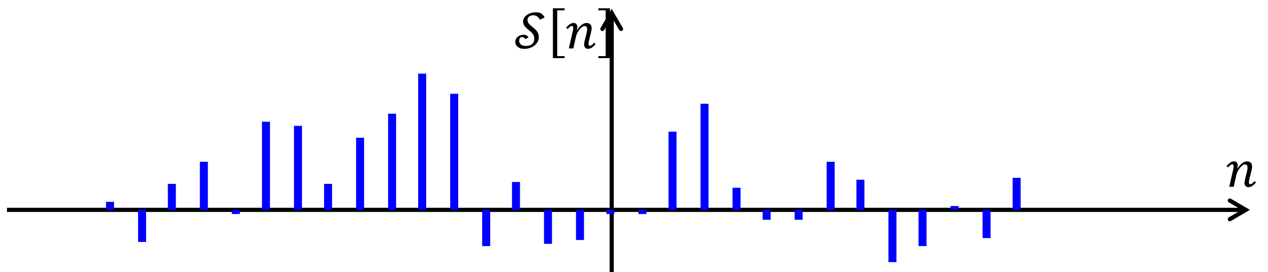
$$\hat{S}(\varphi) = \sum_{n=-\infty}^{\infty} S_n e^{-j2\pi\varphi n}$$

- where $\varphi = f\Delta$ is a normalizes frequency. The inverse DTFT gives the initial realization of the discretized stochastic process:

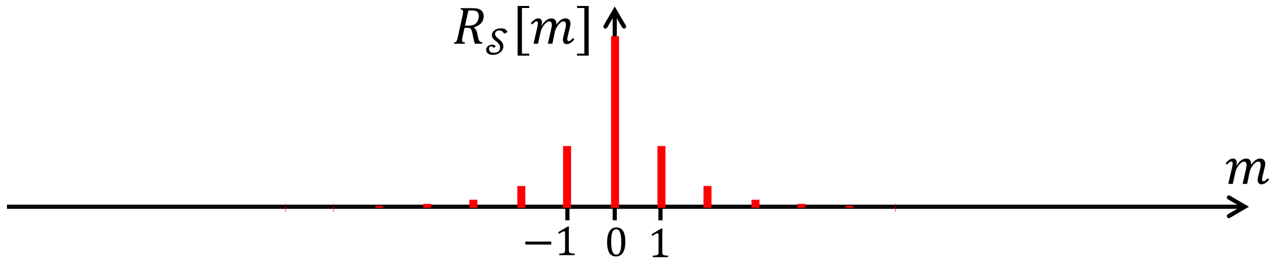
$$S[n] = \int_{-1/2}^{1/2} \hat{S}(\varphi) e^{j2\pi\varphi n}$$

Discrete-time stationary process

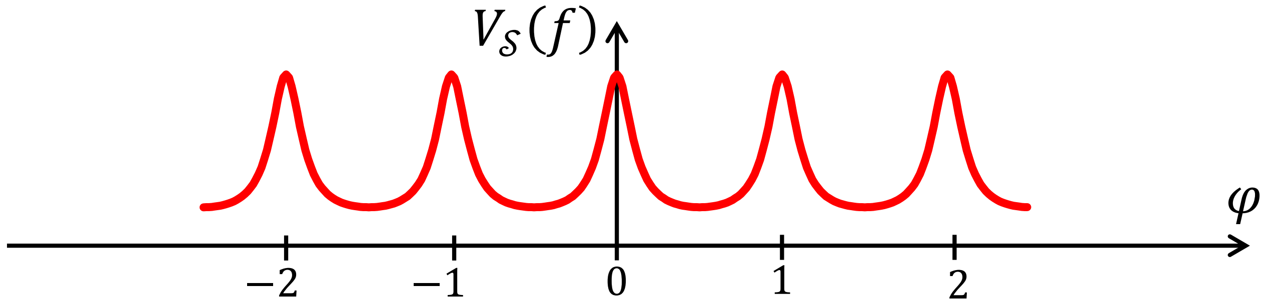
- Discrete time process



- Discrete autocorrelation function



- Periodical power spectral density



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Characteristics of Discrete random process

- The second order discretized stochastic process is ***harmonizable*** and can be characterized in the frequency domain if its autocorrelation function can be expressed by the Fourier-Stieltjes integral:

$$E\{S[n_1]S[n_2]\} = \int_{[-1/2,1/2]^2} e^{j2\pi(\varphi_1 n_1+\varphi_2 n_2)}\, \mathrm{d}\mathcal{G}_S(\varphi_1,\varphi_2)$$

- The ***spectral correlation function*** (Loève bifrequency spectrum) of the harmonizable discretized stochastic process is defined as:

$$\mathcal{G}_S(\varphi_1,\varphi_2) = E\left\{\hat{S}(\varphi_1)\hat{S}(\varphi_2)\right\}$$

- The relation between the autocorrelation function and the spectral correlation function is defined by two-dimensional DTFT:

$$E\{S[n_1]S[n_2]\} = \int_{[-1/2,1/2]^2} \mathcal{G}_S(\varphi_1,\varphi_2) e^{j2\pi(\varphi_1 n_1+\varphi_2 n_2)}\, \mathrm{d}\varphi_1 \mathrm{d}\varphi_2$$

$$\mathcal{G}_S(\varphi_1,\varphi_2) = \sum_{n_1=-\infty}^{\infty}\sum_{n_2=-\infty}^{\infty} E\{S[n_1]S[n_2]\} e^{-j2\pi(\varphi_1 n_1+\varphi_2 n_2)}$$

Characteristics of stationary RP

- **Distribution of the probability over RVs of the random process doesn't depend of time**
- **Ensemble averaging of the random process by using expectation operator**
- **Statistical mean and autocorrelation function doesn't depend of time**
- **Spectral correlation function in a Fourier transform of the ACF**
- **Discretization of the stationary RP gives the periodic PSD and discrete ACF**

Cyclostationary random process

- The cyclostationary random process $\mathcal{S}(t)$ is a non-stationary stochastic process whose statistical properties are periodically vary with respect to time. The period T_0 is called a **cycle**, and its inverse $\alpha = 1/T_0$ is a **cyclic frequency**. More generally, the process is **almost-cyclostationary (ACS)** if its statistical properties can be represented by a superposition of periodic functions with distinct cyclic frequencies $\alpha \in \mathcal{A}$.

- The **autocorrelation function** of the ACS stochastic process posses the periodicity in time and can be expressed by the Fourier series expansion[1]:

$$\mathcal{R}_{\mathcal{S}}(t,\tau) = E\{S(t)S(t+\tau)\} = \sum_{\alpha \in \mathcal{A}} R_{\mathcal{S}}(\alpha,\tau)e^{j2\pi\alpha t}$$

- ✓ where $E\{\cdot\}$ is the operator of ensemble averaging:

$$E\{S(t)S(t+\tau)\}_{T_0} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} S(t+T_0)S(t+\tau+T_0)$$

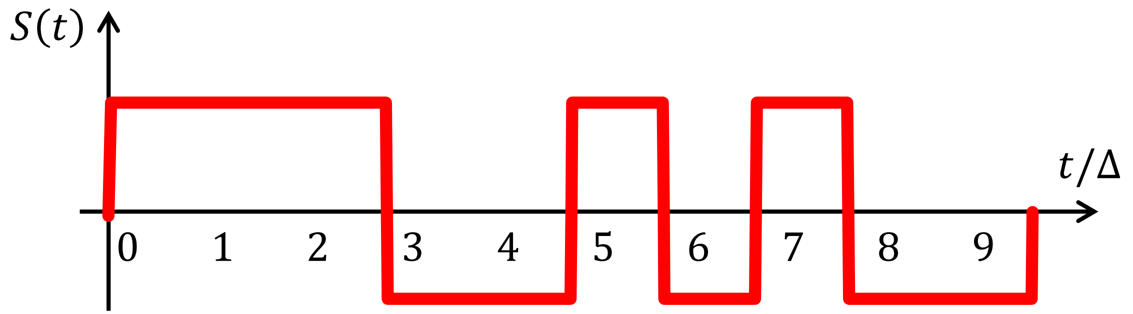
- The Fourier coefficients of $\mathcal{R}_{\mathcal{S}}(t,\tau)$ are called ***cyclic autocorrelation functions*** and can be defined by a time domain cyclic averaging for each known cyclic frequency:

$$R_{\mathcal{S}}(\alpha,\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \mathcal{R}_{\mathcal{S}}(t,\tau)e^{-j2\pi\alpha t}\mathrm{d}t$$

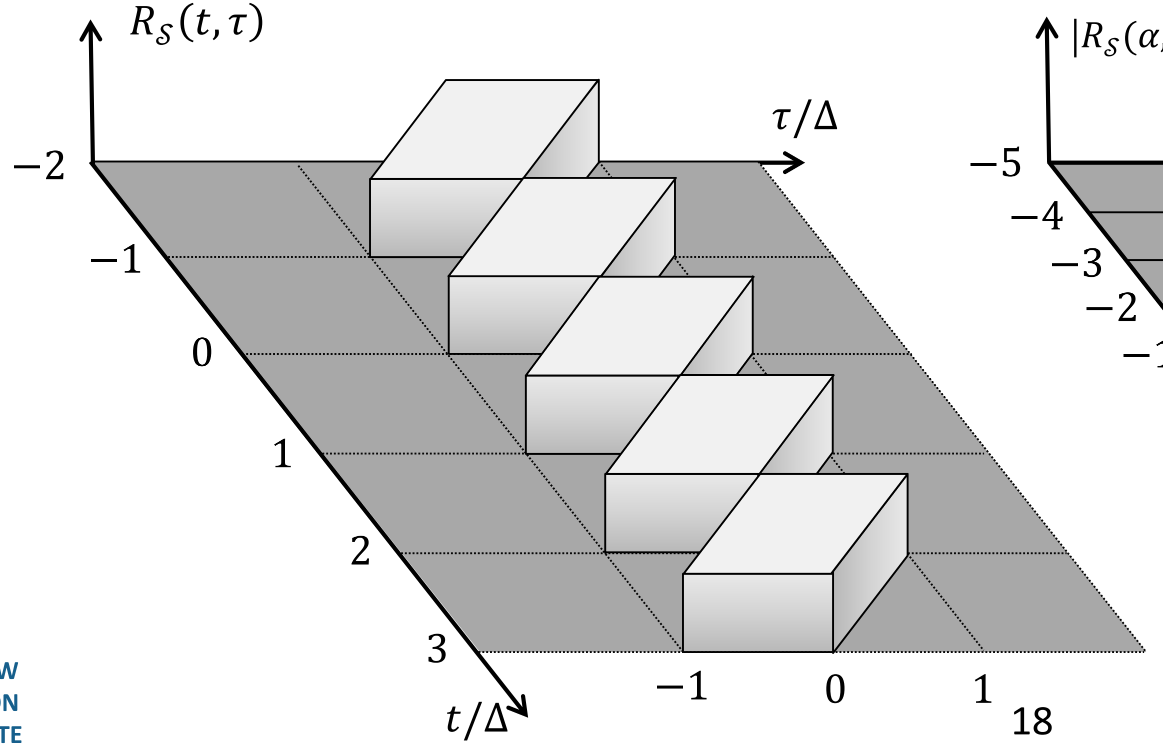
[1] W. A. Gardner, *Introduction to Random Processes with Applications to Signals and Systems*. Macmillan, New York, 1985 (2nd Edition McGraw-Hill, New York, 1990).

Pulse Amplitude Modulation

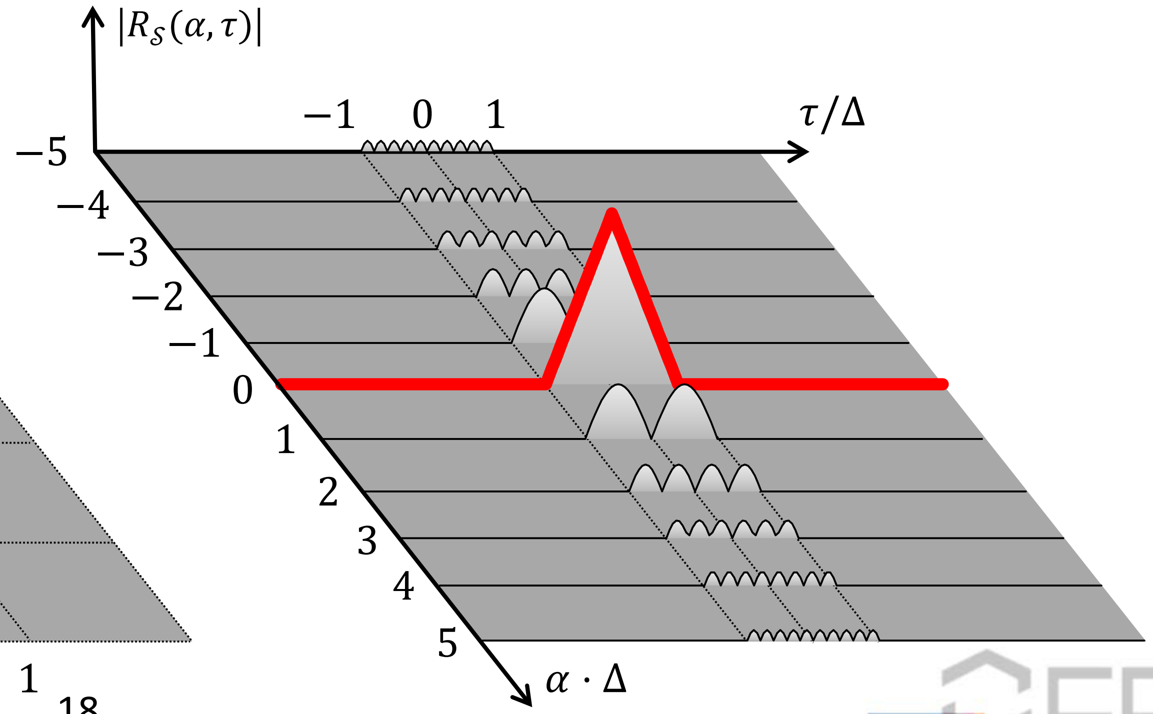
➢ **Pulse amplitude modulated (PAM) signal**



➢ **Autocorrelation function**



➢ **Cyclic autocorrelation function**



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Cyclic correlation function

- The magnitude and phase of $R_S(\alpha,\tau)$ represent amplitude and phase of the additive complex harmonic component at frequency α for time lag τ contained in the autocorrelation function of the ACS stochastic process $\mathcal{R}_S(t,\tau)$. For $\alpha = 0$ the cyclic autocorrelation function reduces to the autocorrelation function of the stationary random process $R_S(\tau)$.
- For a zero-mean stochastic process $m_S(t) = E\{S(t)\} = 0$ the magnitude of the cyclic autocorrelation functions $\left|R_S(\alpha,\tau)\right| \to 0$ as $\tau \to \infty$. If the mean function of the stochastic process $m_S(t) = E\{S(t,\omega)\} \neq 0$, then some $R_S(\alpha,\tau)$ contain additive sinusoidal functions of τ, which arise from the products of sinusoidal terms in $m_S(t)$. Such ACS processes are called unpure and need to be processed accounting on such property of the process[1].
- For cyclo-ergodic stochastic process the pure cyclic autocorrelation function called autocovariance function can be evaluated by synchronize removing the deterministic mean function from realization of the ACS process:

$$C_S(\alpha,\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} [E\{S(t)S(t+\tau)\} - E\{S(t)\}E\{S(t+\tau)\}] e^{-j2\pi\alpha t} \mathrm{d}t$$

[1] J. Antoni, “Cyclostationarity by examples,” *Mechanical Systems and Signal Processing* 23(4), 2009, pp. 987–1036.

