

# Electromagnetic Modelling Techniques

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# OUTLINE:

- Introduction
- Numerical Modelling and Physical Experimentation
- Generic Classification of Numerical Methods
- Multiscale problems, Complexity and Uncertainty
- Conclusions and Outlook

# Introduction:

- The application of Computational Electromagnetics (CEM) to EMC and other EM design and analysis problems presents special difficulties: very **broadband** characterization (TD or FD models); large differences of electrical scale (**multi-scale models**); changing material properties (f-dependent **material models**); extreme **complexity** (embedded models, hybrid models); parameter uncertainty (**stochastic models**)...
- Experiments at high-frequencies and over a wide bandwidth are also very challenging, as calibration and the impact of environmental factors can affect the accuracy of measurements

# Numerical Modelling and Physical Experimentation:

- There is an unfortunate dichotomy between practitioners on the one hand who pride themselves as “practical, based on experience, hands on” and those on the other hand who pride themselves as “theoreticians”
- This, in my view is a false dichotomy. A good “theory” is a very “practical tool” as it encapsulates in a small number of postulates and general principles this essence of numerous experiences. Thus, it provides generality and the capacity to extrapolate from current experiences to future possibilities...a very powerful tool.
- BUT, our capacity to function securely as engineers cannot alone be build on abstractions-we need to temper our theoretical modelling work by observations and careful experiments. The key is to exploit the **synergies between experiments and simulation!**

# Some common aphorisms!

- Everyone believes the results of an experiment except the person who did it!
- No one believes the results of a simulation except the person who did it!

**Data alone does not confer knowledge...** it is the interpretation of the data and their placement into a wider context (a theory) that extracts meaning and allows us to formulate a proper response.

***...We need a broader context, a coherent framework, in which to interpret our experiences...***

**The formation of an engineer is and must be a multi-faceted affair, a blend of practical, experimental and theoretical “experiences”.**

Some examples of recurrent problems from EMC are given next:

## Issues with EMC tests:

- **Many aspects of the test environment are difficult to control**
- Screened rooms can never be completely damped but worthwhile improvements can be made
- **Partially damped rooms are a compromise between cost and accuracy**
- There are size limitations for TEM and GTEM cells
- **Open area test sites suffer from environmental problems and must be well designed**
- Proximity effects influence the calibration of antennas
- **Near-field effects make extrapolation difficult**

# Issues related to EMC Simulations:

- Relatively easy to isolate factors and establish what is critical in the response of a systems
- Possible to do “what if” experiments
- You can study system response well before you have a prototype thus anticipating problems
- Full diagnostics (a ring-side seat!)
- Can do numerical experiment which in real life are too expensive or dangerous to attempt
- It is easy to overlook significant factors which affect performance
- You can be overwhelmed by information!
- *Experimentation and simulation **together** can improve the understanding of EMC!*



**Practical experience is great**, except that in science and engineering we are often confronted with unfamiliar things and it is then very easy to make mistakes! We thus need a theoretical framework, a numerical model, to back up or otherwise our intuitions...

But, we must be aware that **models are devices for capturing particular insights**, not full-proof systems for predicting everything under all circumstances..."

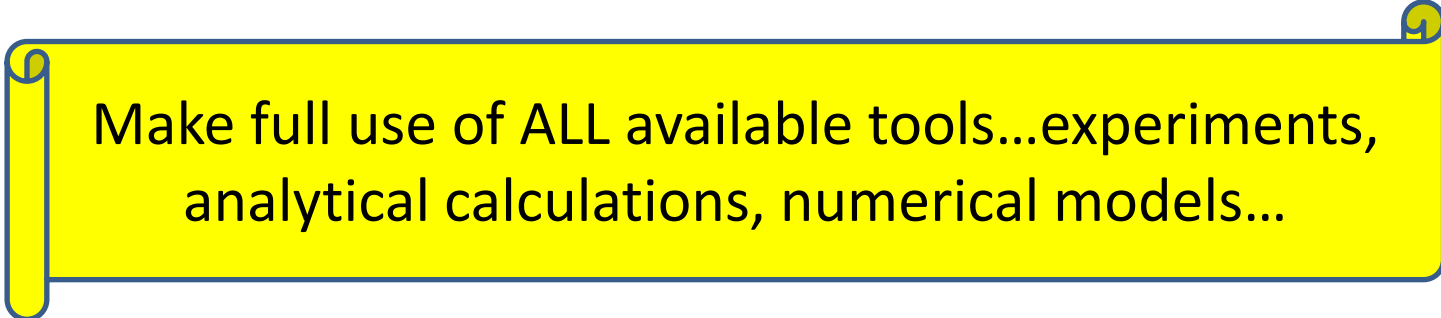
*All models are 'wrong', but some are useful!"*

The need to control costs, achieve designs right first time, and to minimise time to market, imply a greater use of simulation for certification and compliance.

**...relying on numerical models alone is unacceptable**

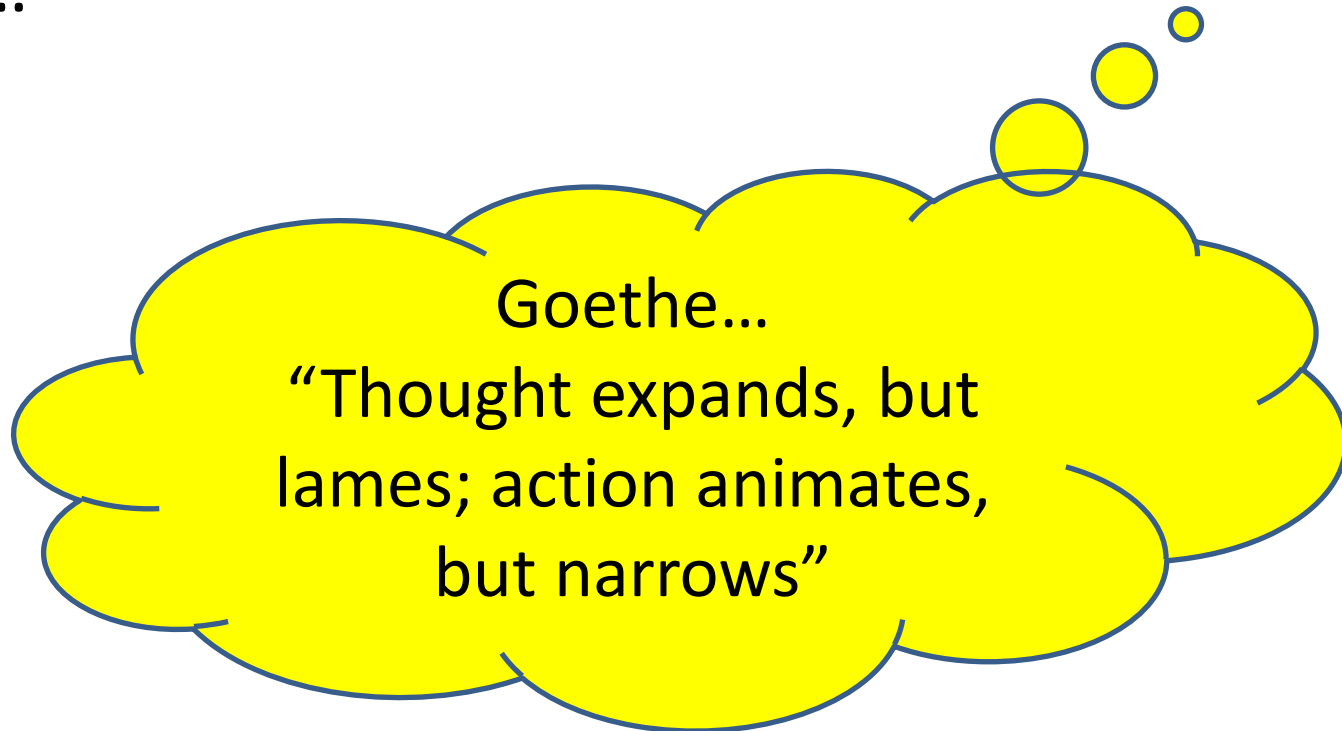
**...relying on experiments alone is impractical**

We will increasingly rely more on simulation and we must validate the software against carefully conducted basic experiments AND validate the models against whole system experimental testing



Make full use of ALL available tools...experiments, analytical calculations, numerical models...

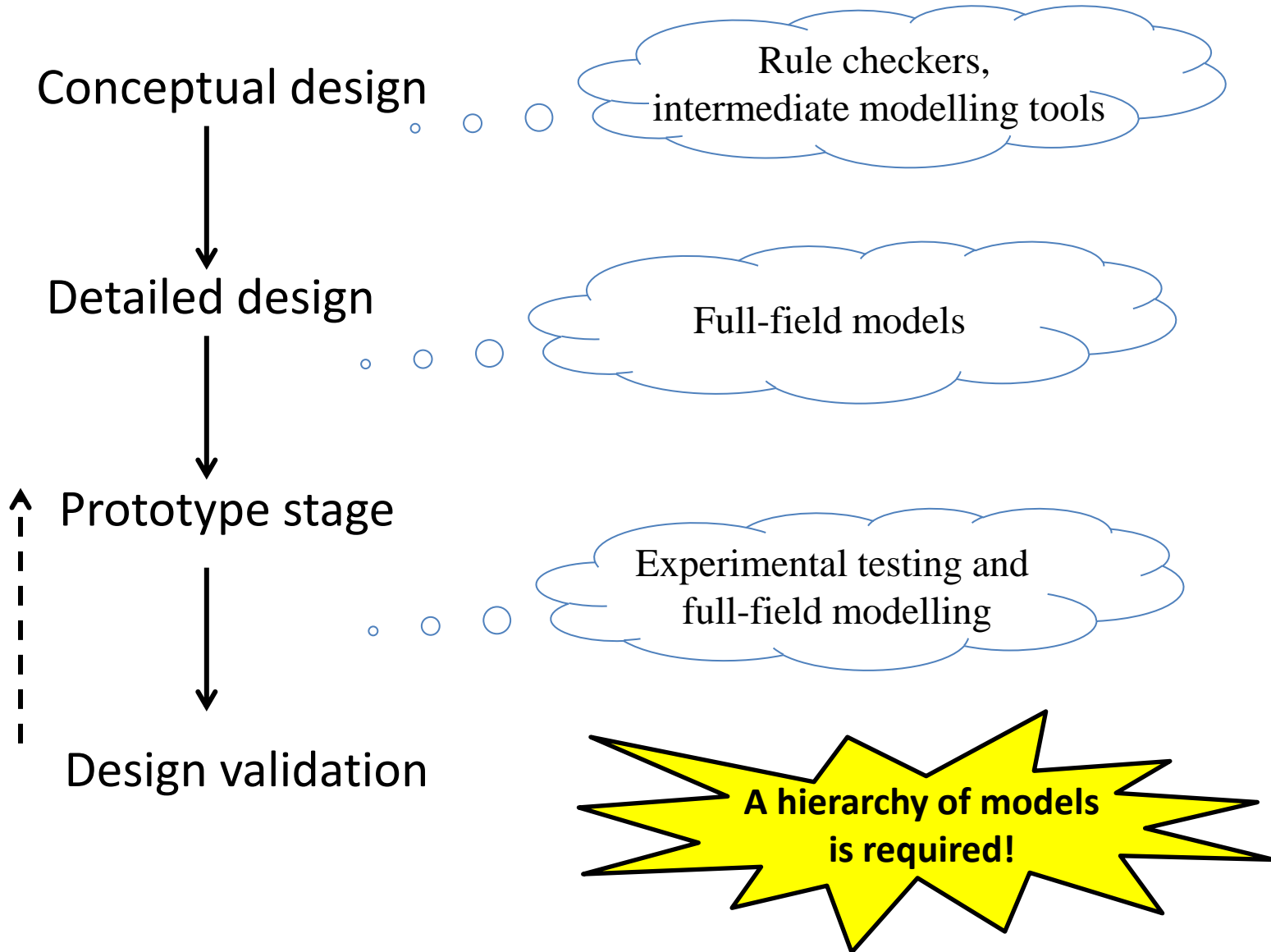
We need to observe, experiment, model, and think sometimes abstract thoughts to gain a better understanding of the behaviour of complex systems and to engage in creative design. Sticking with only experiments, or, only simulations is not a good idea...



Goethe...

"Thought expands, but lames; action animates, but narrows"

# The Design Process:



...BEGINNINGS.....MIDDLES.....ENDS...

# The Modelling Process:

CONCEPTUALISATION

Relate observations to basic physical principles

FORMULATION

Formulate problem in mathematical form

NUMERICAL IMPLEMENTATION

Formulate problem for solution by digital computer

COMPUTATION

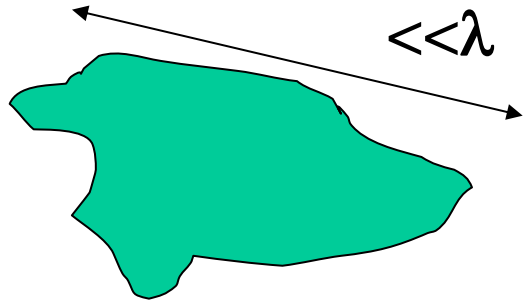
Implement computational algorithm

VALIDATION

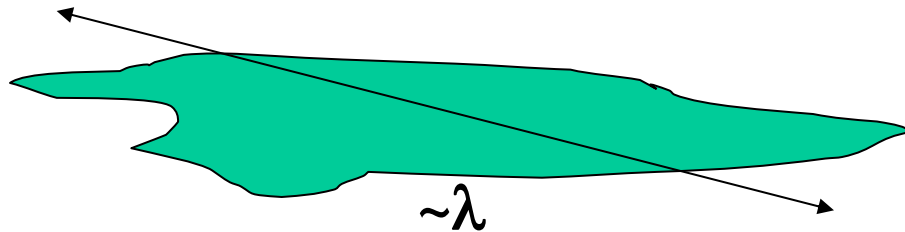
Check against other results/ physical reasonableness etc

...BEGINNINGS.....MIDDLES.....ENDS...

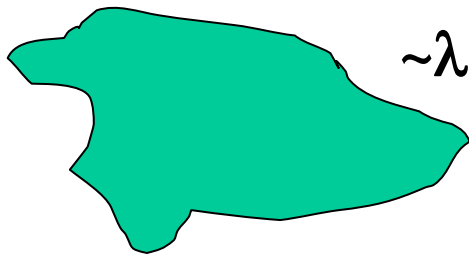
All models are not the same!...



Network approach  
(KVL, KCL)



Transmission  
line theory may  
be adequate  
(Telegrapher's  
equations)



Full-field theory  
(Maxwell's  
equations)

# Generic Classification of Numerical Methods:

- Time/frequency domain techniques
- Integral/differential equation techniques
- Some other very high frequency techniques (e.g. ray methods)

## **Classification Criteria:**

- according to the domain of the operator (differential DE, integral IE)
- according to the domain of the variable (time TD, frequency FD)
- other categories (e.g. ray methods)

## Example of a formulation in the time- and frequency-domains:

Consider an R-L series circuit. Its solution can be tackled into different ways.

Time-domain differential equation formulation

$$V_0 \cos(\omega t) = i(t)R + L \frac{di(t)}{dt}$$

Frequency-domain formulation

$$V_0 = \bar{I}(R + j\omega L)$$

## Example of formulation in integral- and differential form:

Consider the solution of an electrostatic problem. It can be formulated in two different ways.

Either using Gauss's Law

$$\int_S \bar{D} d\bar{s} = \int_V \rho dv$$

Or, using Poisson's equation

$$\nabla^2 \varphi = -\rho / \epsilon$$



# INTEGRAL METHODS:

## **Good!**

- open boundary problems
- only active region is discretised
- wire-like electrically long geometries

## **Not so good!**

- Dielectrics, lossy materials
- large matrix manipulations
- complex electrically small conductors and thin plates

# DIFFERENTIAL METHODS:

## **Good!**

- highly inhomogeneous media
- highly complex electrically small conductors and thin plates
- time-varying problems, non-linear media

## **Not so good!**

- entire problem space must be discretised
- open-boundary problems

# Generic Solvers(1)

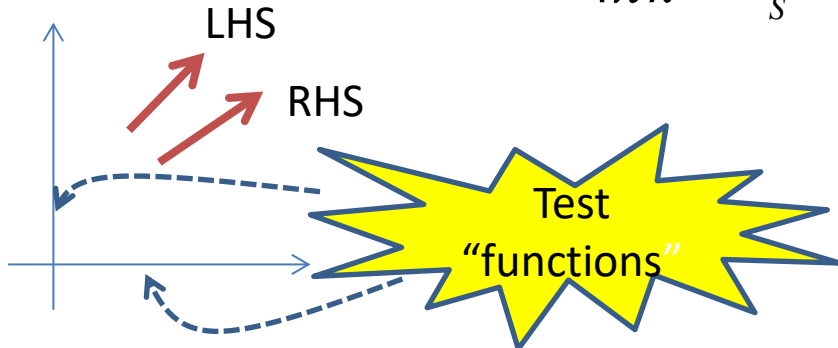
- *Method of Moments (MoM)*

Well established technique, belonging to the class of frequency-domain, integral methods. Used as standard for checking other methods. Good for problems with wire-like structures. Common commercial package is NEC. Uses known expansion functions with unknown coefficients to approximate unknown fields.

$$\hat{n} \times (\underline{\underline{E}}^i + \underline{\underline{E}}^s) = 0 \quad \longrightarrow \quad \frac{j\eta}{4\pi k} \hat{n} \times \int_s \underline{\underline{J}}(\vec{r}') \cdot (k^2 [I] + \nabla \nabla) \frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} dS' = \hat{n} \times \underline{\underline{E}}^i(\vec{r})$$

$$\mathcal{L}\{\underline{\underline{J}}(\vec{r})\} = \hat{n} \times \underline{\underline{E}}^i(\vec{r}) \quad \text{EFIE}$$

$$\mathcal{L}\left\{\sum_{j=1}^N \alpha_j \underline{\underline{J}}_j(\vec{r}')\right\} = \hat{n} \times \underline{\underline{E}}^i(\vec{r})$$

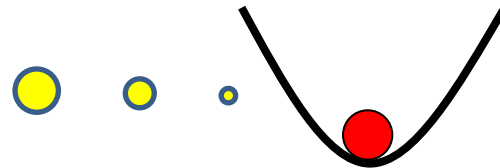


# Generic Solvers(2)

- *Finite Element Method (FEM)*

Well established technique, belonging to the class of frequency-domain, integral methods. A variation of this technique is the Boundary Element Method (BEM) which uses discretisation on surfaces only. Very popular technique especially at low frequencies. Many commercial packages available. Uses energy minimisation principles to arrive at solutions.