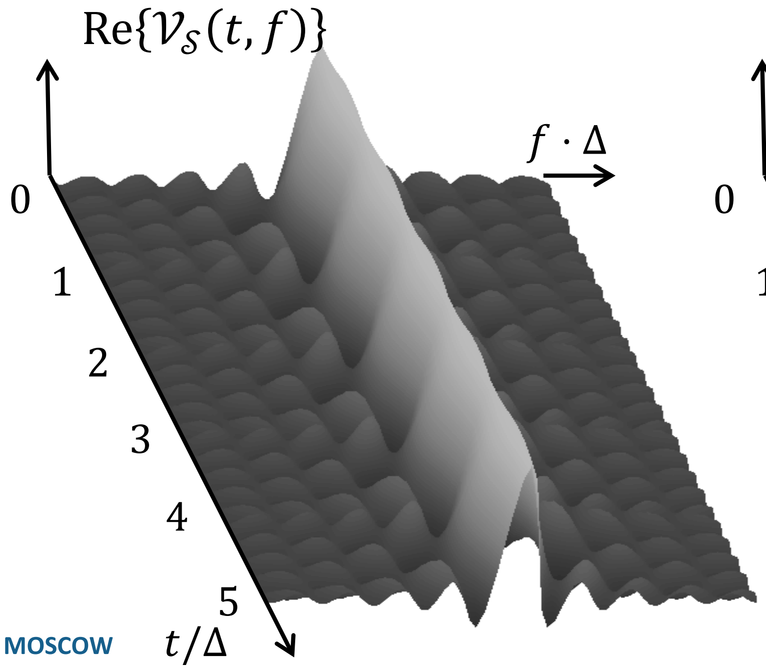
Time-frequency Relation

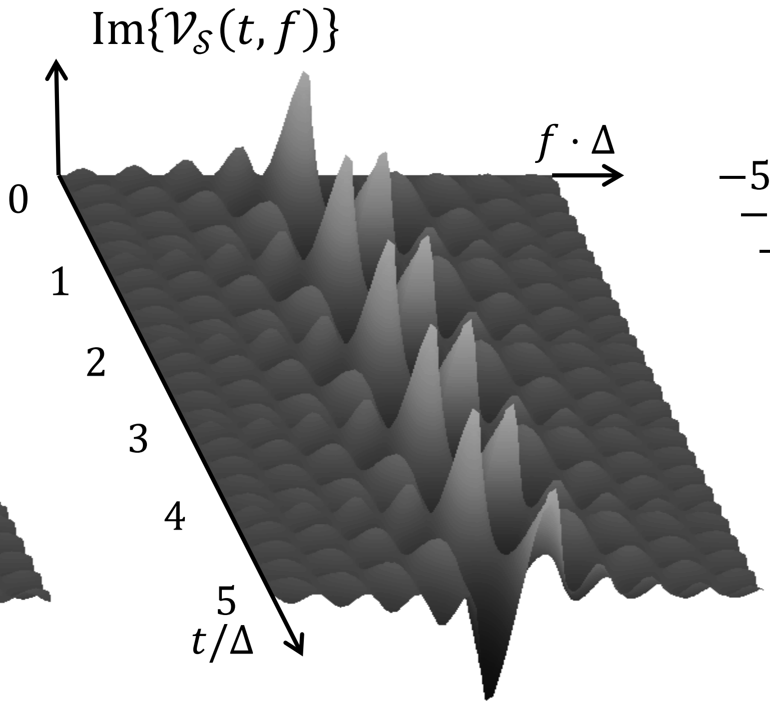
- The ***almost-periodic time-variant spectrum*** of the ACS process is the Fourier transform of the cyclic autocorrelation function $\mathcal{R}_{\mathcal{S}}(t,\tau)$ with respect to the lag parameter τ:

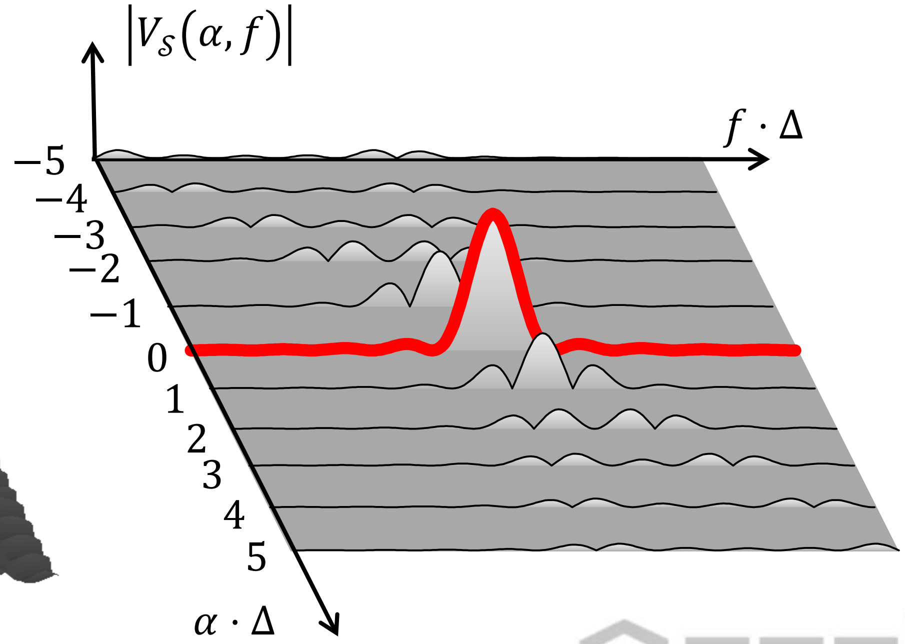
$$\mathcal{V}_{\mathcal{S}}(t,f) = \sum_{\alpha\in\mathcal{A}} V_{\mathcal{S}}(\alpha,f)e^{j2\pi\alpha t}$$

- ✓ where the ***cyclic spectrum correlation function*** $V_{\mathcal{S}}(\alpha,f)$ can be defined by the Fourier transform of the autocorrelation function $R_{\mathcal{S}}(\alpha,\tau)$ with respect to the lag parameter τ:

$$V_{\mathcal{S}}(\alpha,f) = \int_{\mathbb{R}} R_{\mathcal{S}}(\alpha,\tau)e^{-j2\pi f\tau}\,\mathrm{d}\tau$$







Cyclic Spectrum

- The alternative approach for the evaluation of the cyclic spectrum correlation function is the averaging of the short-time Fourier transforms (STFT) of the stochastic process realizations $S(t)$:

$$X_{1/\Delta f}(t, f) = \int_{t-1/\Delta f}^{t+1/\Delta f} S(\xi) e^{-j2\pi f \xi} \mathrm{d}\xi$$

- The averaging of $X_{1/\Delta f}(t, f)$ can be implemented by two strictly ordered successive limits:

$$V_{\mathcal{S}}(\alpha, f) = \lim_{\Delta f \to 0} \lim_{T \to \infty} \frac{\Delta f}{T} \int_{-T/2}^{T/2} E\{X_{1/\Delta f}(t, f) X_{1/\Delta f}(t, \alpha - f)\} \mathrm{d}t$$

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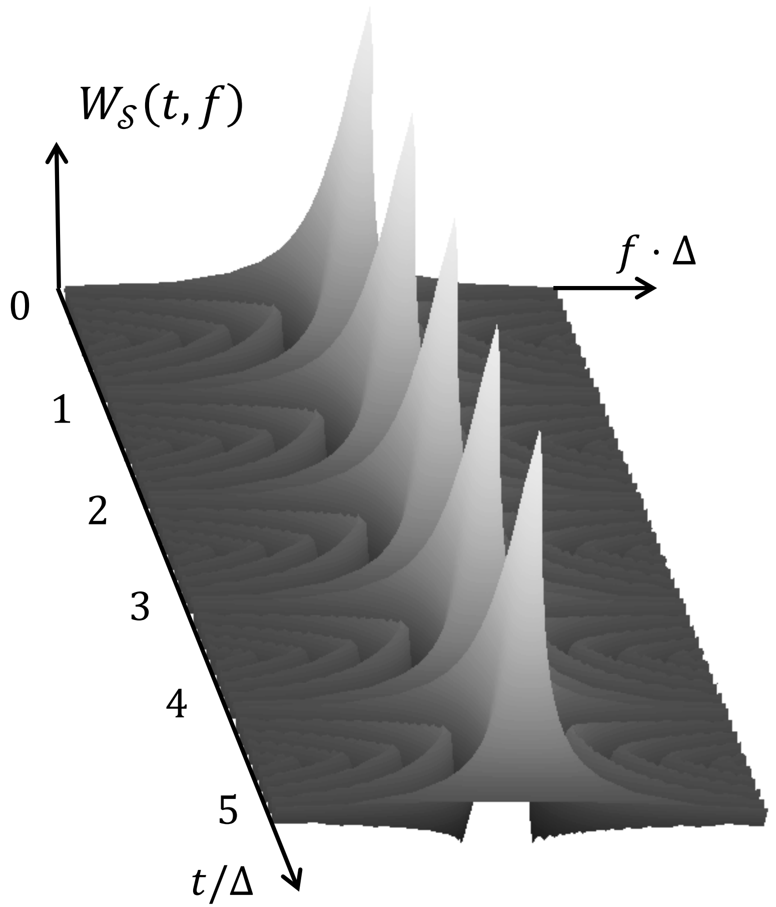
Wigner-Ville Cyclic Spectrum

- The ***Wigner-Ville spectrum*** for ACS stochastic processes:

$$W_{\mathcal{S}}(t,f)=\sum_{\alpha\in\mathcal{A}} V_{\mathcal{S}}(\alpha, f+\alpha/2)e^{j2\pi\alpha t}$$

The Wigner-Ville spectrum of the ACS stochastic process can be expressed by the Fourier series expansion over the time t with frequencies $\alpha\in\mathcal{A}$ and Fourier-series coefficients $V_{\mathcal{S}}(\alpha, f+\alpha/2)$.

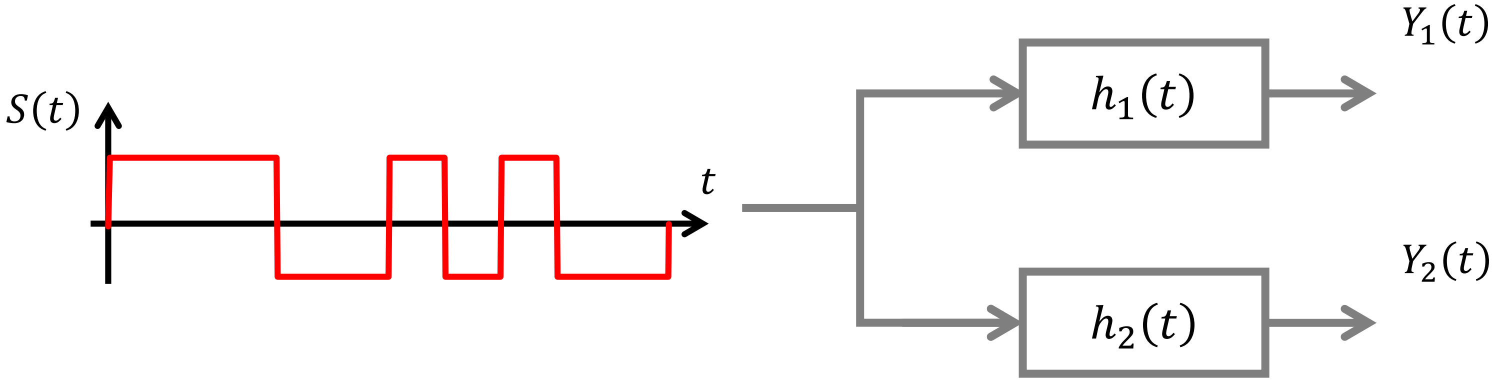


- The ***spectral correlation function*** (Loève bifrequency spectrum) of the ACS stochastic process is defined as:

$$\mathcal{G}_{\mathcal{S}}(f_1,f_2)=\sum_{\alpha\in\mathcal{A}} V_{\mathcal{S}}(\alpha, f_1)\delta(f_2+f_1-\alpha)$$

It concentrated in the countable set of lines with slope +1 on the bi-frequency plane. It means that ACS process have distinct spectral components that are correlated only if the spectral separation belongs to a countable set of cycle frequencies.

Cross-Correlation Function



$$\mathcal{R}_{Y_1Y_2}(t_1,t_2) = E\{Y_1(t_1)Y_2(t_2)\} = \iint_{\mathbb{R}^2} h_1(t_1-\tau_1)h_2(t_2-\tau_2)E\{S(\tau_1)S(\tau_2)\}d\tau_1 d\tau_2 =$$
$$\iint_{\mathbb{R}^2} h_1(t_1-\tau_1)h_2(t_2-\tau_2)\mathcal{R}_S(\tau_1,\tau_2)d\tau_1 d\tau_2$$

- For two different ACS stochastic processes $\mathcal{S}_1(t)$ and $\mathcal{S}_2(t)$ are said to be jointly correlated if the ***second-order cross-correlation function***

$$\mathcal{R}_{Y_1Y_2}(t,\tau) = E\{Y_1(t)Y_2(t+\tau)\} = \sum_{\alpha\in\mathcal{A}_{12}} R_{Y_1Y_2}(\alpha,\tau)e^{j2\pi\alpha t}$$

at cycle frequencies $\alpha \in \mathcal{A}_{12}$ is defined by non-zero ***cyclic cross-correlation functions***

$$R_{Y_1Y_2}(\alpha,\tau) = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\mathcal{R}_{Y_1Y_2}(t,\tau)e^{-j2\pi\alpha t}\mathrm{d}t$$

Cross-Correlation Spectrum

- The ***cyclic spectrum cross-correlation*** $V_{Y_1Y_2}(\alpha, f)$ can be defined by the Fourier transform of the cyclic cross-correlation function $R_{Y_1Y_2}(\alpha, \tau)$ with respect to the lag parameter τ:

$$V_{Y_1Y_2}(\alpha, f) = \int_{\mathbb{R}} R_{Y_1Y_2}(\alpha, \tau) e^{-j2\pi f\tau}\,\mathrm{d}\tau$$

- It can be independently evaluated by the averaging of the short-time Fourier transforms (STFT) of the stochastic process realizations $Y_1(t)$ and $Y_2(t)$:

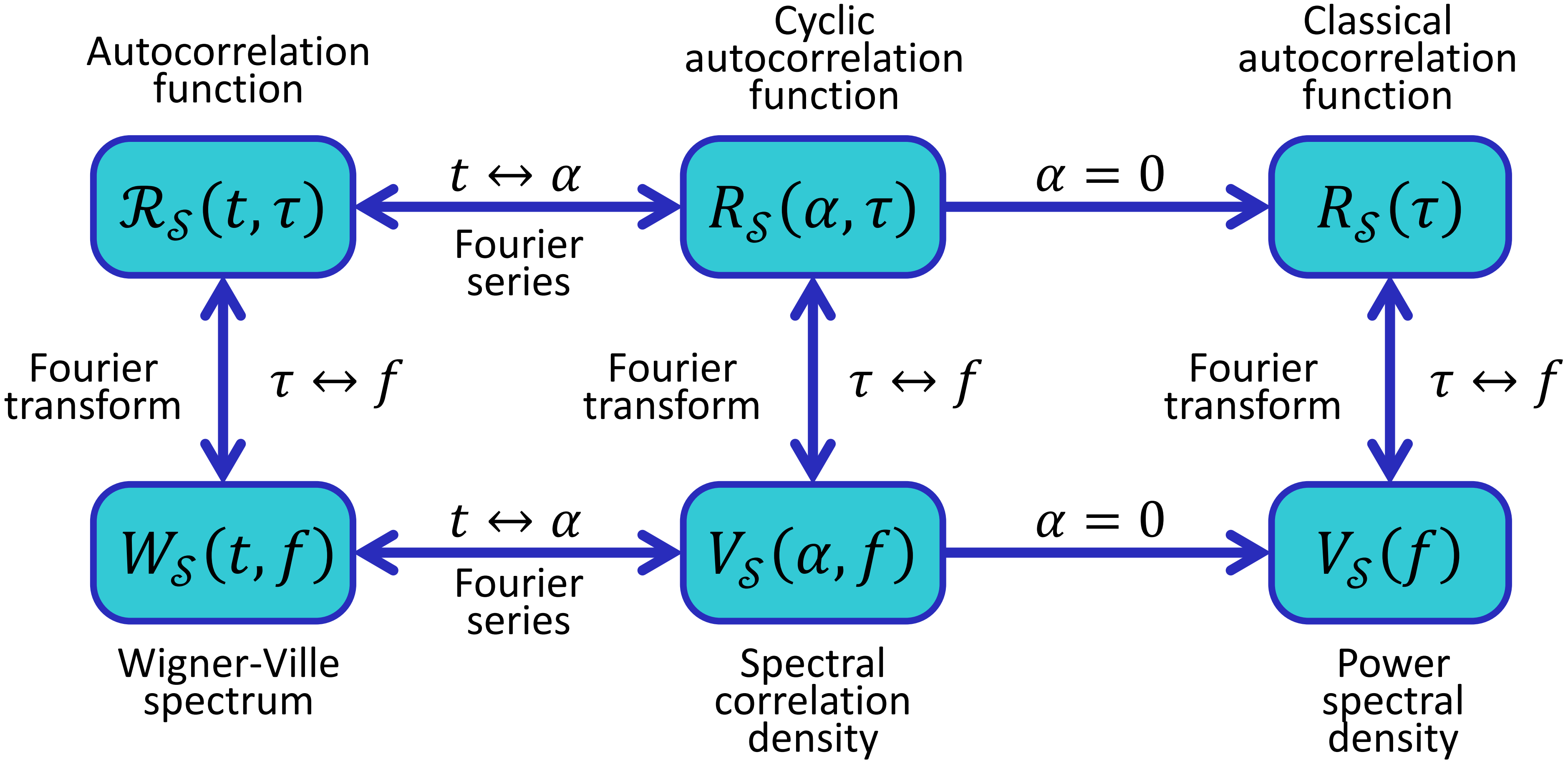
$$V_{Y_1Y_2}(\alpha, f) = \lim_{\Delta f\to 0} \lim_{T\to\infty} \frac{\Delta f}{T} \int_{-T/2}^{T/2} E\{X_{1,1/\Delta f}(t, f) X_{2,1/\Delta f}(t, \alpha - f)\}\mathrm{d}t$$

where

$$X_{i,1/\Delta f}(t, f) = \int_{t-1/\Delta f}^{t+1/\Delta f} Y_i(\xi) e^{-j2\pi f\xi}\mathrm{d}\xi\,;\quad i = 1,2$$

- For the estimation of second-order statistical functions of ACS stochastic process need to have finite or “effectively finite” memory. It means that ACS process need to be a zero-mean stochastic process $m_S(t) = E\{S(t)\} = 0$.

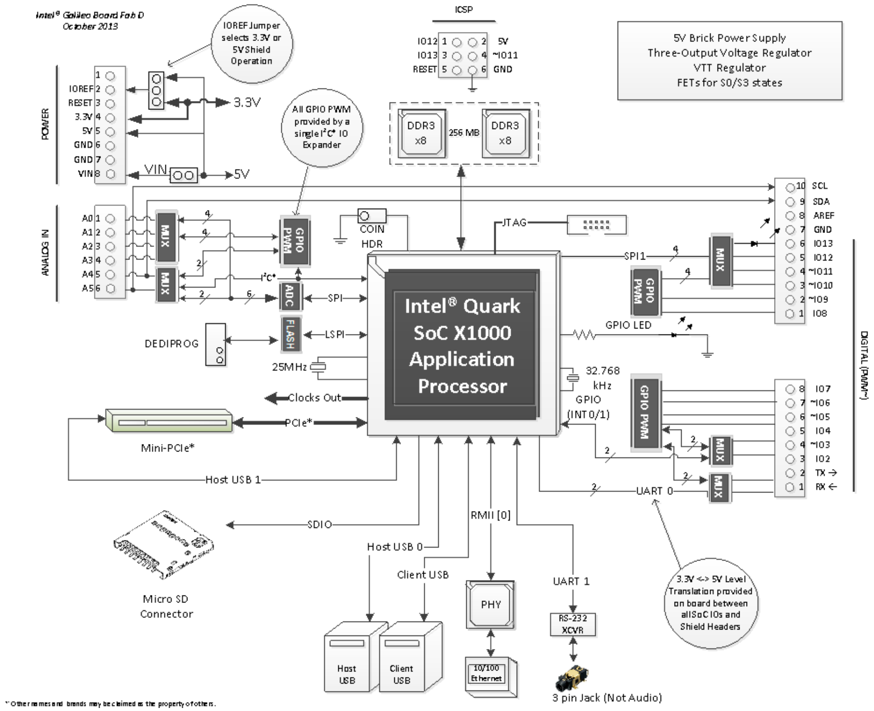
Relation between ACS Characteristics



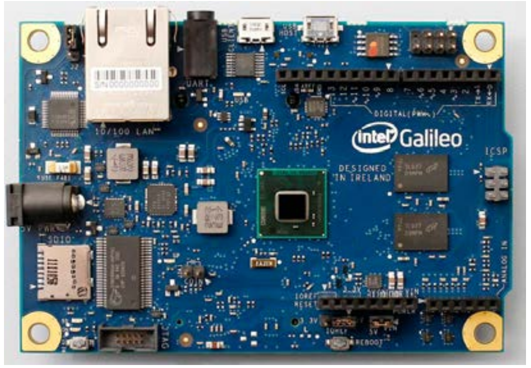
Characteristics of CS Process

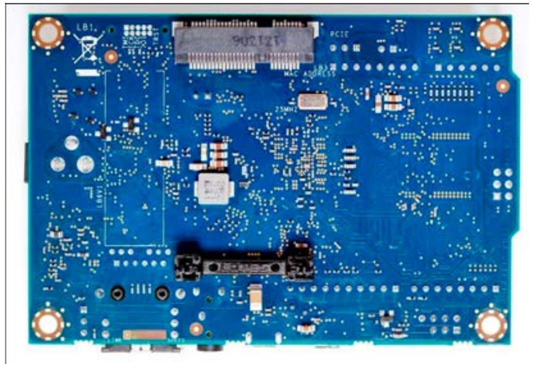
- **Mean function and 2D autocorrelation function of the CS process can be expressed by a superposition of periodic functions with different periods**
- **Cyclic autocorrelation function (ACF) can be obtained by the time-domain cyclic averaging of the non-linear time-shift transformation**
- **To obtain the pure cyclic ACF the mean function need to be removed from the realizations of the random process**
- **Cyclic spectral correlation function can be evaluated by the frequency-domain averaging of the frequency-shifted Fourier transforms of the measured realizations**

Device under test



➢The Intel® Galileo Board





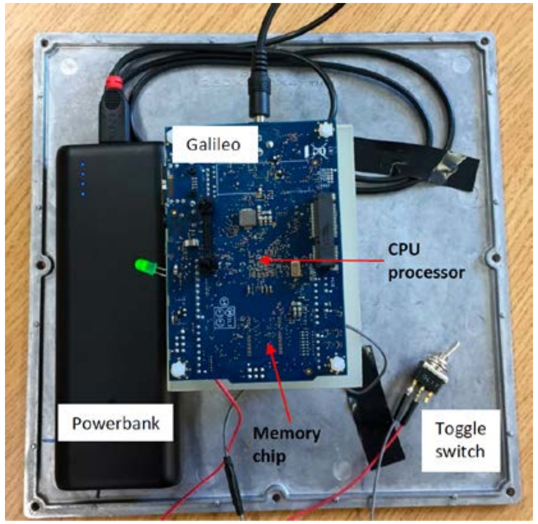
✓400MHz 32-bit Intel® Pentium processor

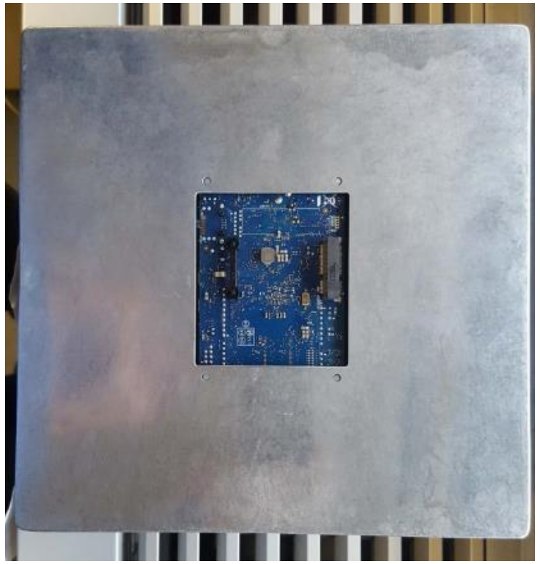
✓10/100 Ethernet connector

✓Full PCI Express* mini-card slot

✓USB 2.0 Host connector

Device under test



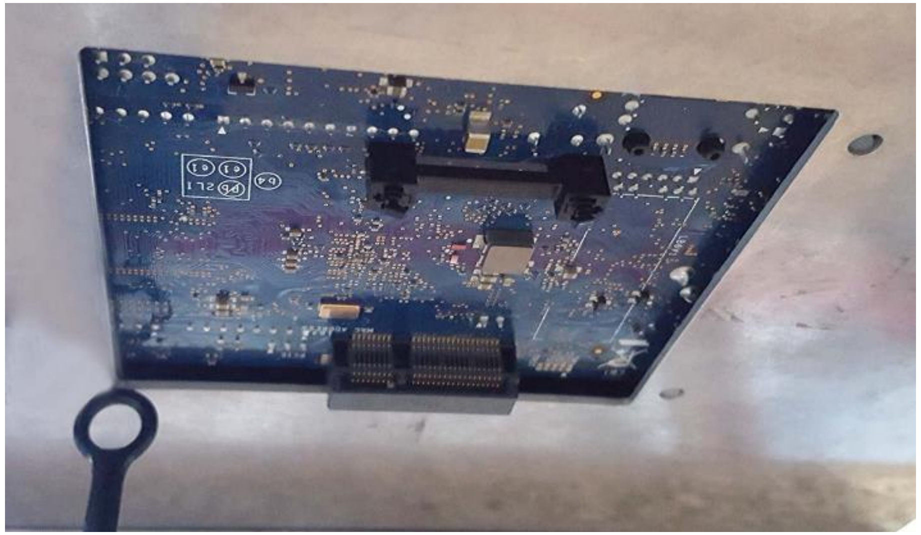


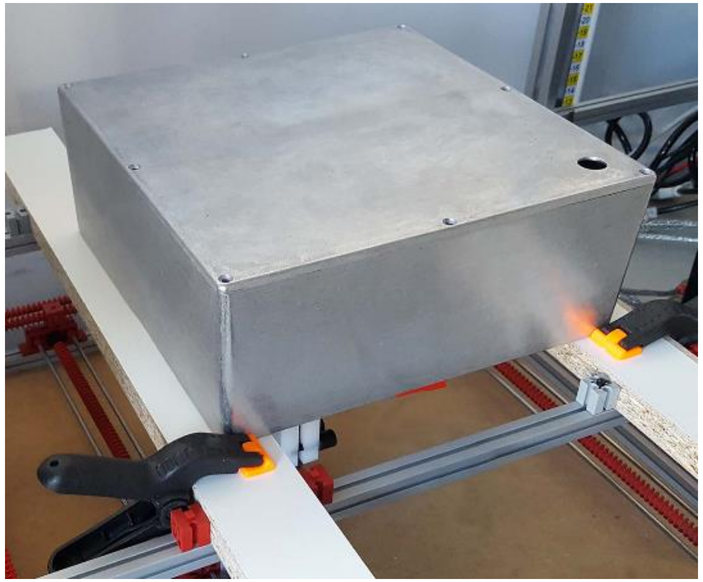
➢ Test modes

✓Memory test OFF

✓Memory test ON. Memory intensive process where random integer numbers are generated and will be saved in a random element in a large array allocated in the memory

Near-field measurement setup



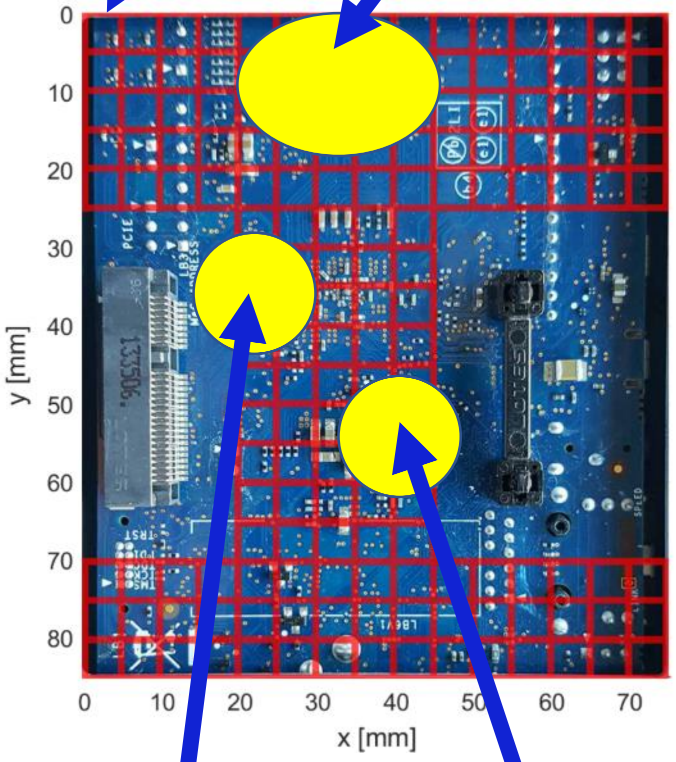


- ✓Langer near-field 10 mm probe
- ✓Two polarization of the probe: H_X and H_Y
- ✓Scanning area 75 x 85 mm
- ✓5 mm scanning step
- ✓4 mm distance between PCB and probe
- ✓13 GHz Oscilloscope LeCroy SDA 813Zi-A
- ✓2.5 GSa/s sampling frequency
- ✓5 MSa data length