Sphere settles at the point of minimum energy!

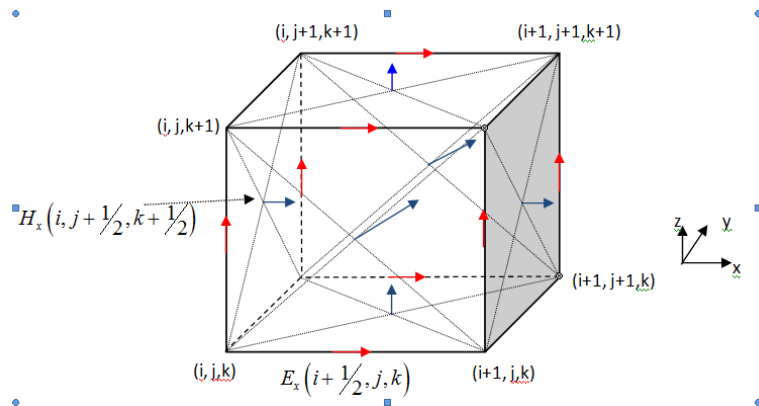


# Generic Solvers(3)

- *Finite Difference Time Domain Method (FDTD)*

Well established technique , belonging to the class of time-domain, differential methods. Very popular technique especially at high frequencies. Many commercial packages available (also FIT).

Approximates derivatives by differences and uses marching in time algorithm to arrive at solutions.



- E, calculated at  $n\Delta t$ , H at  $(n+1/2)\Delta t$
- Also displacement in space
- Stability,  $\Delta t \leq \frac{\Delta \ell}{c\sqrt{3}}$

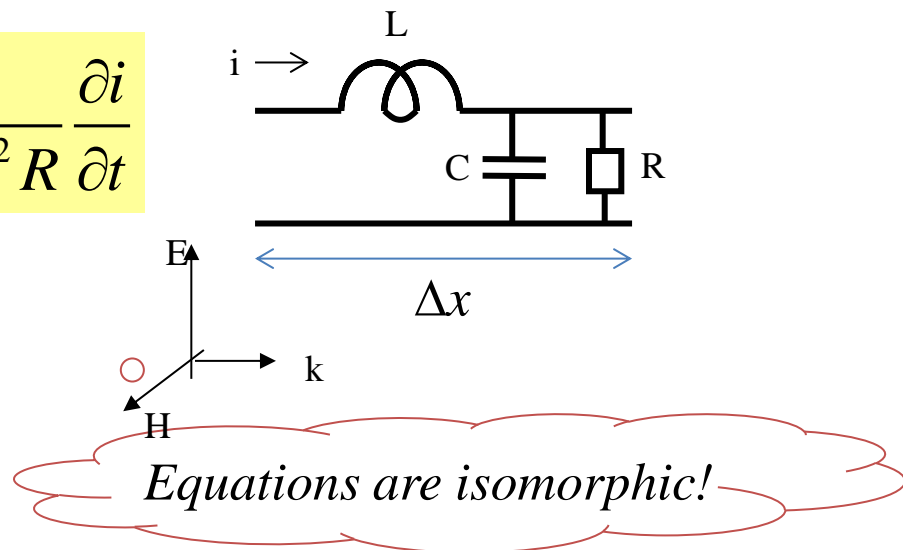
# Generic Solvers(4)

- *Transmission-Line Modelling (or Matrix) Method (TLM)*

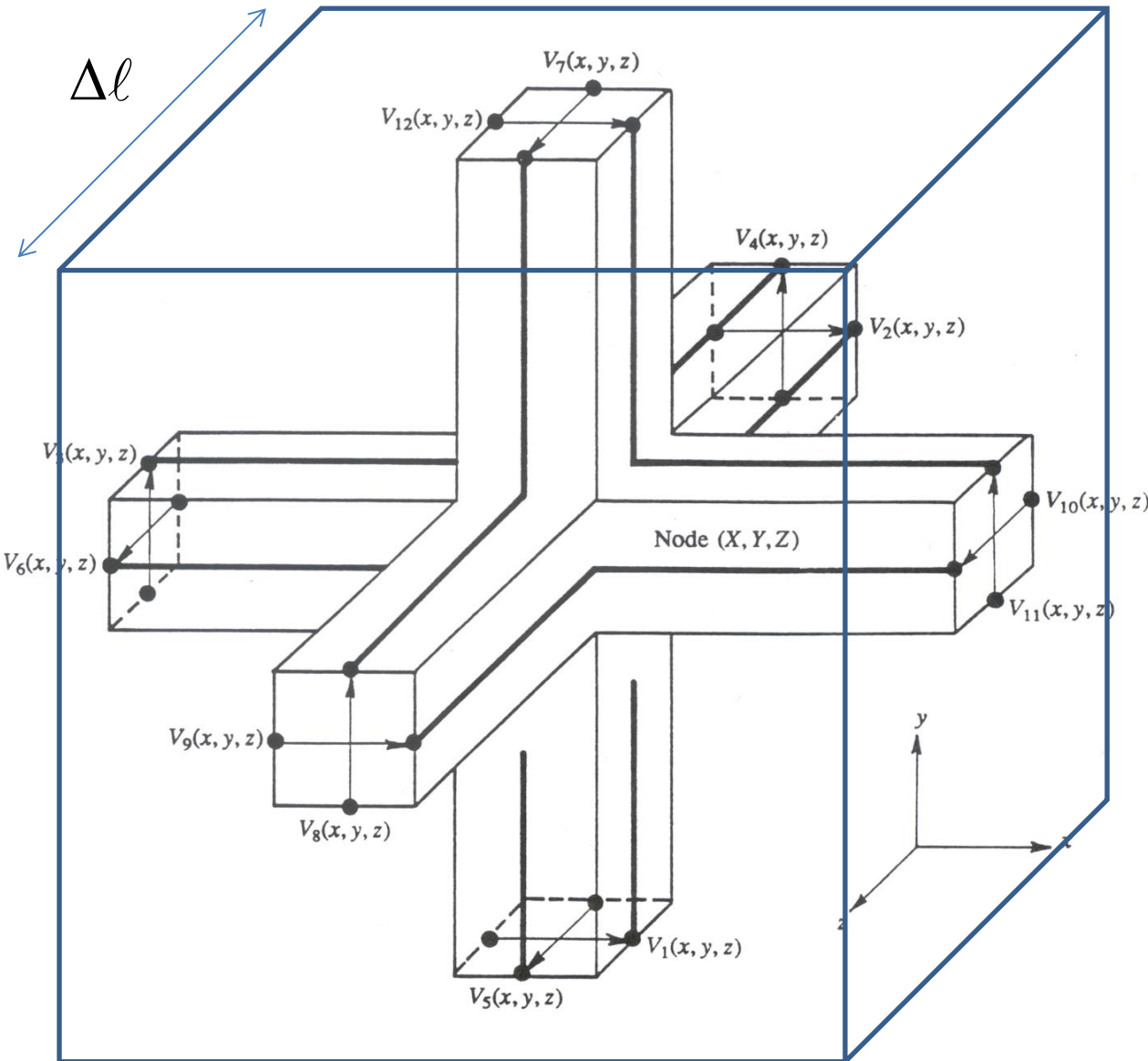
Well established technique, belonging to the class of time-domain, differential methods. Popular technique especially at high frequencies. Example of a commercial package is Micro-Stripes. Uses circuit equivalents to model fields.

$$\frac{\partial^2 i}{\partial x^2} = \frac{LC}{(\Delta x)^2} \frac{\partial^2 i}{\partial t^2} + \frac{L}{(\Delta x)^2 R} \frac{\partial i}{\partial t}$$

$$\frac{\partial^2 j}{\partial x^2} = \mu\epsilon \frac{\partial^2 j}{\partial t^2} + \mu\sigma \frac{\partial j}{\partial t}$$



# TLM(4)-continued: 3D node and associated cell



Scattering:

$$V^r = [S]V^i$$

Connection:

$${}_{k+1}V^i = [C]_k V^r$$

$$\Delta t = \frac{\Delta l}{2c}$$

$$\Delta l \leq \frac{\lambda_{\min}}{10}$$

# Generic Solvers(5)

- *Semi-Analytical Methods*

Examples are, the Method of Lines, Mode Matching etc. They are efficient and accurate but only suited to problems with some degree of symmetry.



# Generic Solvers(6)

- *Hybrid Methods*

Combine the best features of two methods e.g. finite difference and integral methods. Can be very efficient but not many commercial versions are available.



Aficionados of the **mule** claim that they are *"more patient, sure-footed, hardy and long-lived than horses, and they are considered less obstinate, faster, and more intelligent than donkeys."*

However, I do know of several cases of Greek men who have lost their manhood to stroppy kicking mules! I read further in Wikipedia that *"Mules are highly intelligent. They tend to be curious by nature. A mule generally will not let the rider put it in harm's way"*. There is a clue in the last sentence- mules can be awkward animals-you can flog a horse to death but you cannot do this to a mule.