Power hot spots of the DUT

Mesh grid

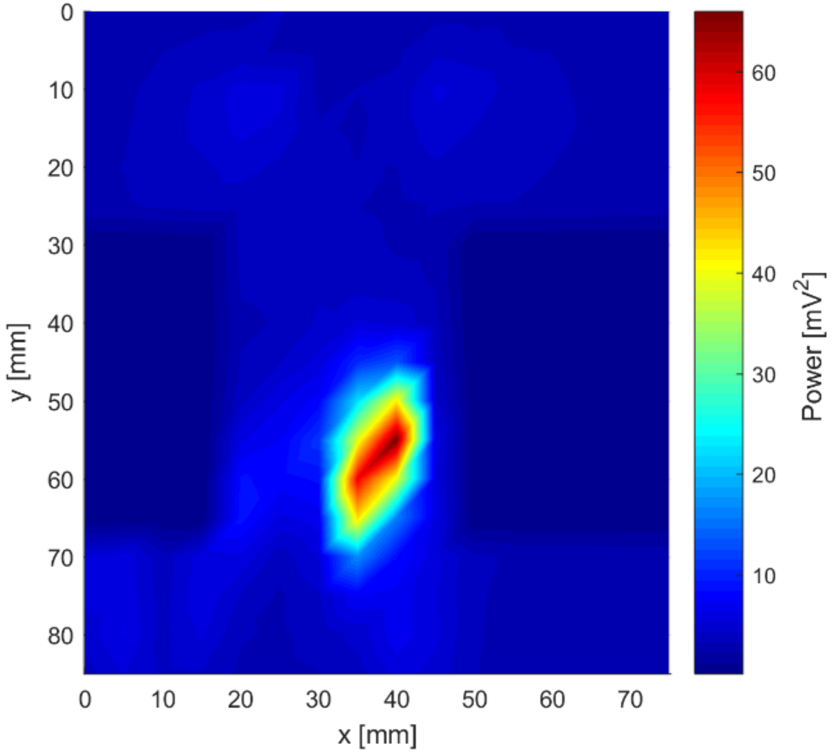
Memory



Clock

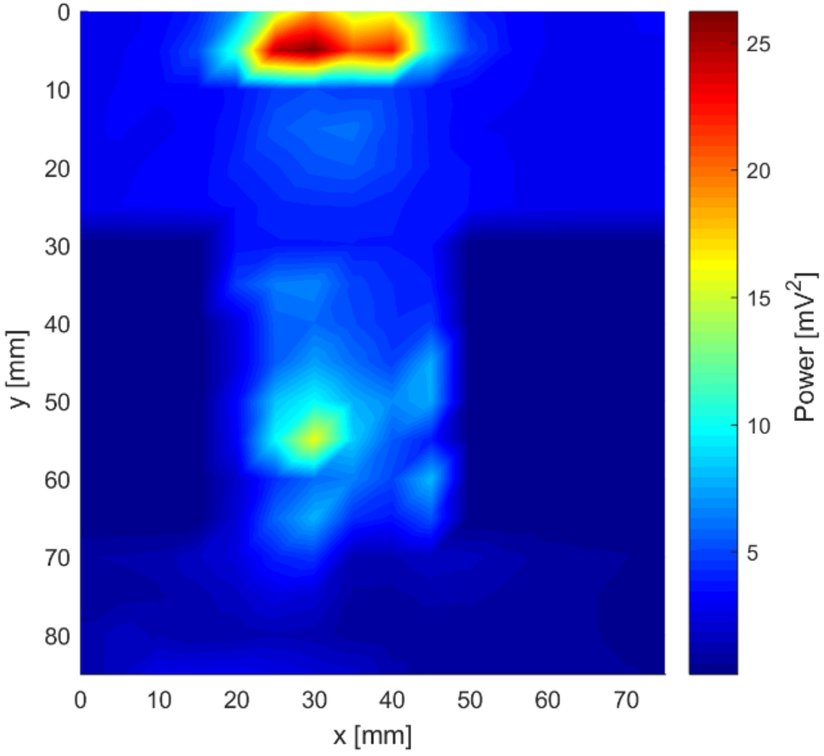
SMPS

➢ H_X polarization



✓ Power level 63 mV^2

➢ H_Y polarization

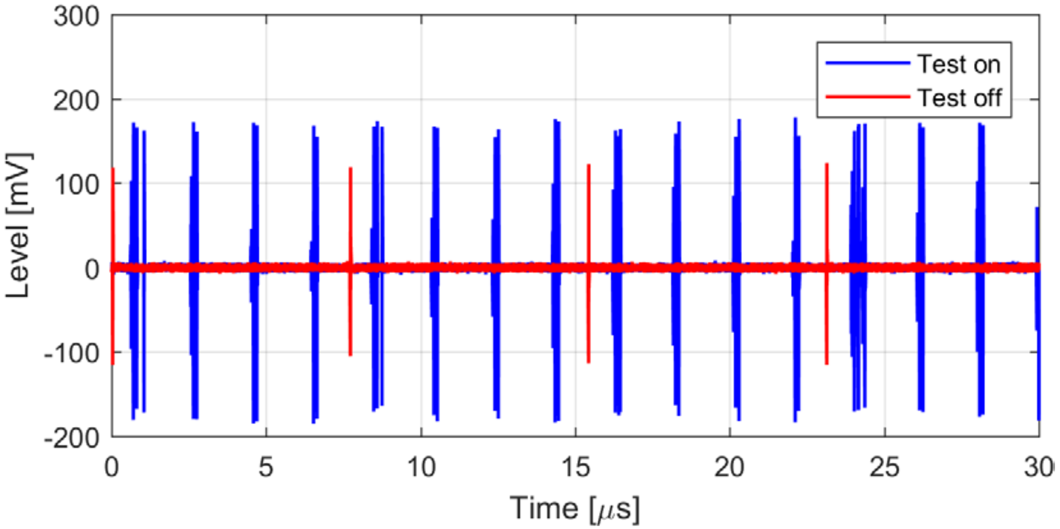


✓ Power level 27 mV^2

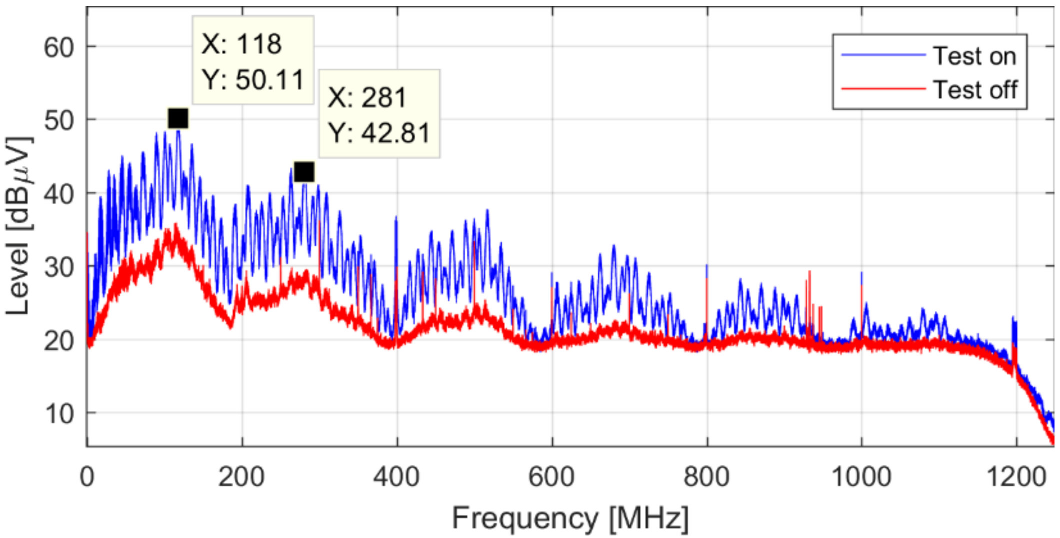
Memory hot spot

- ✓Measured signals are nonperiodic
- ✓Memory test signals are random
- ✓ Maximum of the PS at 118 MHz

➢ Measured signals



➢ Power spectrum



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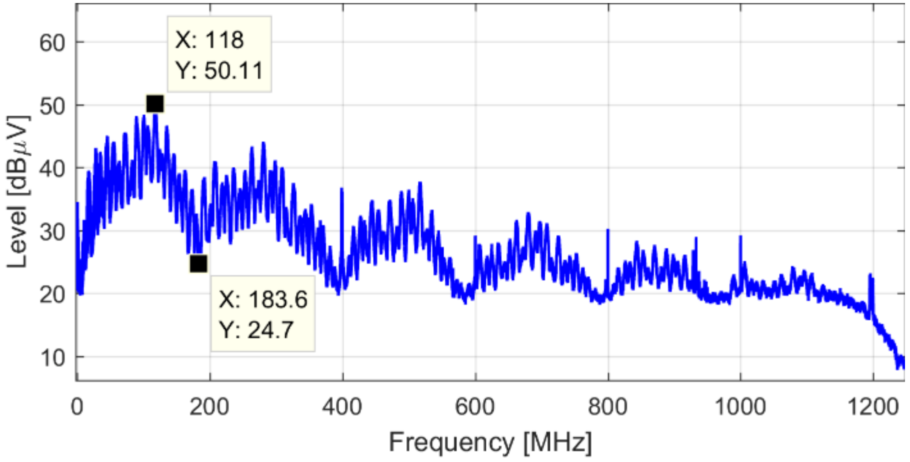
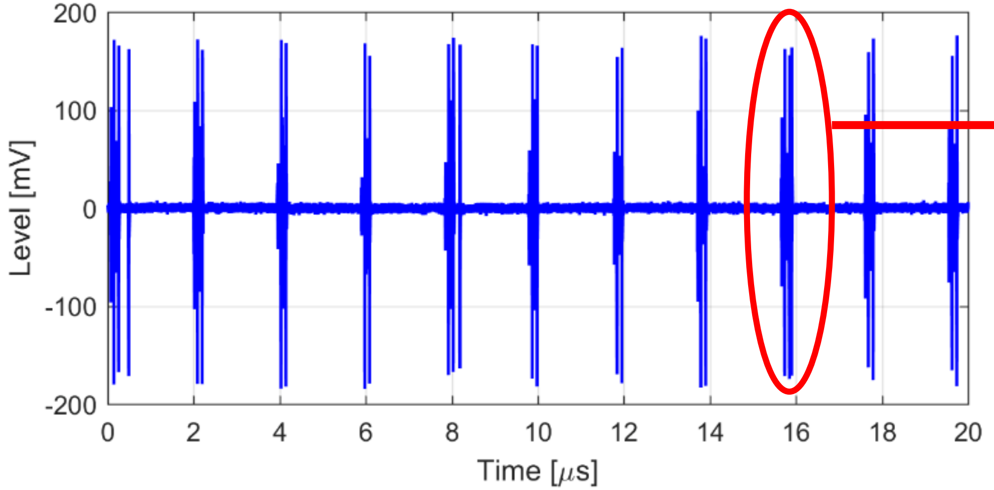
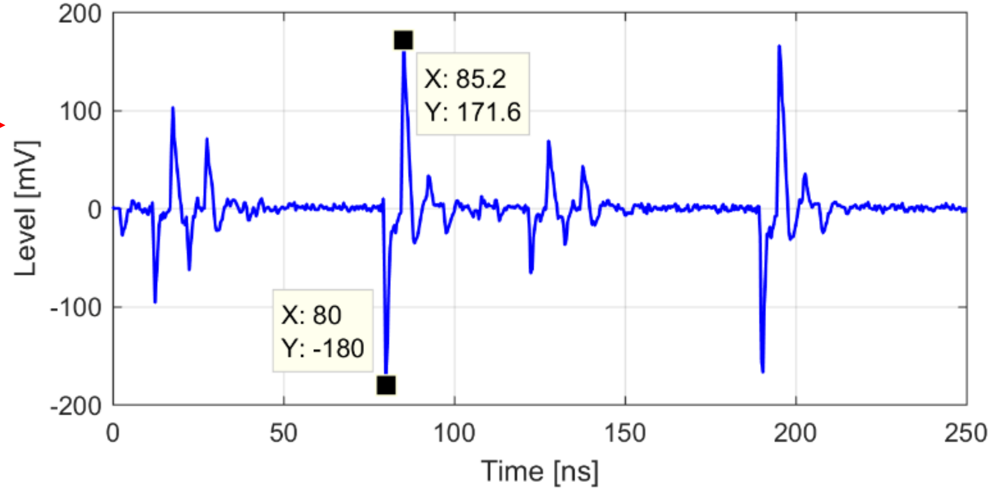
Memory test on



- ✓Bit duration is 5.2 ns
- ✓The shape of pulses is identical
- ✓Memory test process is cyclostationary





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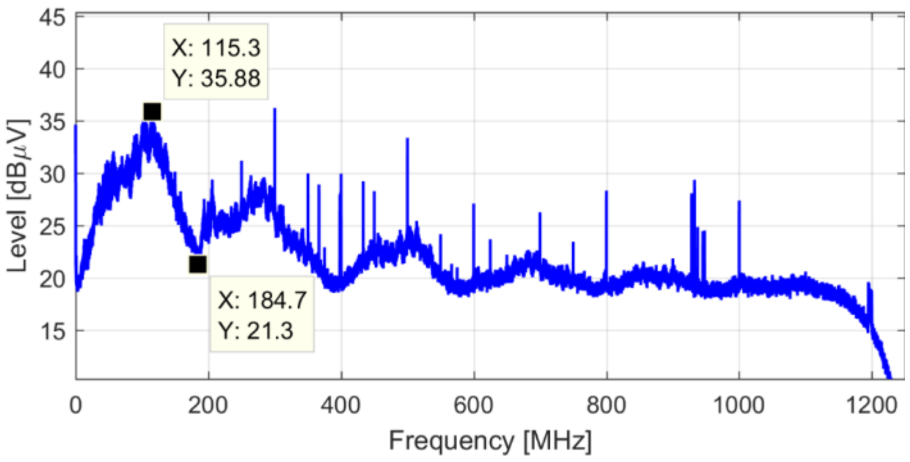
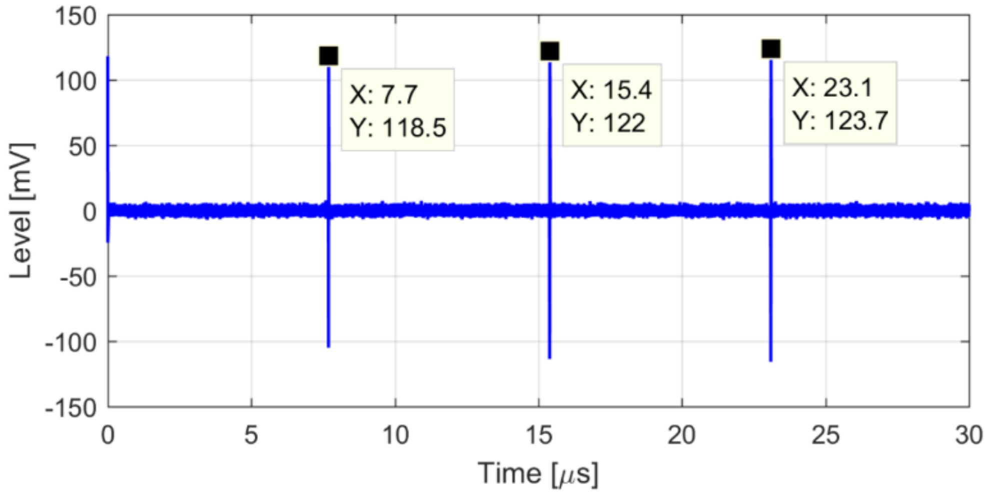
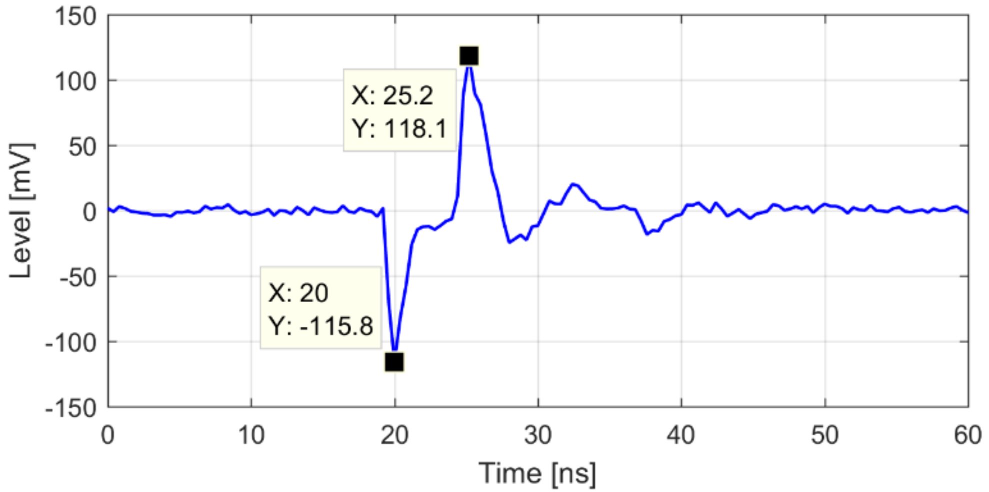
Memory test off



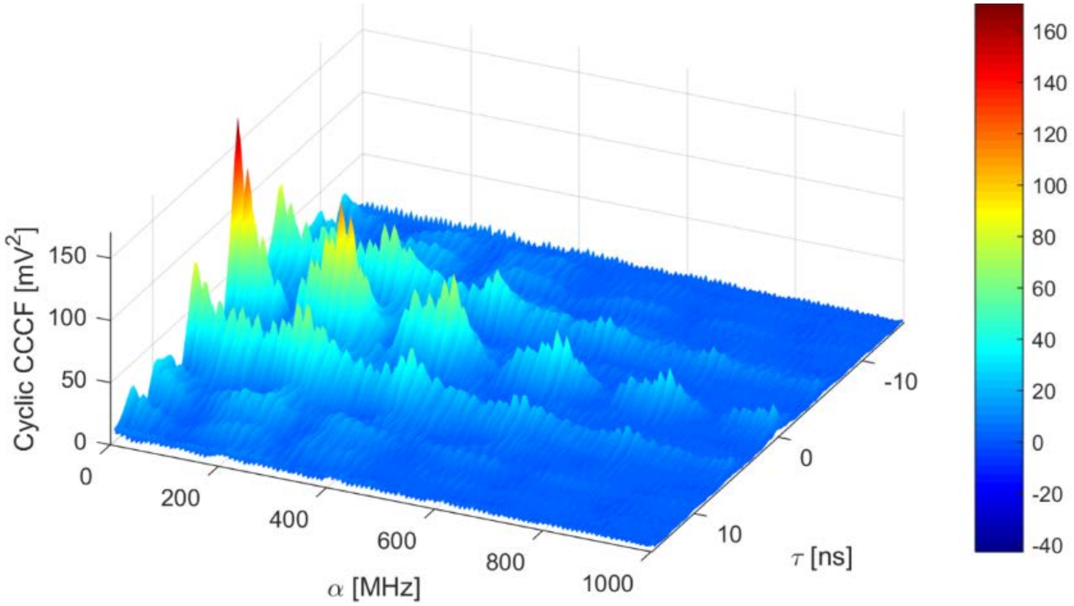
- ✓Pulse duration is 5.2 ns
- ✓Sequence of single pulses
- ✓Period of signal is 7.7 mks