**Hybrid numerical methods have great advantages but are difficult to apply and make them do exactly what you want ...**

The mule syndrome!

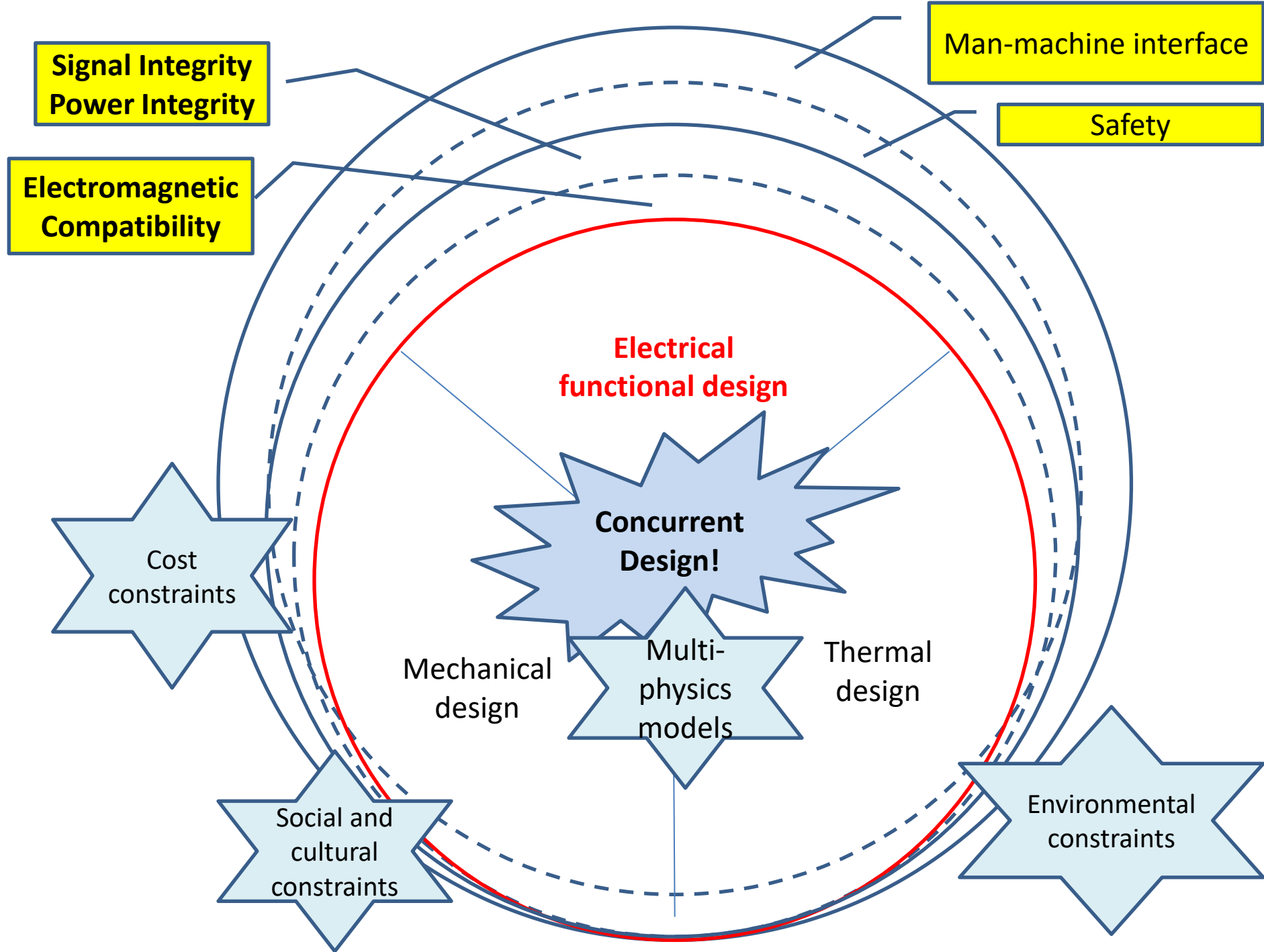
The simpler the model the better it is!

It is never the “real thing” and it should not be.

It should be accurate enough and simple so as to manipulate easily.

Ideally, it should be one that we can sketch on the back of an envelope and experiment with it while on the bus home after work!

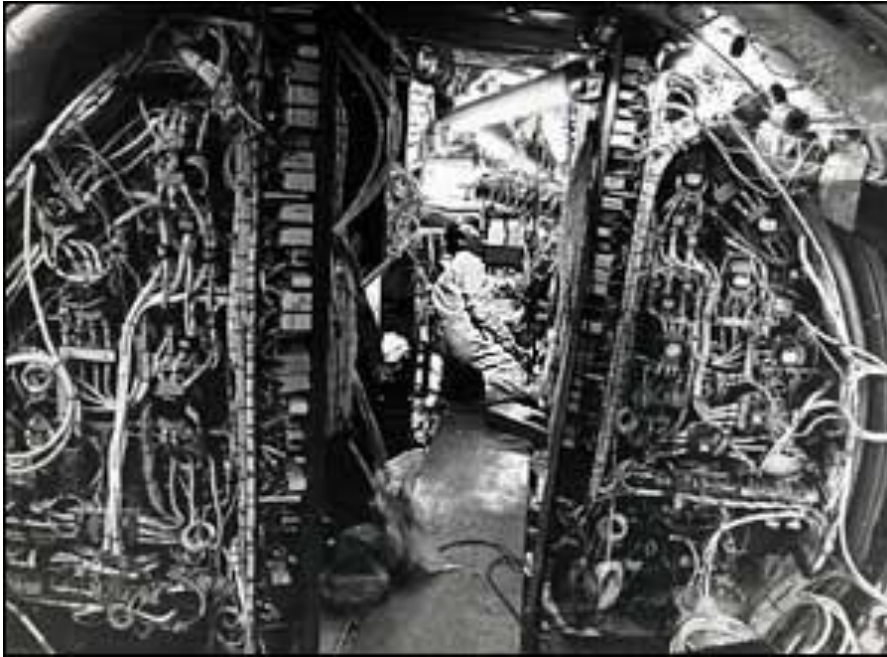
But alas, our systems are far too complex to even approach this ideal!



# Multiscale problems, Complexity and Uncertainty:

- EMC and Signal Integrity (SI) are studied in an environment where high repetition frequency short transition-time pulses propagate. A deep understanding of the relevant interactions required characterization and analysis in the time domain (TD) and inherently over a very wide range of frequencies. Models of materials over a wide frequency range are required and TD simulation codes are essential tools for such studies.
- Probably 75% of simulations in EMC are done in the time domain (TD). Naturally, practice differs in say electrical machine design.
- Material characterization from DC to at least 6 GHz must account for changing properties. New artificial materials with intricate geometrical details and frequency-dependent properties must be efficiently modelled in simulations

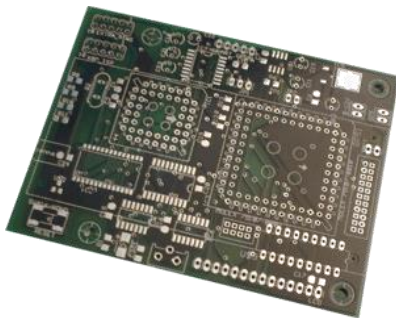
## A panorama of complexity!



Concord wiring!



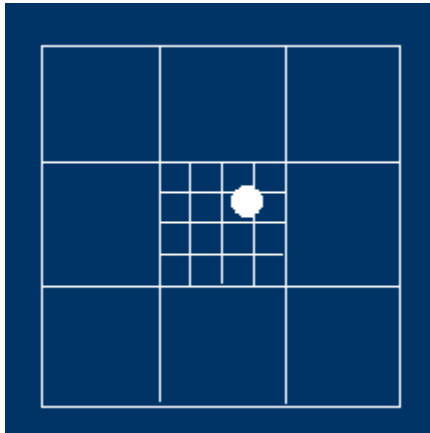
JSF wiring (28km wiring, 20k connectors)!



Typical PCB!

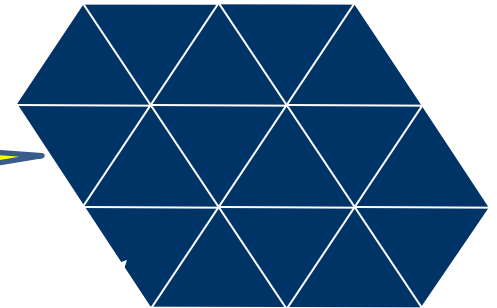
***We are not dealing  
with “canonical”  
problems!...  
de-featuring?***

Multi-grid mesh

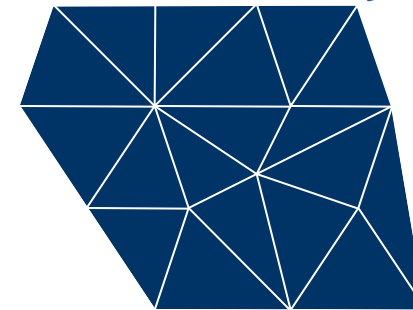


Example: Distort!