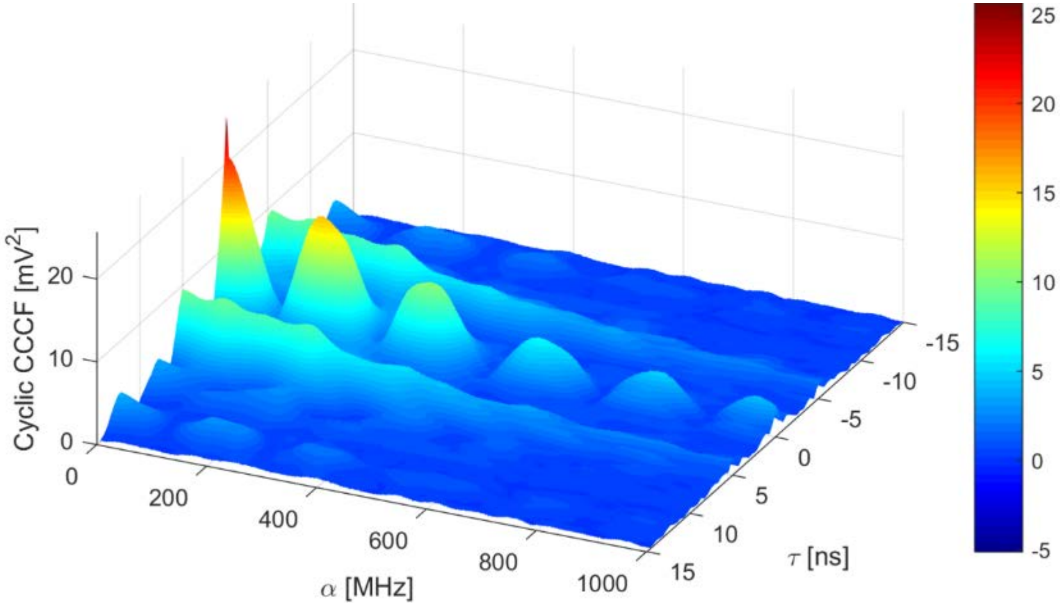




Cyclic auto-correlation cumulant functions

- ➢ Memory test on
- ➢ Memory test off





- ✓ Power level 165 mV^2
- ✓ Power level 25 mV^2

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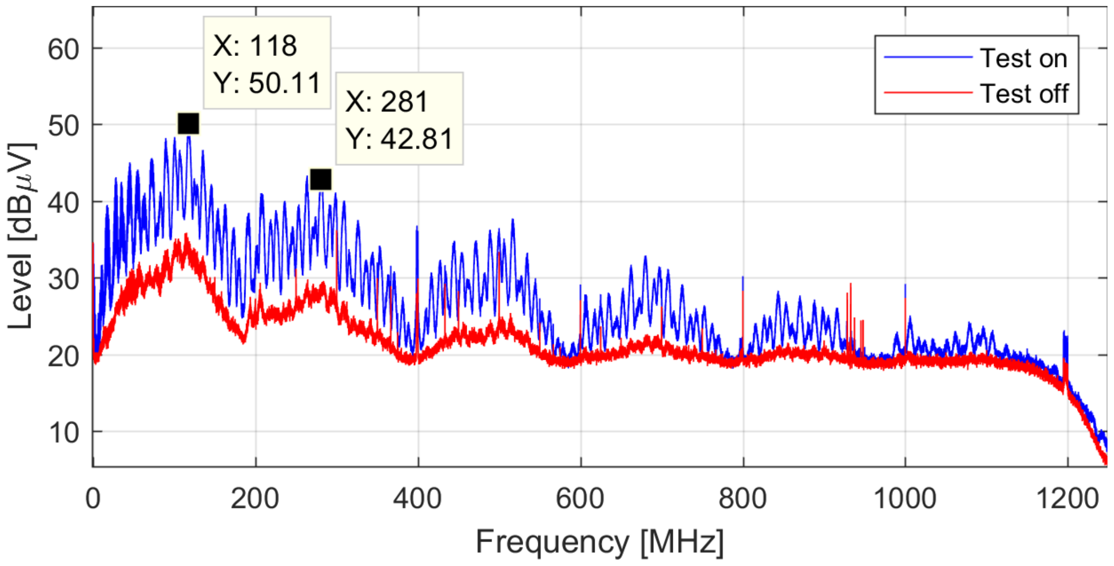
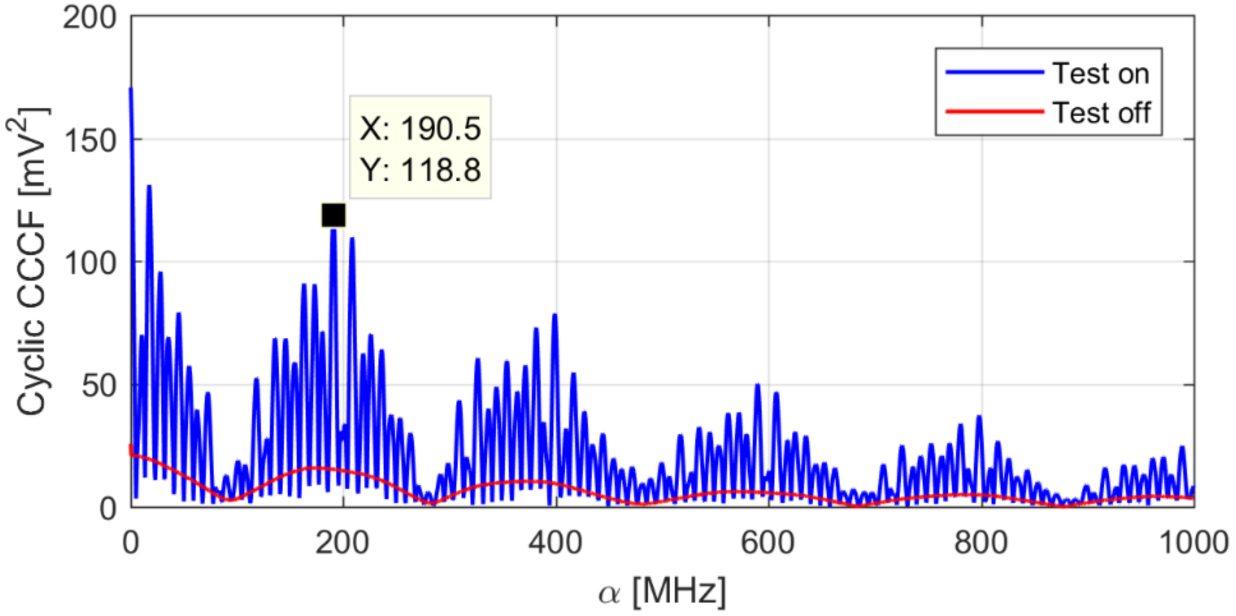
Cyclic auto-correlation cumulant functions

➢ **Power spectrum**



➢ **Cyclic CCCF**



✓ **Maximum of cyclic CCCF corresponds to the cyclic frequency 190.5 MHz**

✓ **Cyclic frequency is suppressed in the power spectrum**

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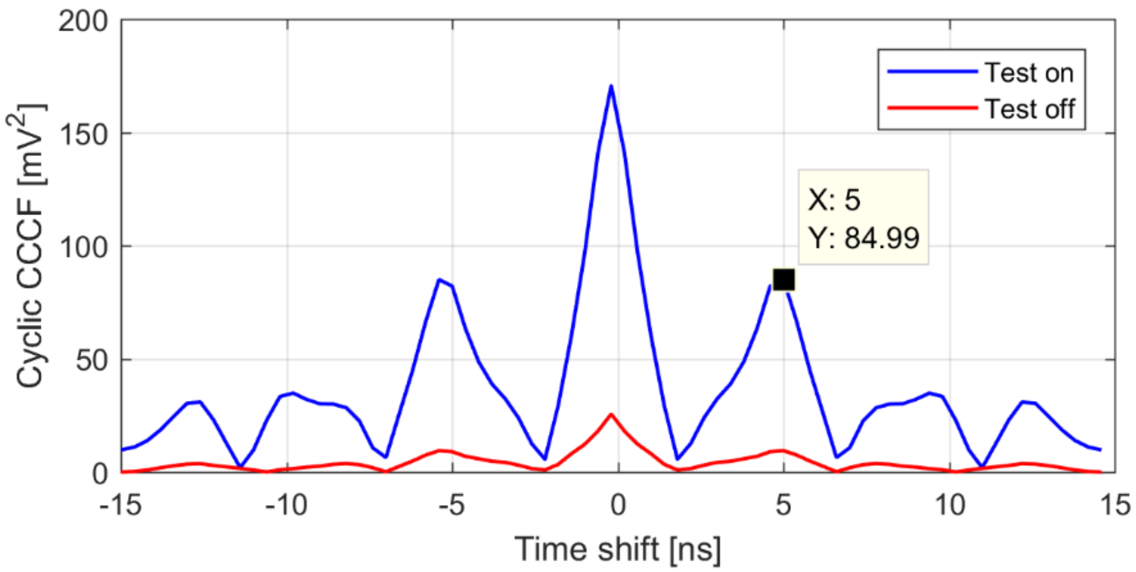
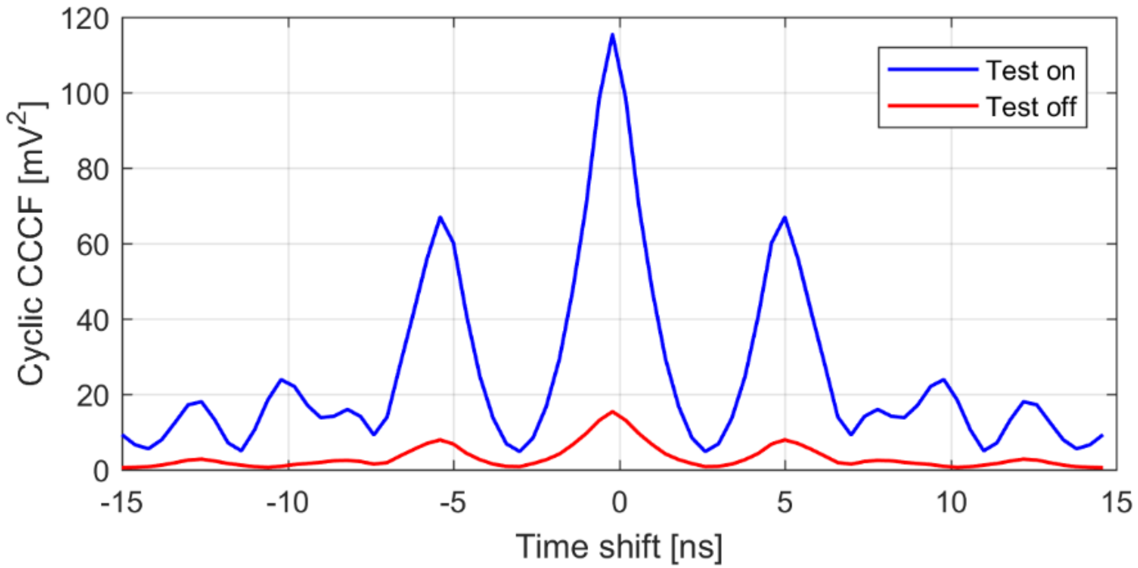
Cyclic cross-correlation cumulant functions

➢ Cyclic frequency α = 0



➢ Cyclic frequency α = 190.5 MHz

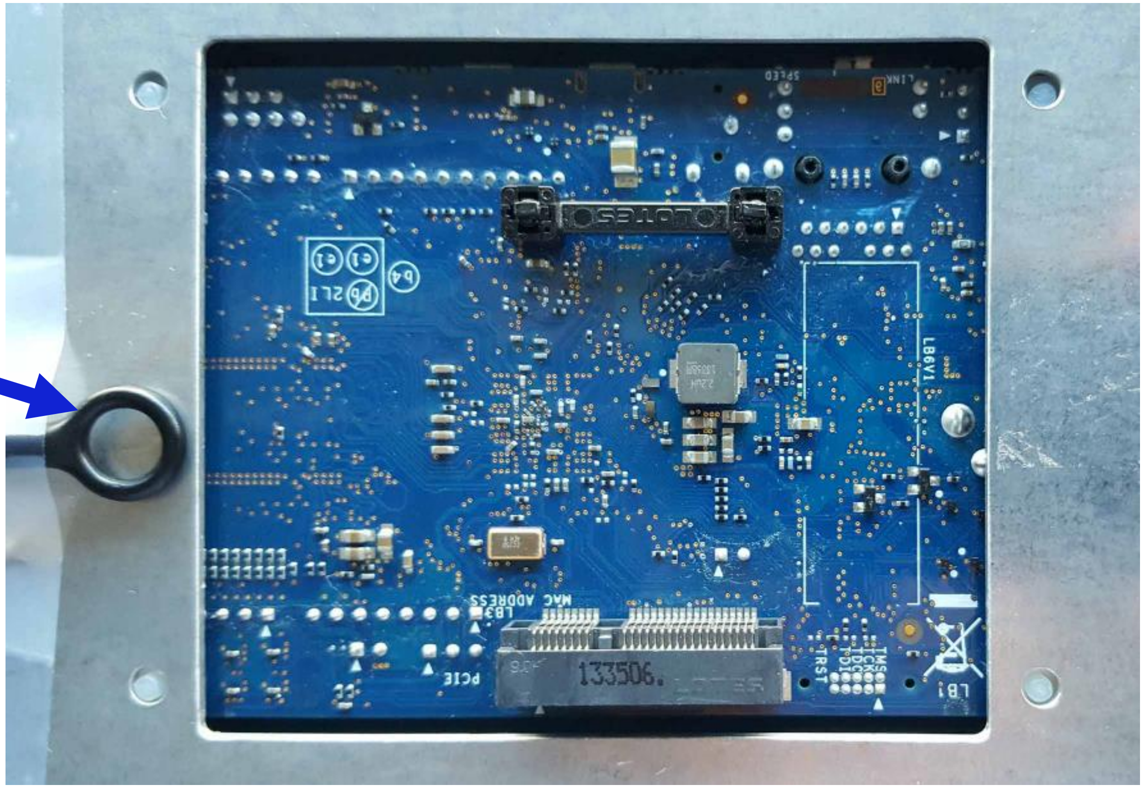


✓Correlation interval corresponds to the pulse duration 5.2 ns

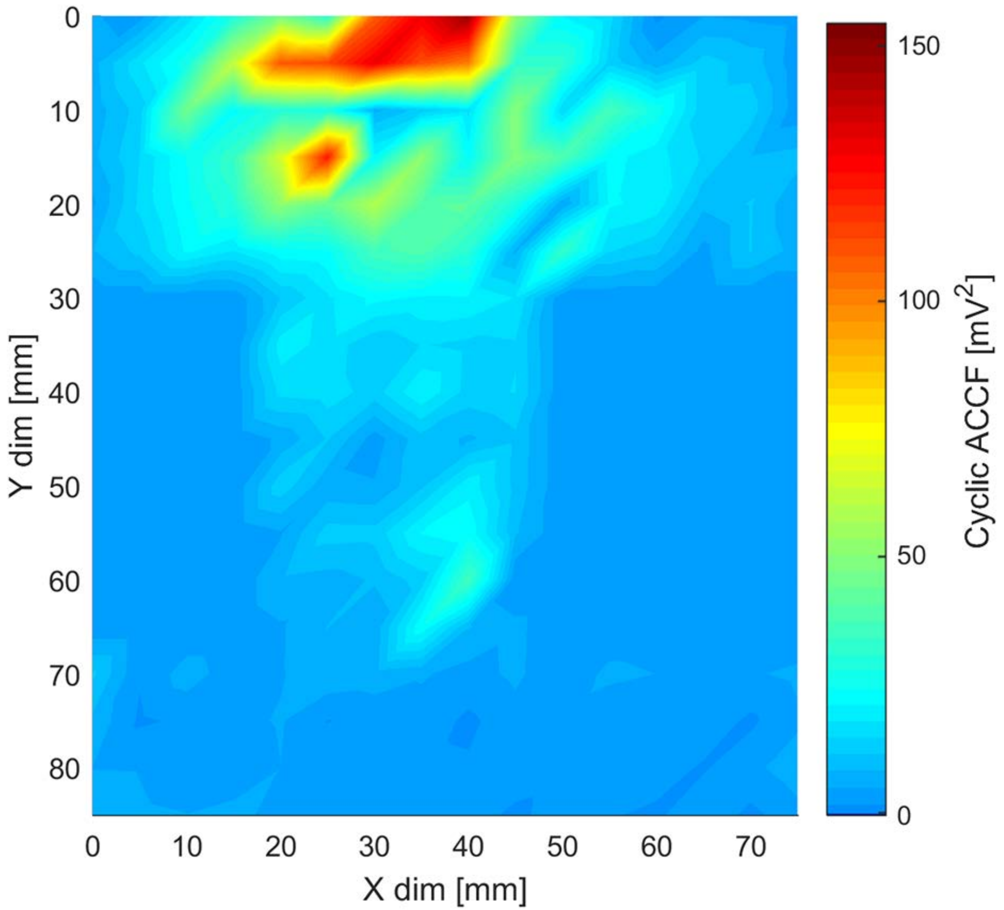
✓Both slices are nearly identical

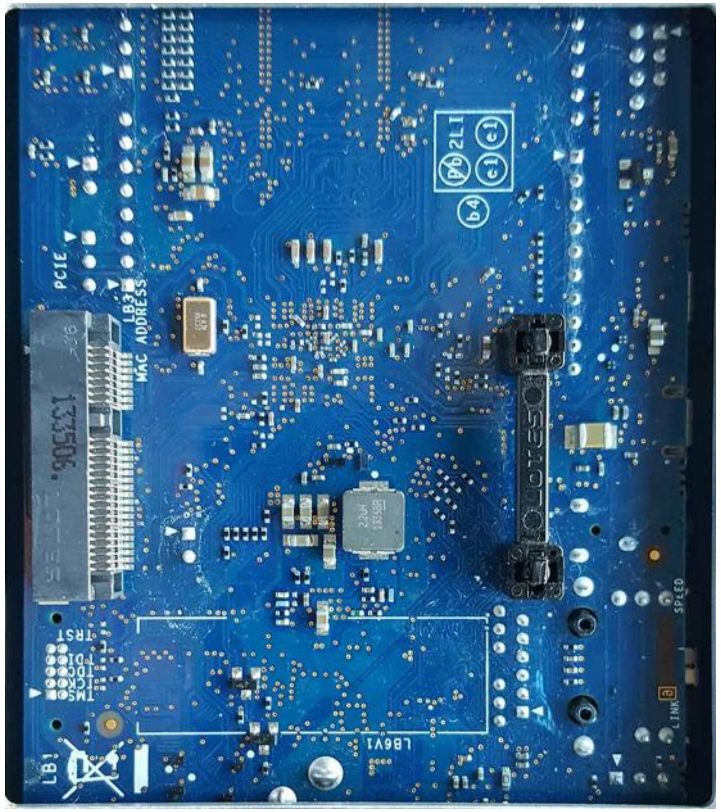
Spatial distribution of the cyclic CCCF

Reference probe



Spatial distribution of the cyclic CCCF



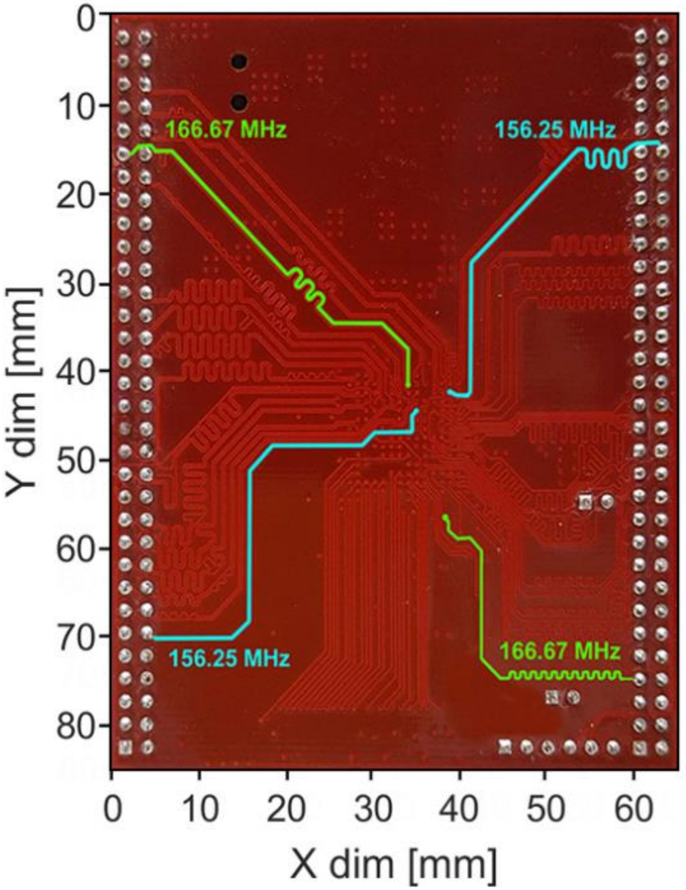


Conclusion Arduino

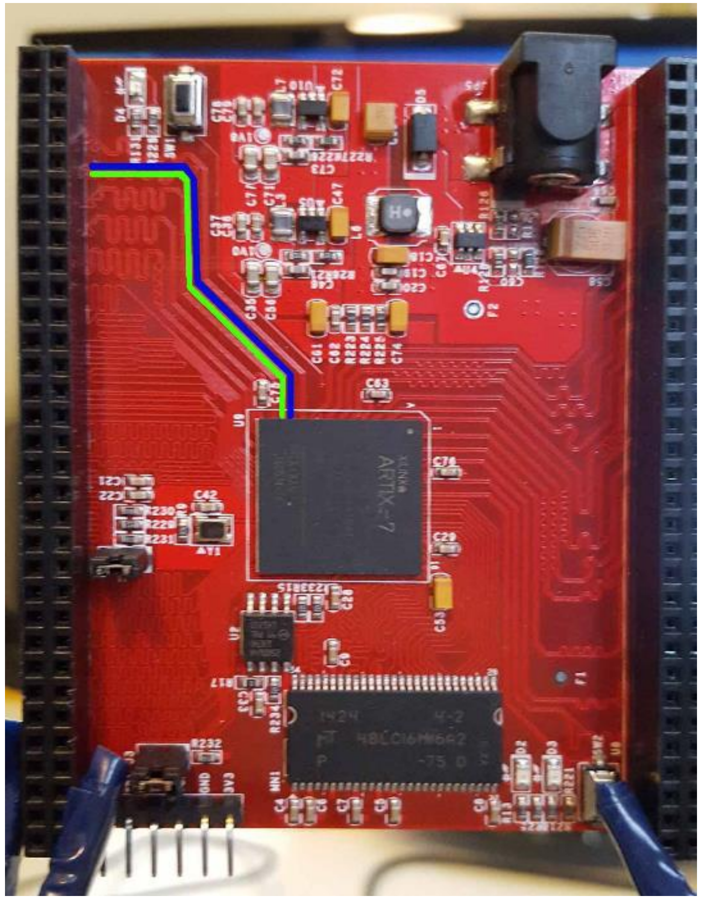
- **Frequency, time and spatial characterization of the physical radiated sources have been obtained**
- **Characterization of the random data signals reveals hidden cyclic frequencies of the sequence**
- **Localization of the physical radiated sources of the DUT was performed**

Device under test

➢ **Xilinx FPGA Development Board Artix-7 XC7A35T**

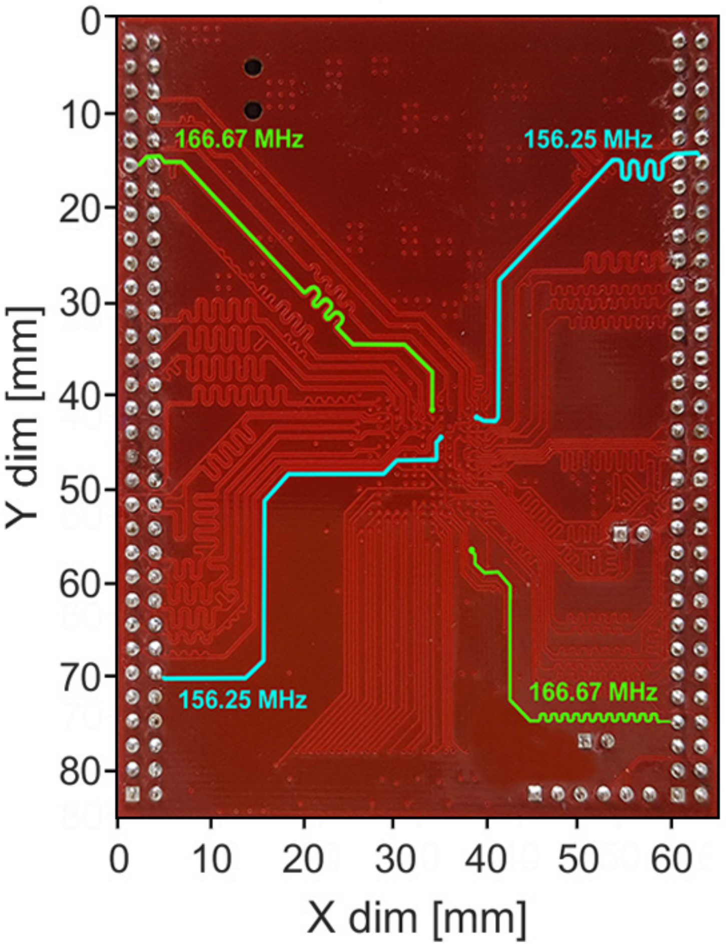


✓**Bottom side**

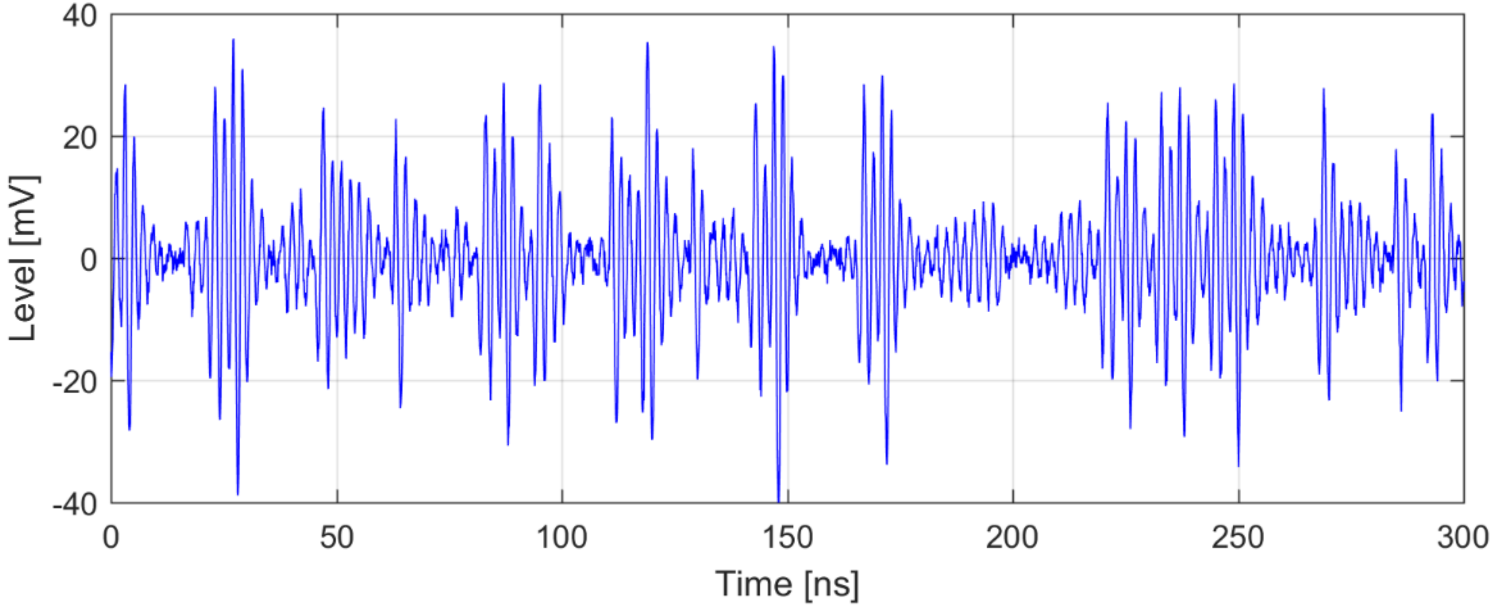


✓**Top side**

Experimental results



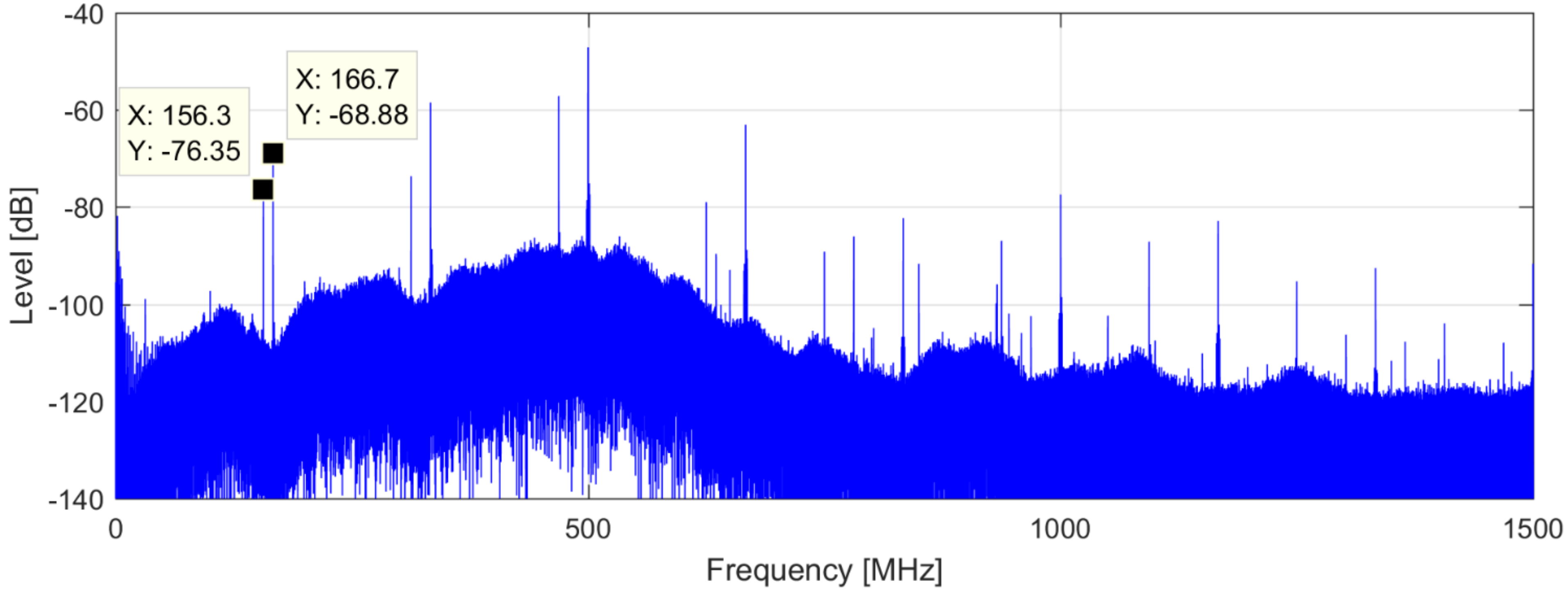
➢ **Near-field measured signal**



✓**Bit frequencies are 166.67 MHz and 156.25 MHz**

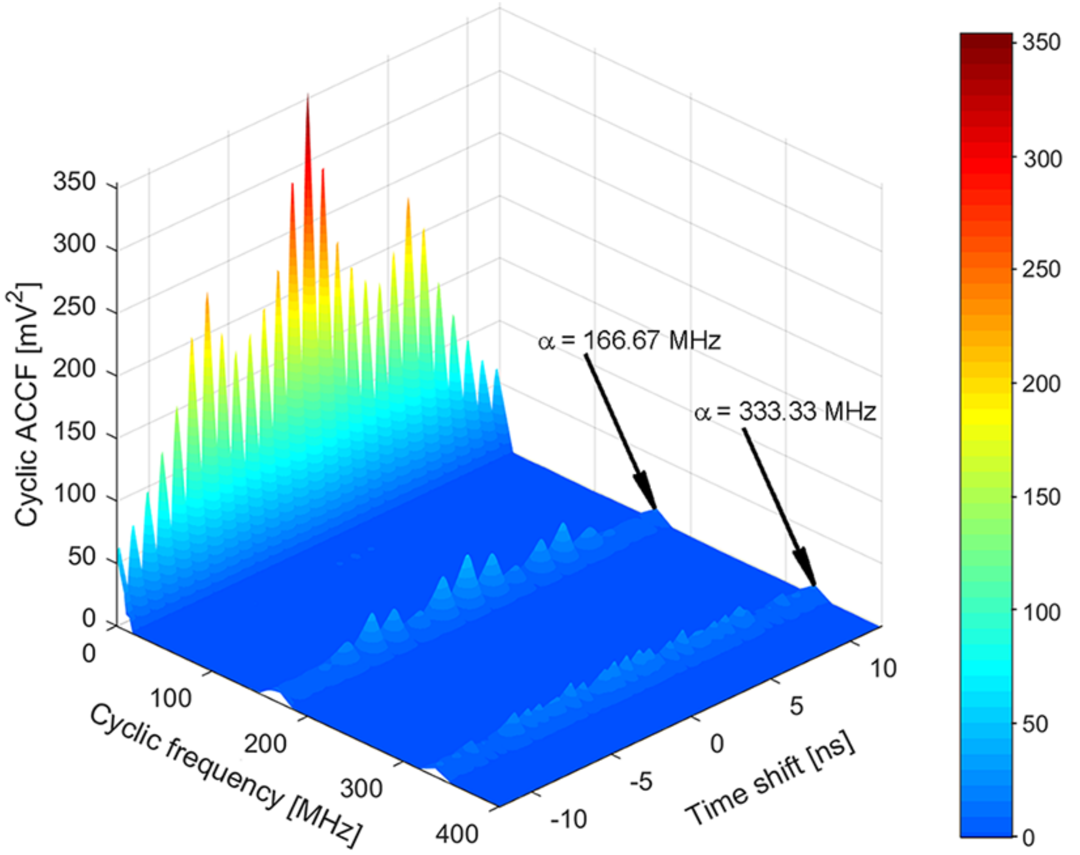
Experimental results

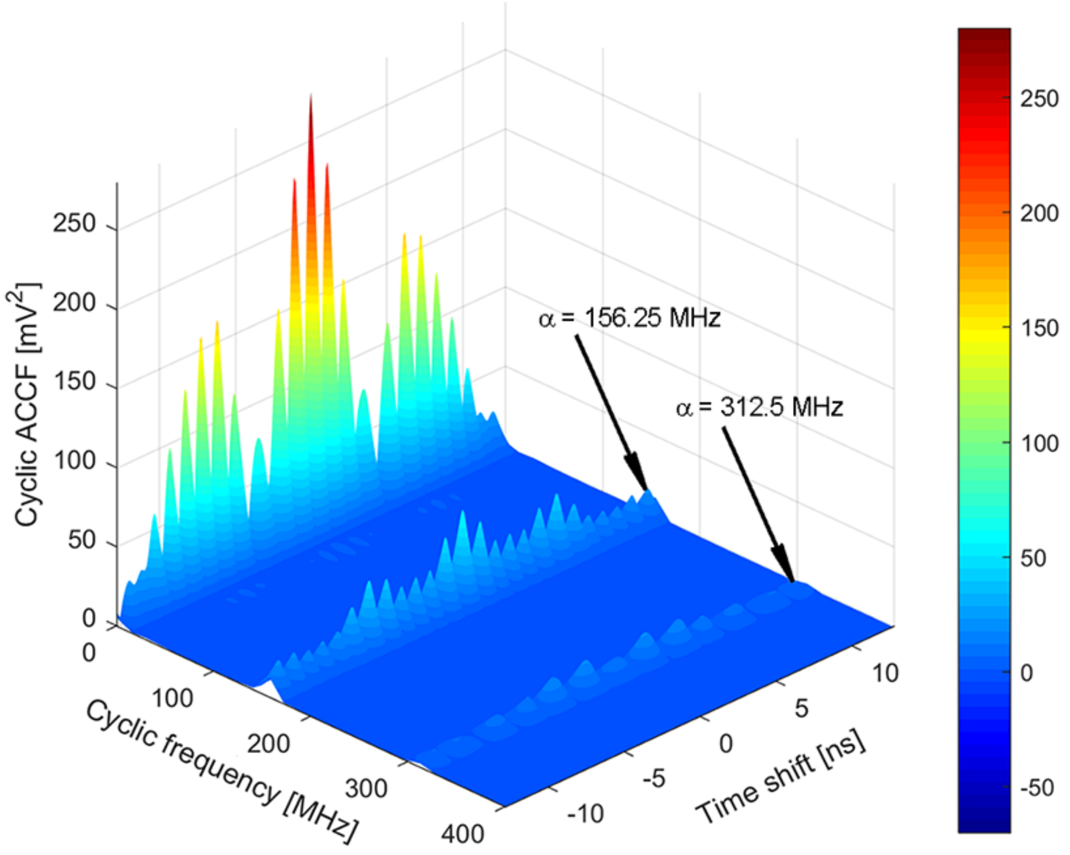
➢ **Amplitude spectrum of the measured signal**