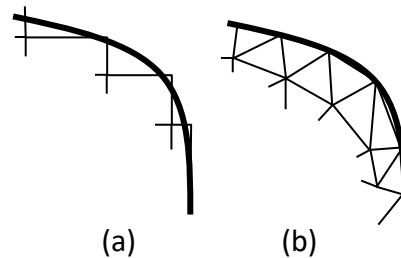
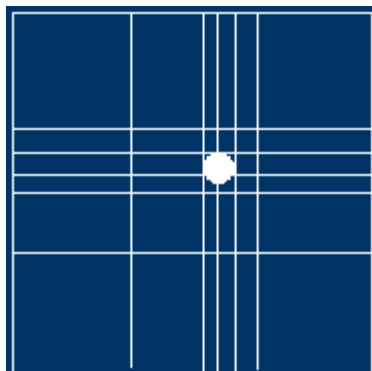
Structured mesh



Un-structured mesh (number of neighbours varies according to demands of the problem)



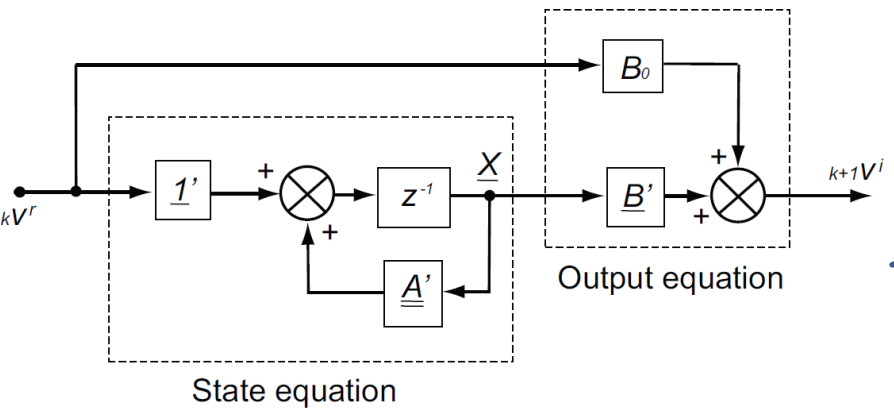
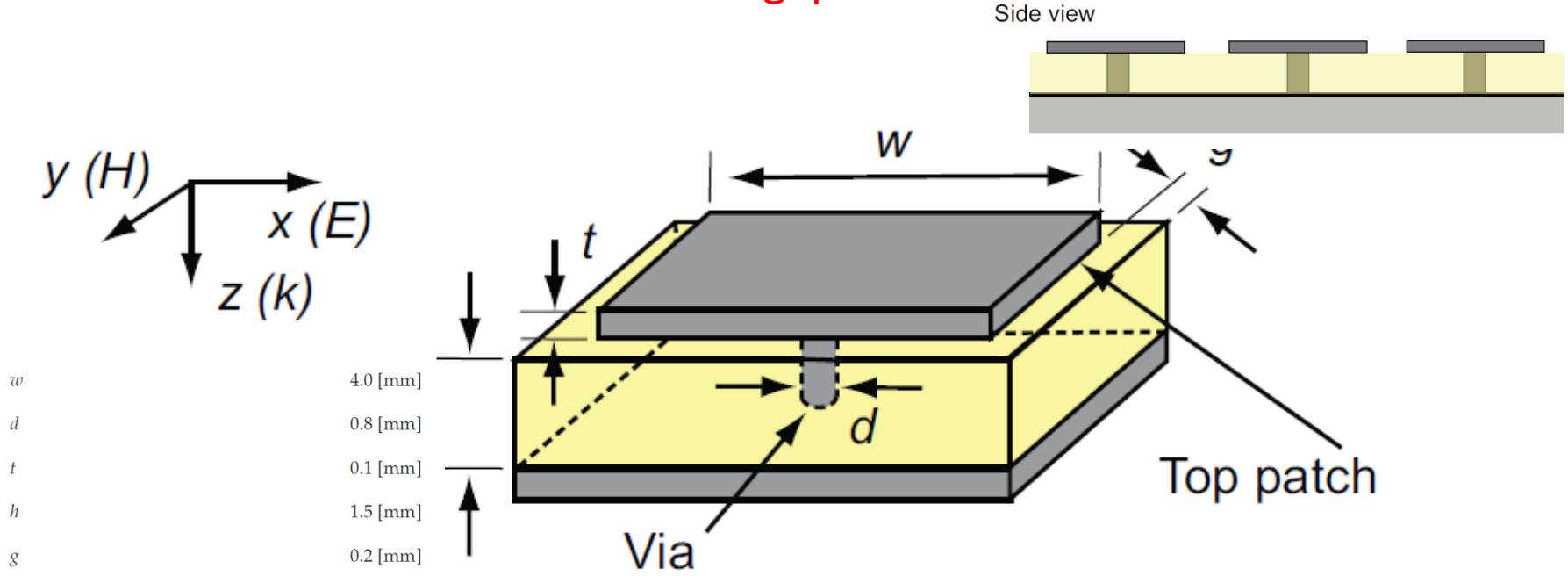
Hybrid mesh



Conformal boundaries (best described by an unstructured mesh)

# Example: Digital filter interface

Extraction of the reflection coefficient from a boundary covered by an electronic **bandgap** structure (EBG)



General form of DF to account for the details of the EBG (the parameters are extracted from the Padé form-see over)



Fine mesh resolution 0.1mm; Coarse mesh resolution 1.0mm  
 The coarse mesh is supplemented with the DF algorithm (2 poles) to account for the EBG

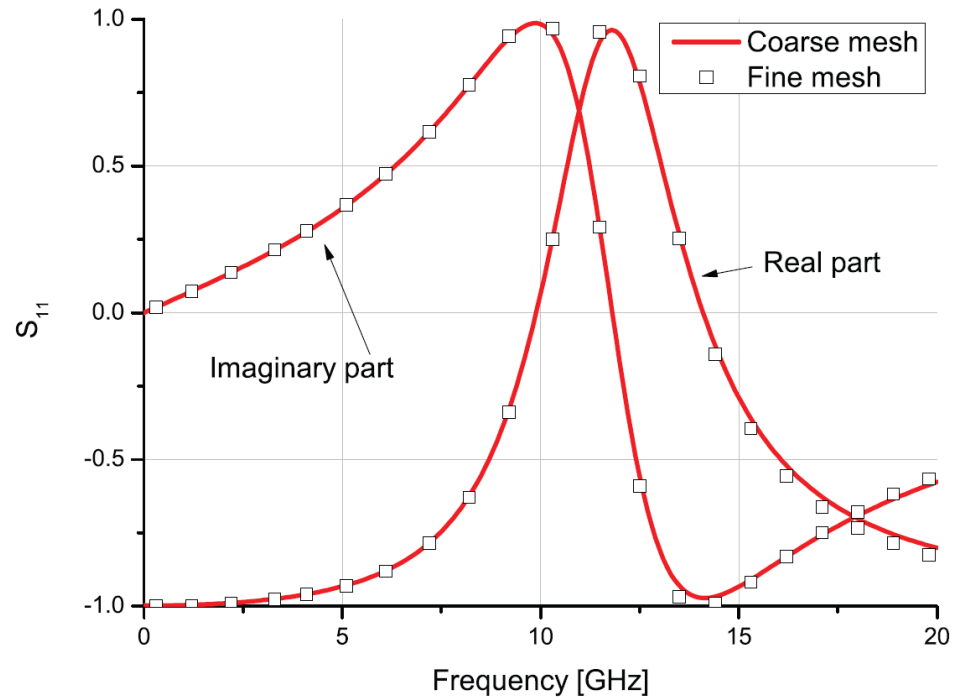
Padé form of reflection coefficient:

$$F(s) = \frac{b_0 + b_1s + b_2s^2}{a_0 + a_1s + s^2}$$

$i$	$a_i$	$b_i$
0	$5.50562 \times 10^{21}$	$-5.49485 \times 10^{21}$
1	$2.69504 \times 10^{10}$	$2.59674 \times 10^{10}$
2		$-9.92844 \times 10^{-1}$

Efficiency gains!