Experimental results

➢ **Cyclic autocorrelation cumulant functions**



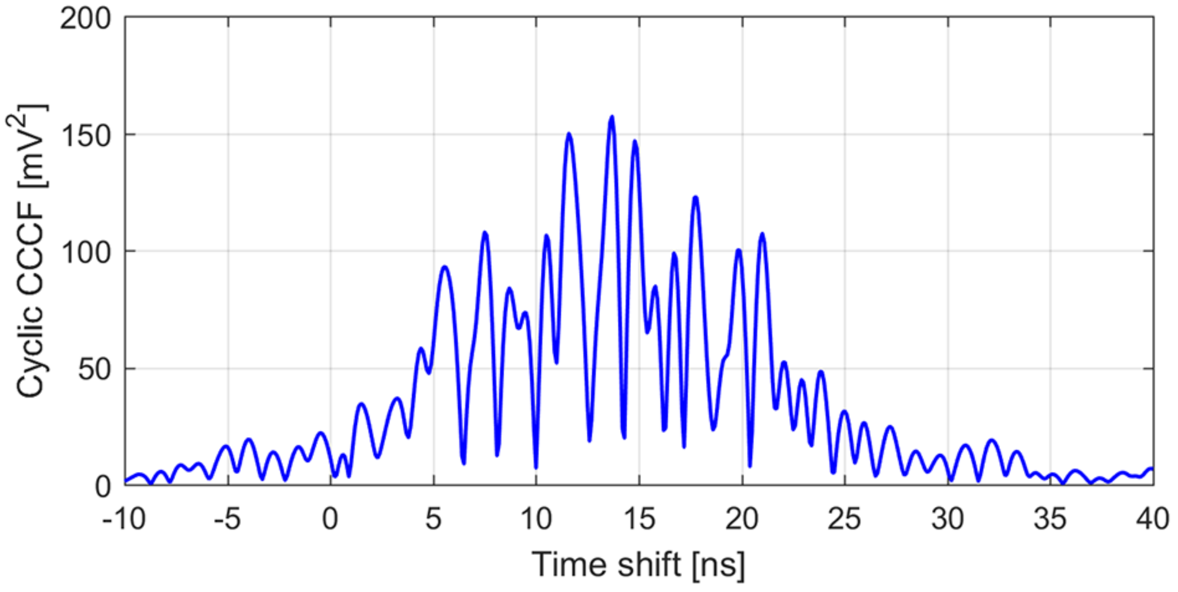


✓ $\alpha_1 = 1/T_{bit1} = 166.67$ MHz

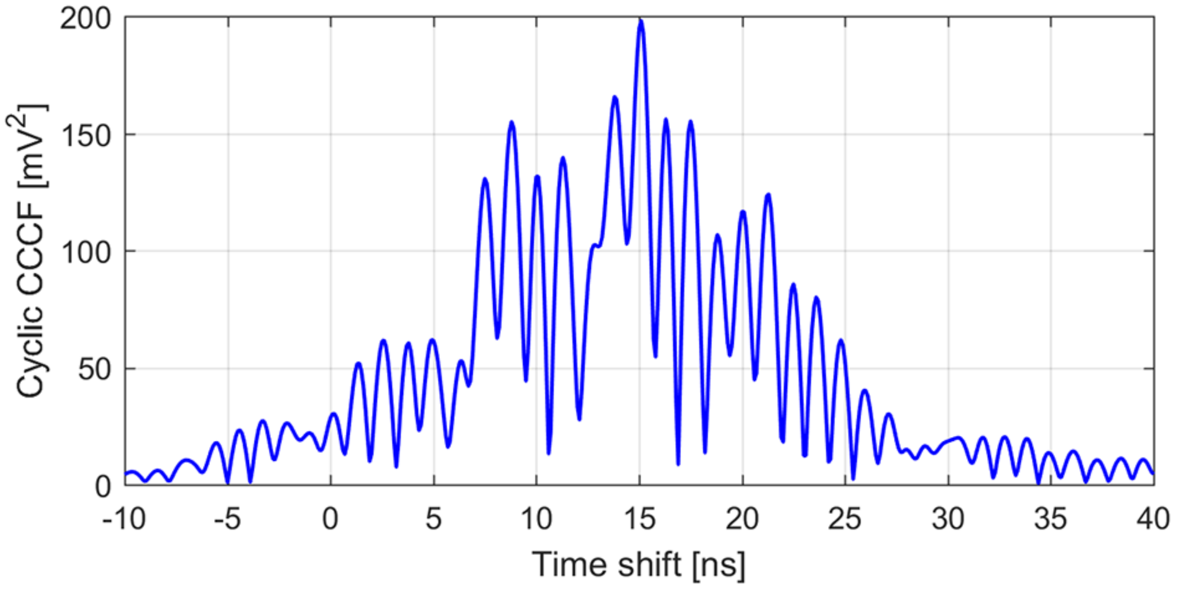
✓ $\alpha_2 = 1/T_{bit2} = 156.25$ MHz

Experimental results

➢ Cyclic cross-correlation cumulant functions



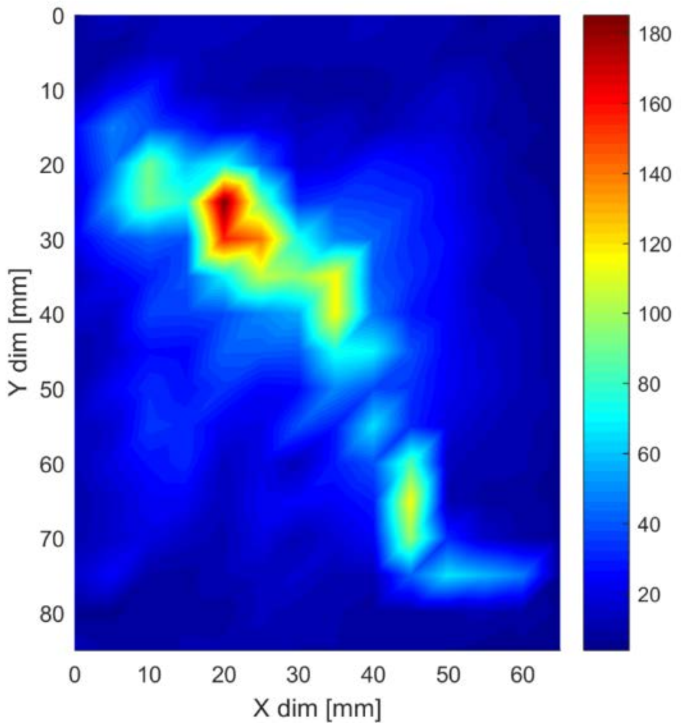
✓α_1 = 166.67 MHz

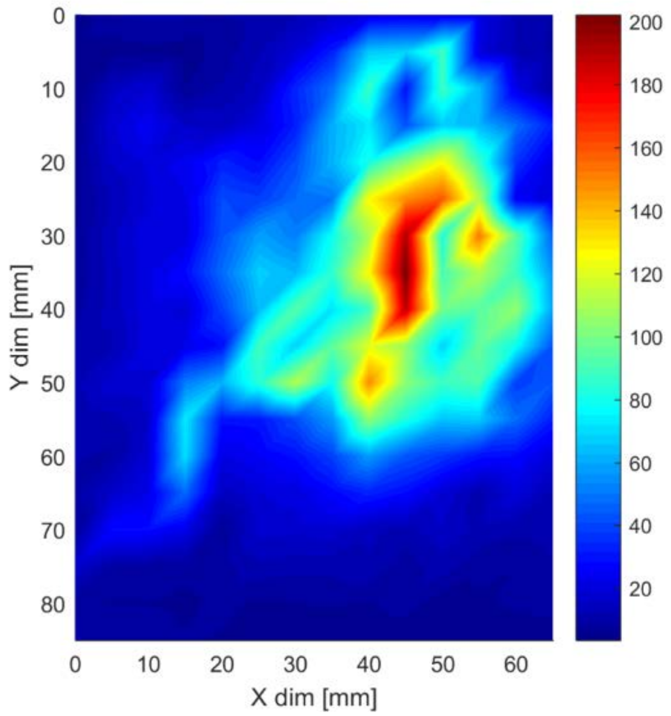


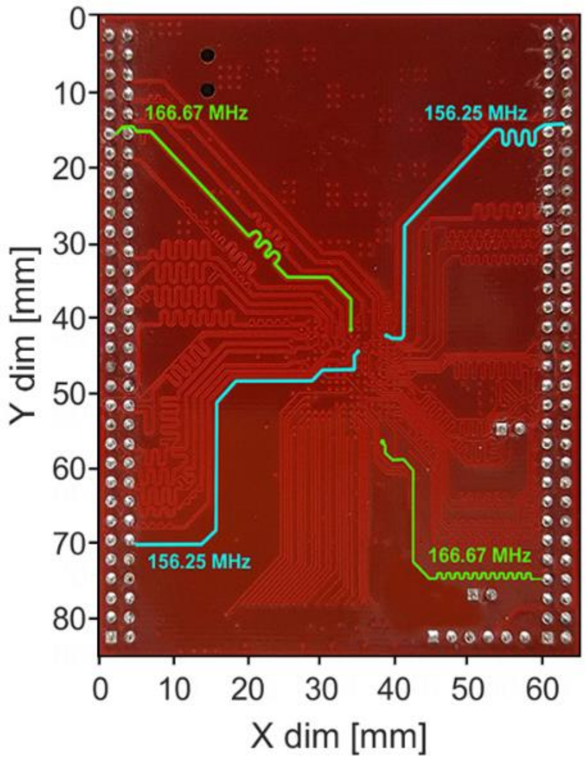
✓α_2 = 156.25 MHz

Experimental results

➢ **Spatial distribution of cyclic CCCF**



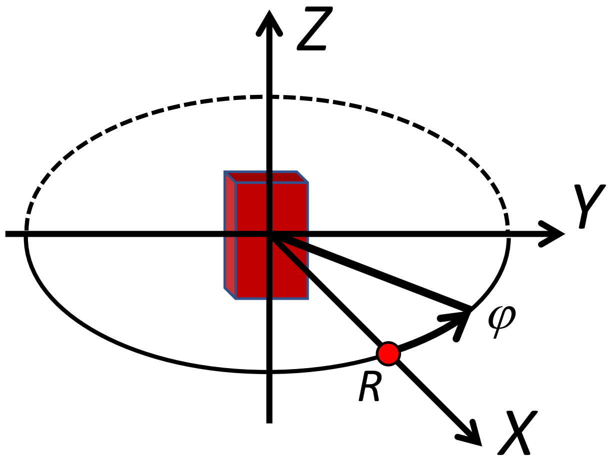


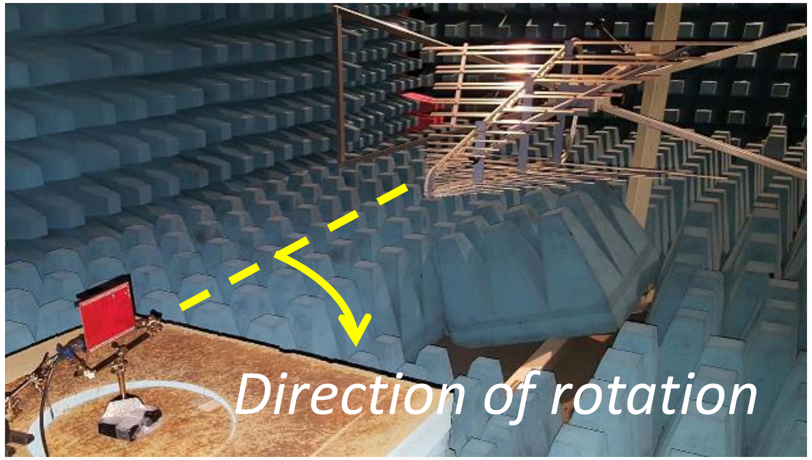


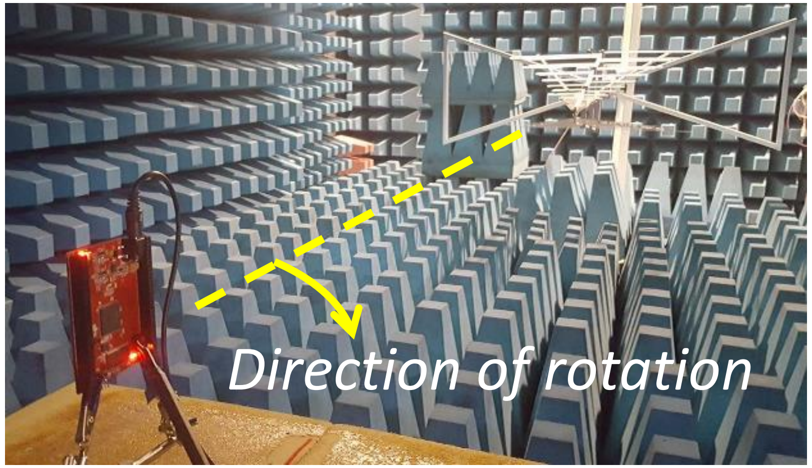
✓ α_1 = 166.67 MHz

✓ α_2 = 156.25 MHz

Far-field measurement setup







✓ **Distance 1 m**

✓ **Distance 4 m**

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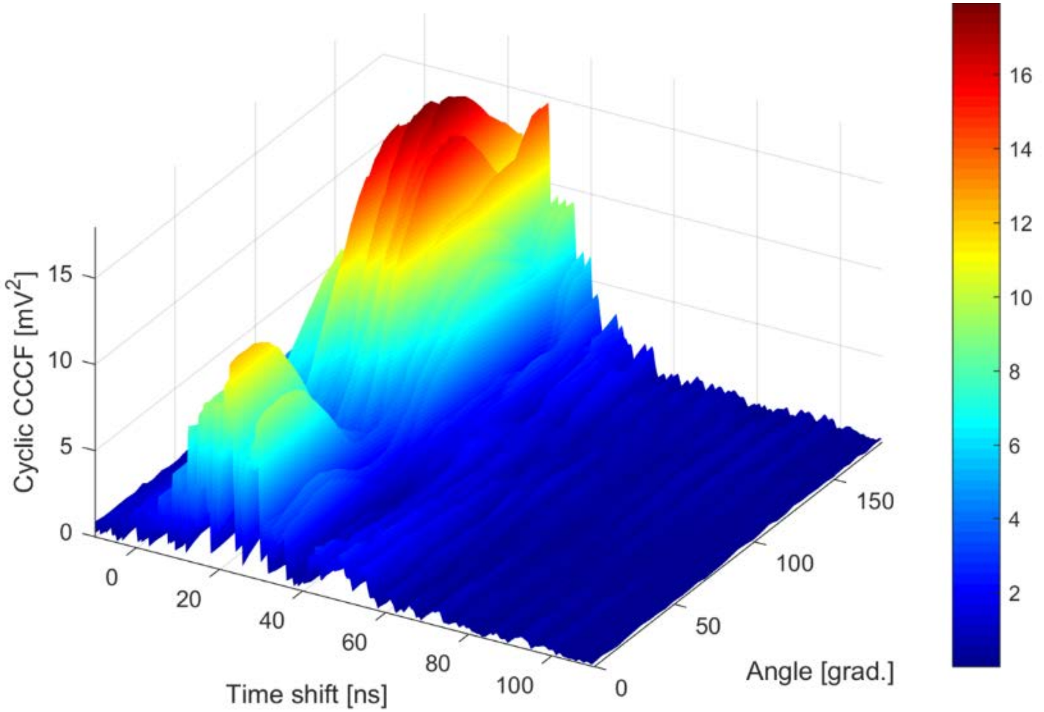
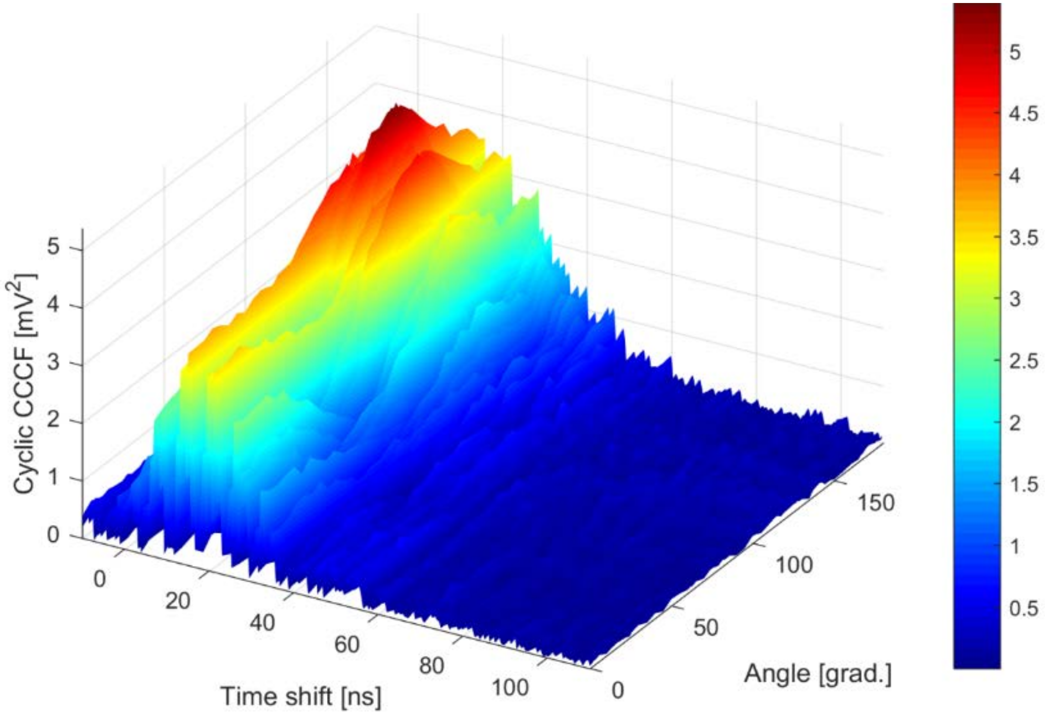
Far-field measurements