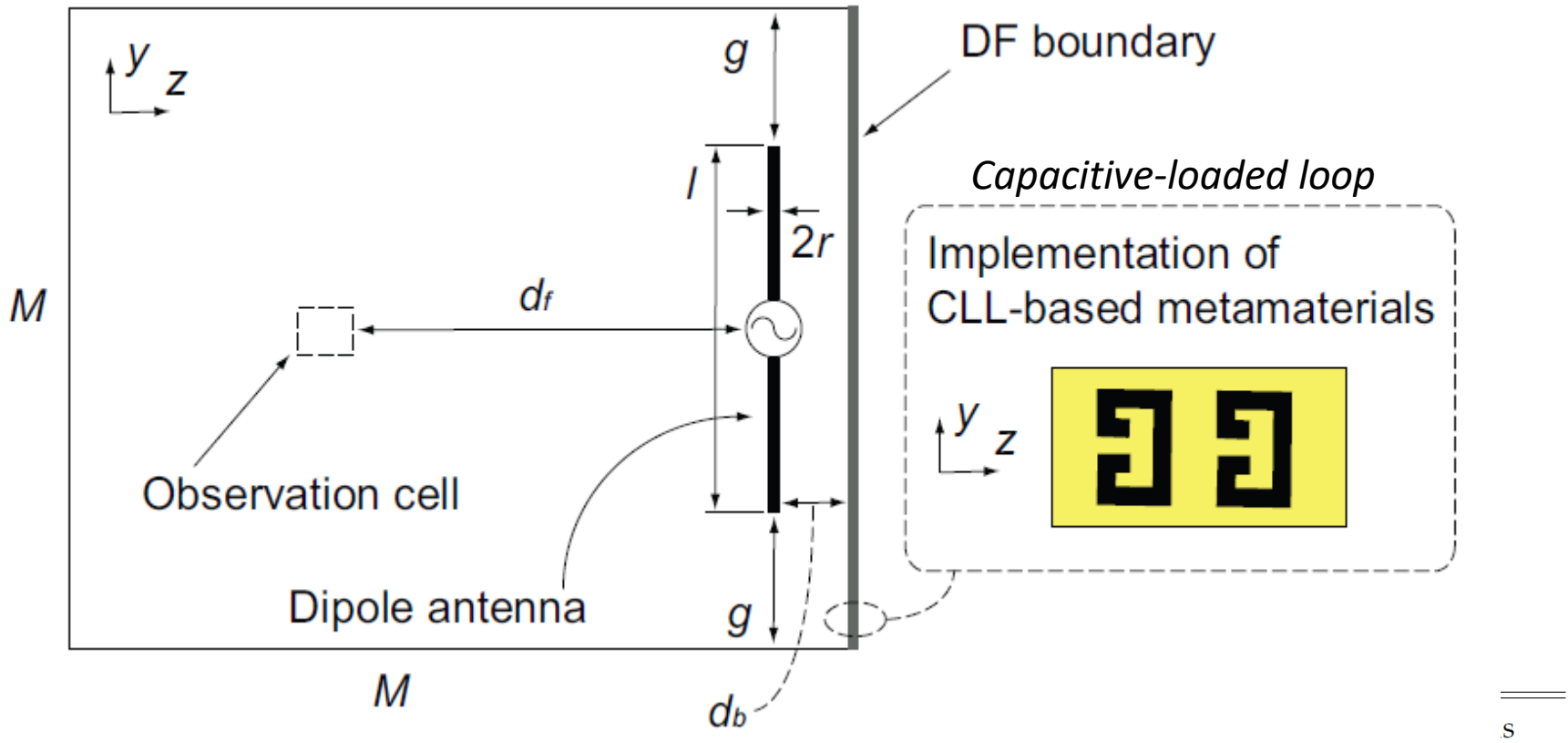
	Coarse mesh	Fine mesh
Cell size [mm <sup>3</sup> ]	$1.0 \times 1.0 \times 1.0$	$0.1 \times 0.1 \times 0.1$
Analysis space size ( $N_x \times N_y \times N_z$ )	$(5 \times 5 \times 44)$	$(44 \times 44 \times 60)$
Calculation time	< 1 minute	3 hours



Note: Implementation in the TD is done by deriving an equivalent DF algorithm (e.g. using bilinear transform). Frequency-dependent materials dealt with in a similar way.

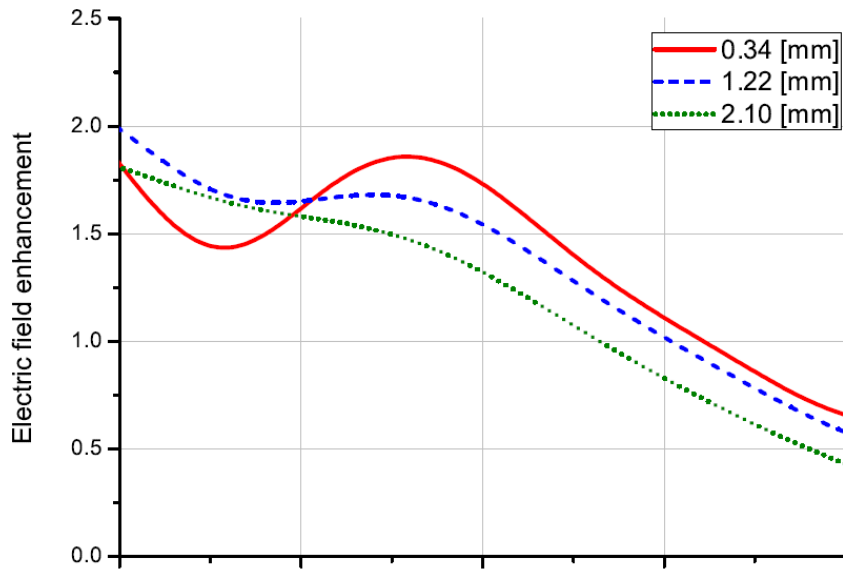


*M: Matched boundary condition*

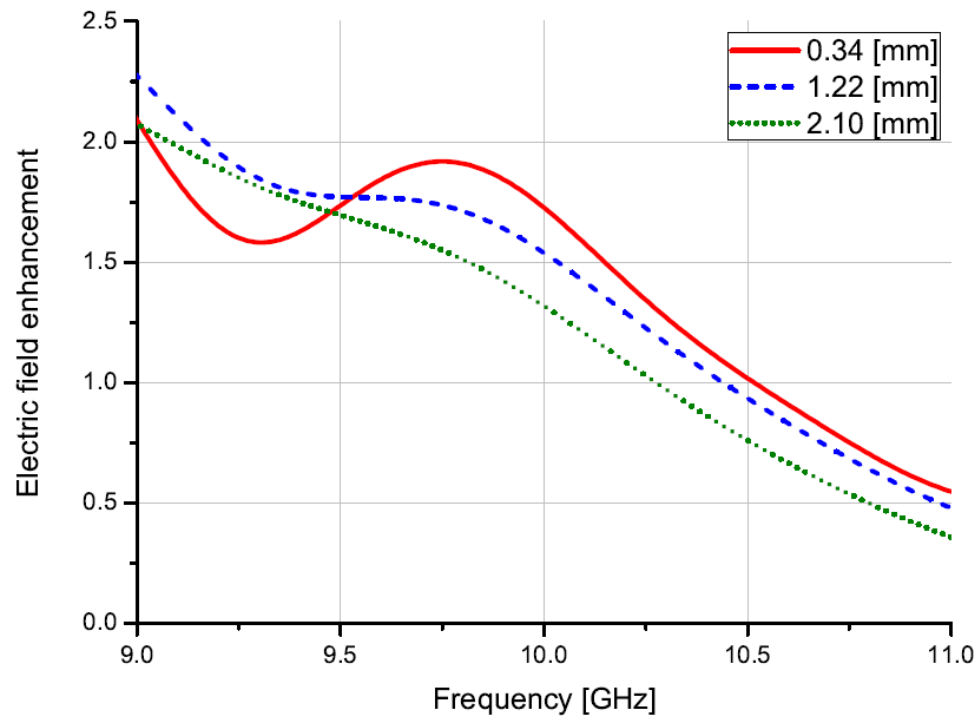


Cylindrical antenna is modelled using a “thin-wire” formulation

TLM unit cell size	$0.88 \times 0.88 \times 0.88$ [mm <sup>3</sup> ]
$d_f$	3.86 [mm]
$d_b$	0.34, 1.22 or 2.10 [mm]
$g$	7.04 [mm]
$r$	0.10 [mm]
$l$	9.68 [mm]
Resistance of centre cell of antenna	50.0 [ $\Omega$ ]



electric field with meta-material wall/electric field with absorbing wall



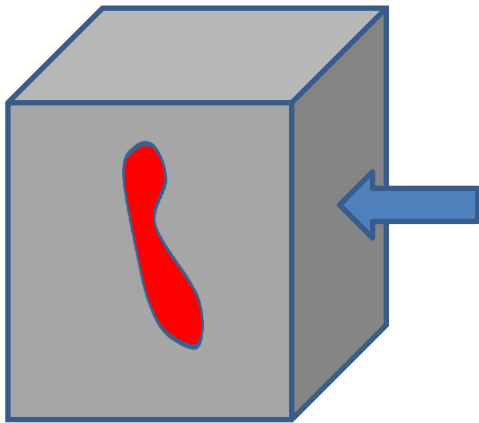
electric field with meta-material wall/electric field with PEC wall

## Example: Embed a cell using modal decomposition

### How do we deal with a cell which contains an unusual object?

Reducing the size of the cell so that we can map directly the geometrical shape of the unusual object is not practical (excessive computational resources required, tedious mapping etc).

The alternative is to deal with this cell in an entirely different way (as a **sub-cell, macromodel**). We need to know the “impedance” seen by signal impinging on each face of the cell so that the rest of the computation encounters there a boundary condition.