➢ α_1 = 166.67 MHz

➢ α_2 = 156.25 MHz





✓ Distance 1 m

✓ Horizontal orientation of antenna

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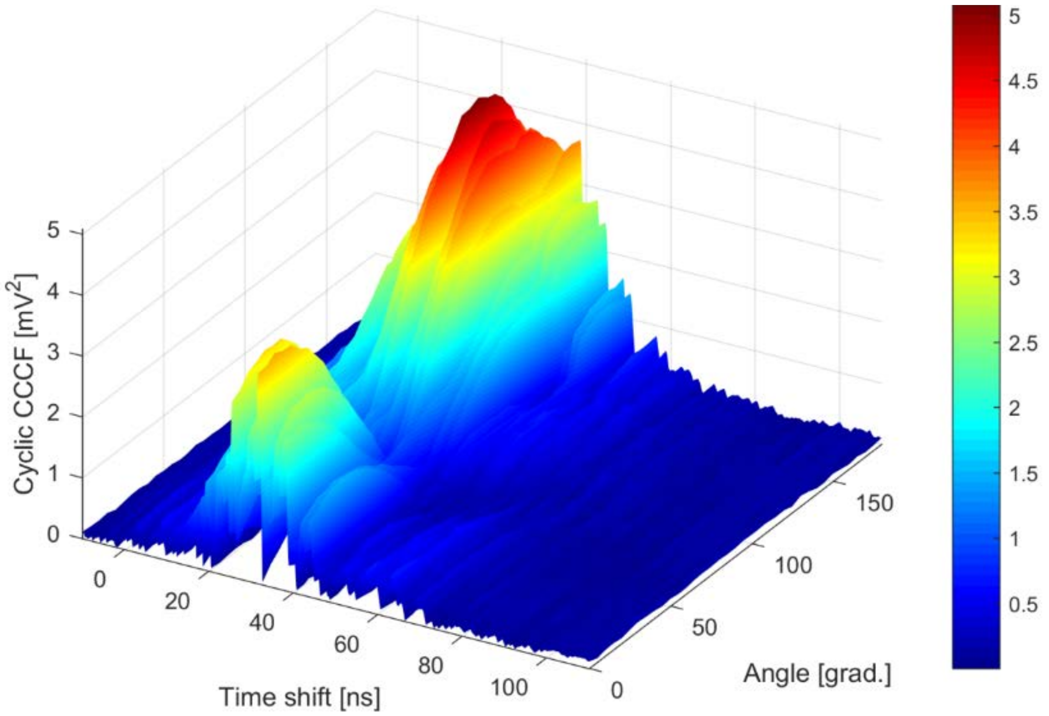
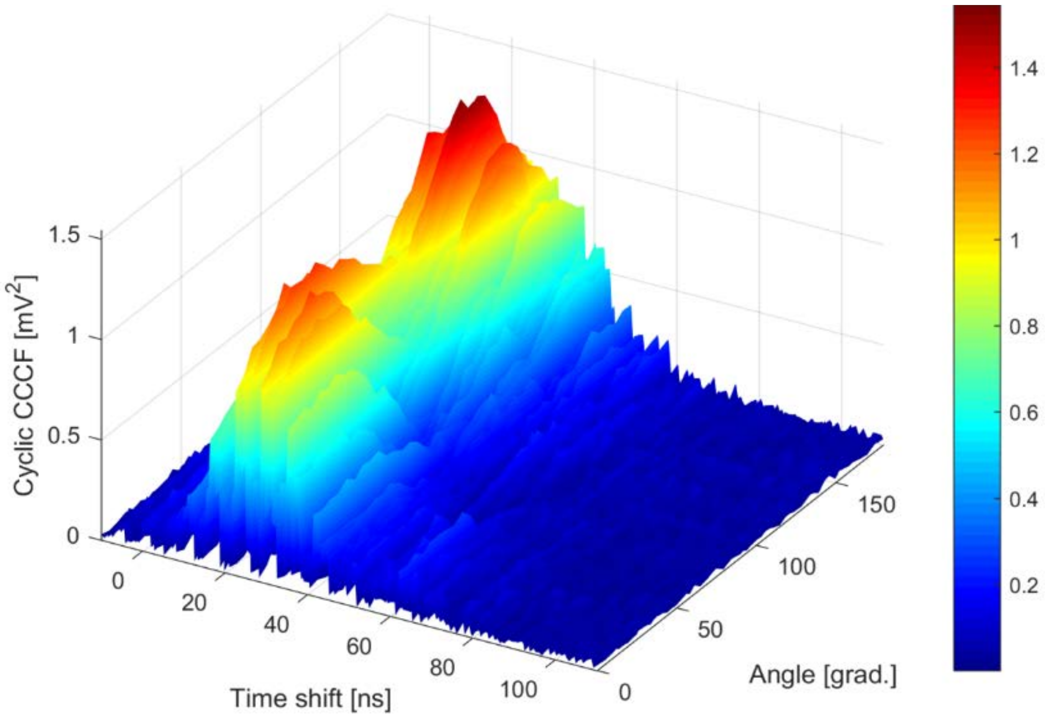
Far-field measurements

➢ α_1 = 166.67 MHz



➢ α_2 = 156.25 MHz



✓ Distance 4 m

✓ Horizontal orientation of antenna

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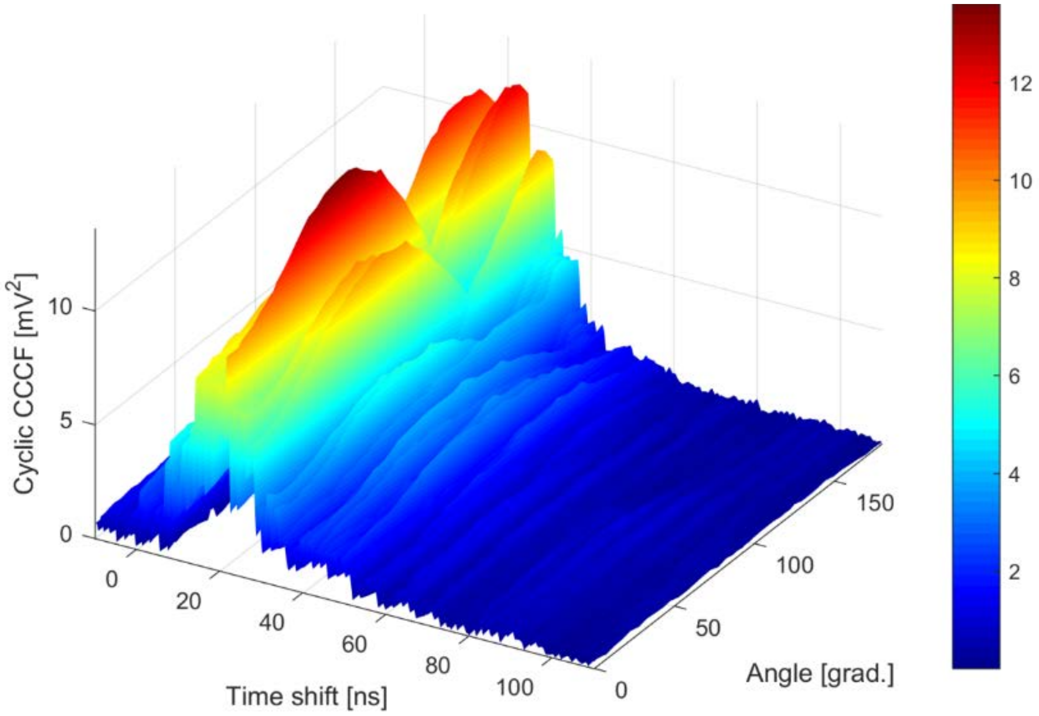
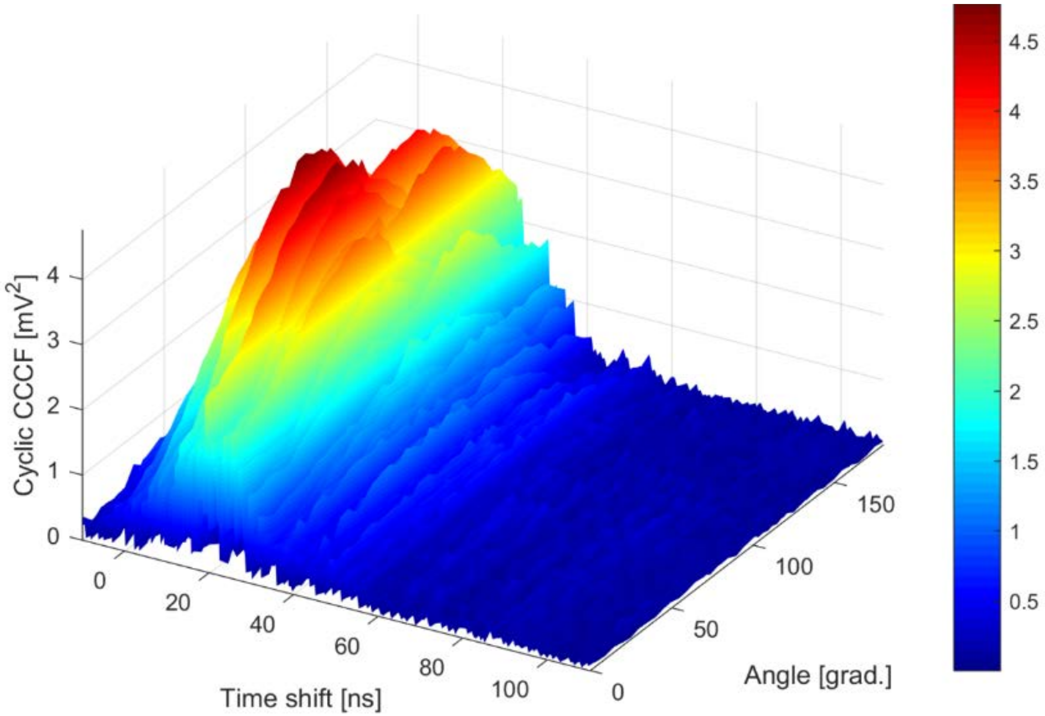
Far-field measurements

➢ α_1 = 166.67 MHz



➢ α_2 = 156.25 MHz

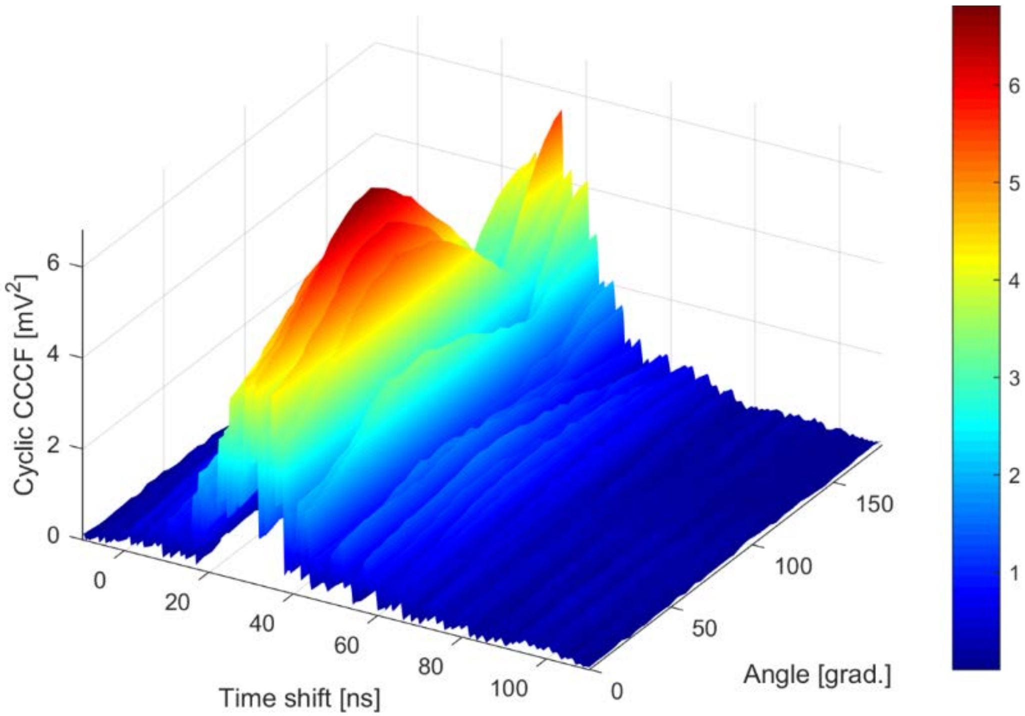


✓ Distance 1 m

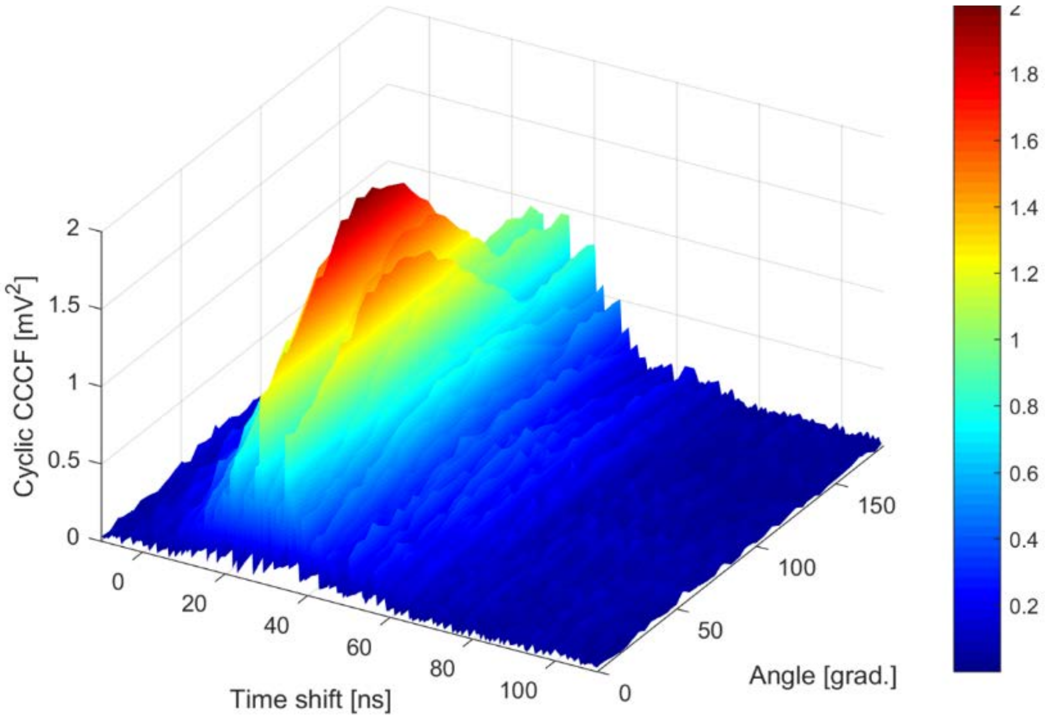
✓ Vertical orientation of antenna

Far-field measurements

➢ α_1 = 166.67 MHz



➢ α_2 = 156.25 MHz



✓ **Distance 4 m**

✓ **Vertical orientation of antenna**

Conclusion Artix

- Cyclic cross-correlation cumulant functions can be used for separation of two different random bit sequences with different cyclic frequencies
- Special-time distribution was used for the localization of the transmission lines over the DUT surface
- For cyclostationary source separation the position of the reference probe need to be chosen for sensing radiations of both sources

Questions?