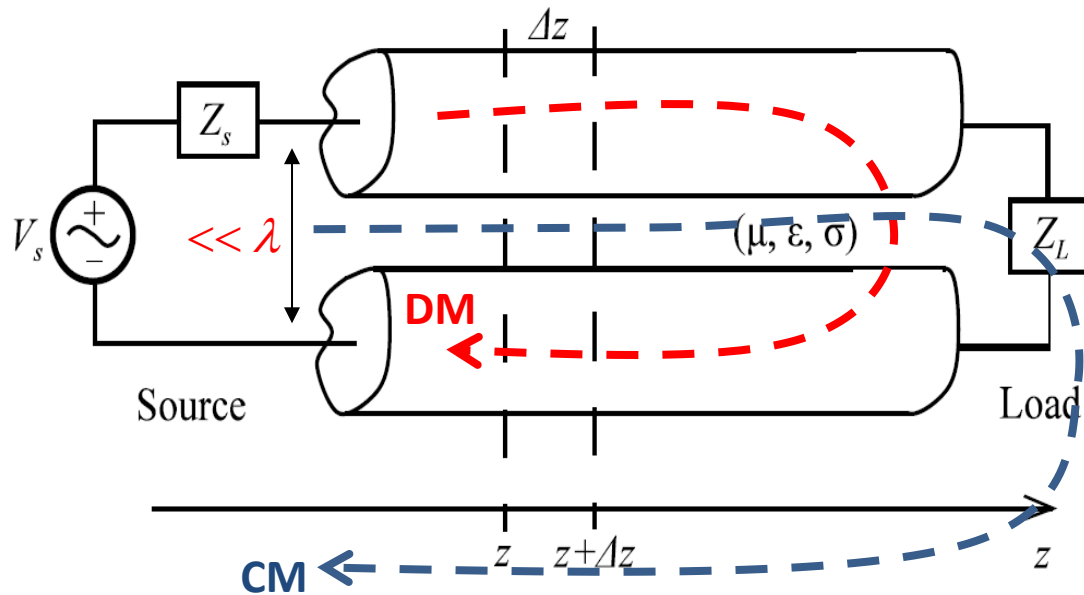


The difficulty here is that the impedance seen on each face depends on the entire 3D environment as there is coupling between all signals impinging on the six facets of the cell.

The calculation becomes far easier if incident signals are decomposed into components which are independent and see a well defined impedance at each face. These components are what we call “**modes**”.

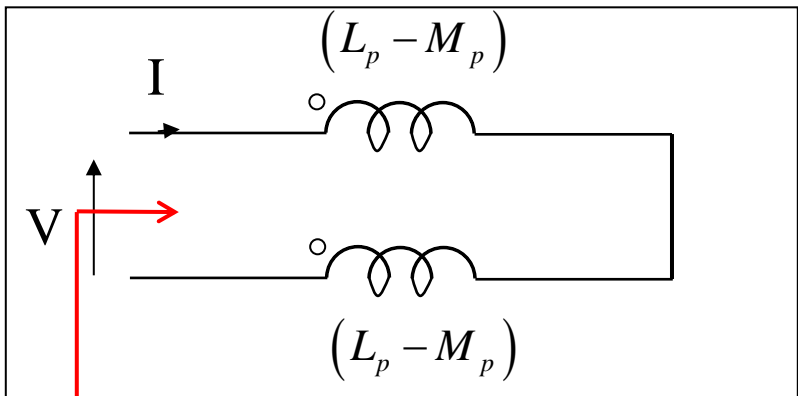
$$\bar{V}^i = [U] \bar{X}^i$$

$$\bar{V}^r = [U] \bar{X}^r \quad [U], \text{ eigenvector matrix}$$

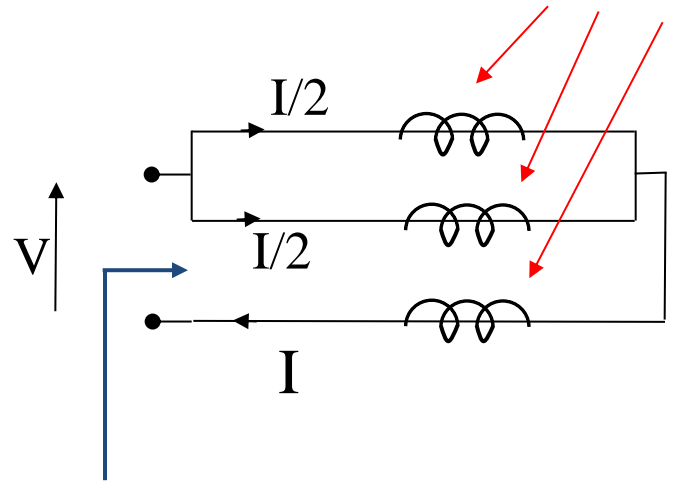


What are "modes" ?

$$L_p - M_p$$



$2(L_p - M_p)$  DM impedance

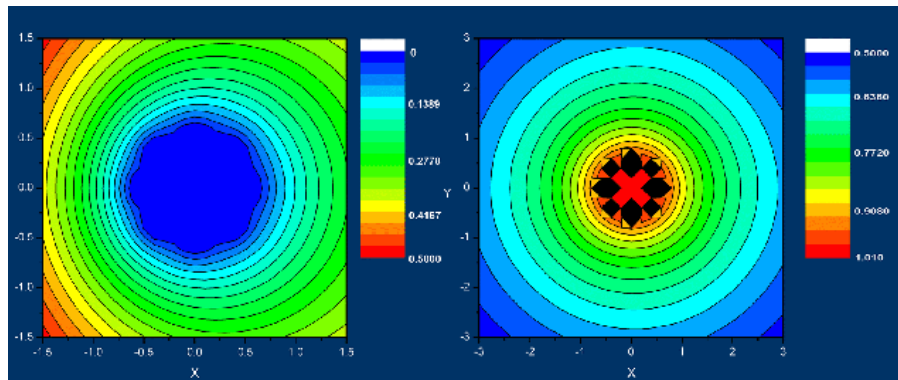
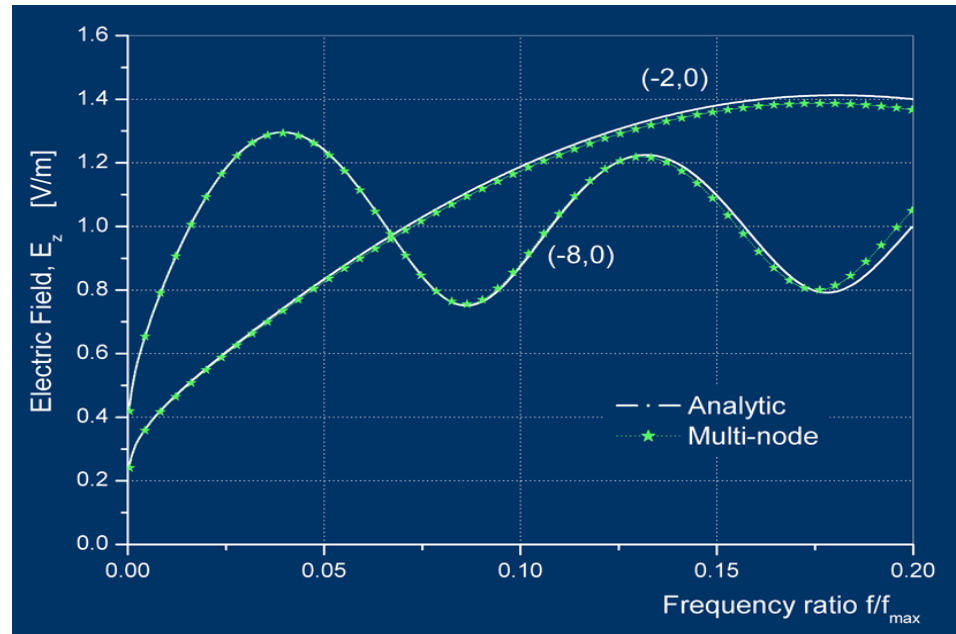
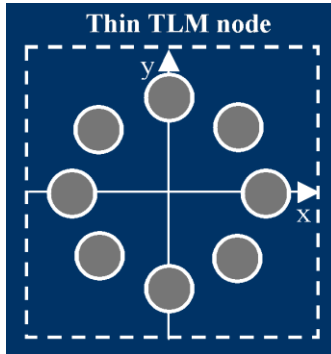


$\frac{3}{2}(L_p - M_p)$  DM impedance

# Embed!

## Modal Expansion Techniques

wire shield



The parameters:

Number of wires = 8,

$a = 0.003125\text{m}$ ,  $r = 0.0125\text{m}$ ,

$\theta = 0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ,$

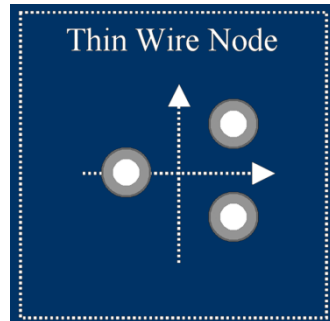
$225^\circ, 270^\circ, 315^\circ$

$$\begin{matrix} \frac{1}{2} & & & \\ \frac{1}{2} & \bullet & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ \frac{1}{2} & & & \end{matrix} \quad \begin{matrix} \frac{1}{2} & & & \\ -\frac{1}{\sqrt{2}} & \bullet & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ \frac{1}{\sqrt{2}} & & & \end{matrix} \quad \begin{matrix} \frac{1}{\sqrt{2}} & & & \\ & \bullet & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ -\frac{1}{\sqrt{2}} & & & \end{matrix} \quad \begin{matrix} -\frac{1}{2} & & & \\ \frac{1}{2} & \bullet & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ \frac{1}{2} & & & \end{matrix}$$

Simplified example of modal decomposition

# Modal Expansion Techniques

## Dielectric coated wires



### Parameters:

Dielectric constant = 100

Number of wires = 3

$a_{w1} = 0.005\text{m}$ ,

$a_{w2} = 0.00625\text{m}$ ,

$a_{w3} = 0.0075\text{m}$ ,

$a_{c1} = a_{c2} = a_{c3} = 0.0125\text{m}$

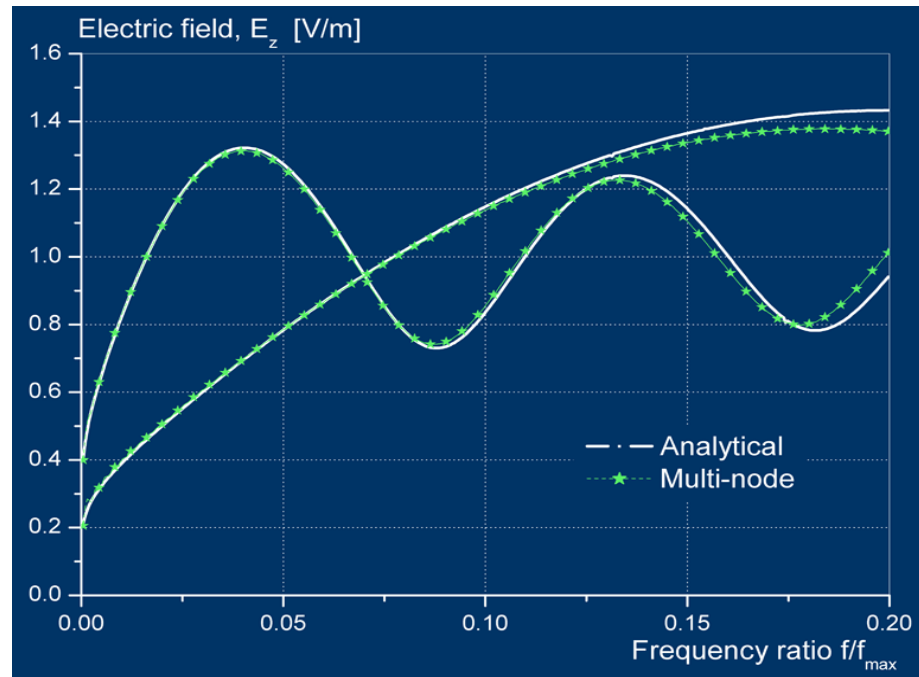
$r_1 = 0.01$ ,  $r_2 = 0.01125$ ,

$r_3 = 0.0125$

$\theta_1 = 45^\circ$ ,  $\theta_2 = -45^\circ$ ,  $\theta_3 = 180^\circ$

node size = 0.05m,

total mesh area 60m by 60m



# Uncertainty:

- Complex systems in particular are characterized by uncertainties in parameter values which make the prediction of their response problematic. The normal **deterministic** studies are useful, but they fail to bring out the full range of potential responses and to take account of the risks involved and the confidence that can be attached to predictions
- Monte-Carlo studies require numerous simulations to span the parameter space and hence are unrealistic in situations where a single simulation requires major computational resources
- A **stochastic** approach to modelling is based on extracting the first few moments (mean and standard deviation, or even the entire pdf) from a relatively small number of simulations...(see the Unscented Transform UT)...

For an output  $g(x)$  which depends on a random variable  $x = \bar{X} + \hat{x}$  where  $\hat{x}$  is a zero mean Gaussian random variable we need 3 simulations at

Example of the applications of UT

$$\bar{X}, \quad \bar{X} + \sigma\sqrt{3}, \quad \bar{X} - \sigma\sqrt{3} \quad \text{Where } \sigma \text{ is the standard deviation of } \hat{x}$$

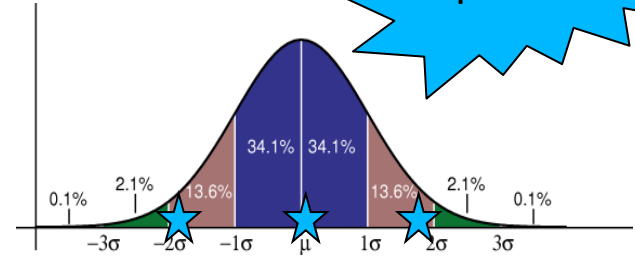
The results of the 3 simulations are:  $g(\bar{X}), \quad g(\bar{X} + \sigma\sqrt{3}), \quad g(\bar{X} - \sigma\sqrt{3})$

We combine the 3 simulations as shown below to get the expected value and variance of the output  $g$ .

Sigma points!

Variance of the output

Expected value of output



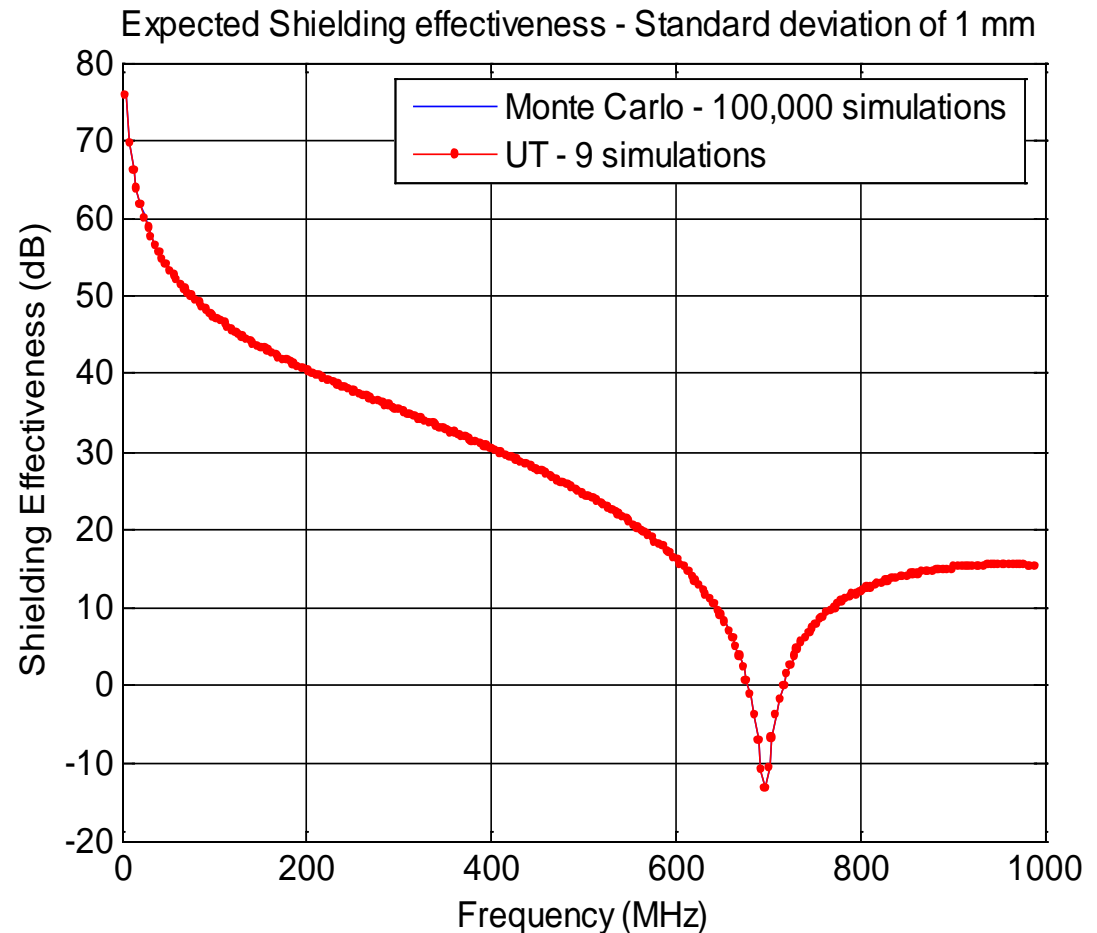
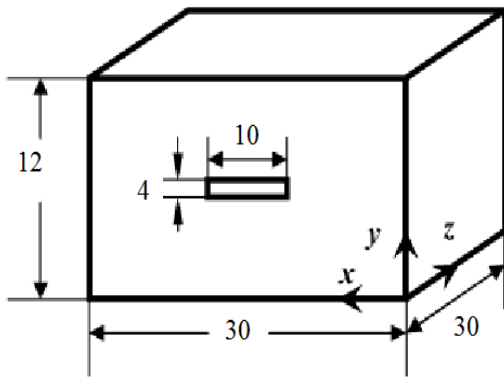
$$\tilde{g} = E\{g(\bar{X} + \hat{x})\} = \frac{2}{3}g(\bar{X}) + \frac{1}{6}g(\bar{X} + \sigma\sqrt{3}) + \frac{1}{6}g(\bar{X} - \sigma\sqrt{3})$$

$$\tilde{\sigma}_g^2 = \frac{2}{3}[g(\bar{X}) - \tilde{g}]^2 + \frac{1}{6}[g(\bar{X} + \sigma\sqrt{3}) - \tilde{g}]^2 + \frac{1}{6}[g(\bar{X} - \sigma\sqrt{3}) - \tilde{g}]^2$$



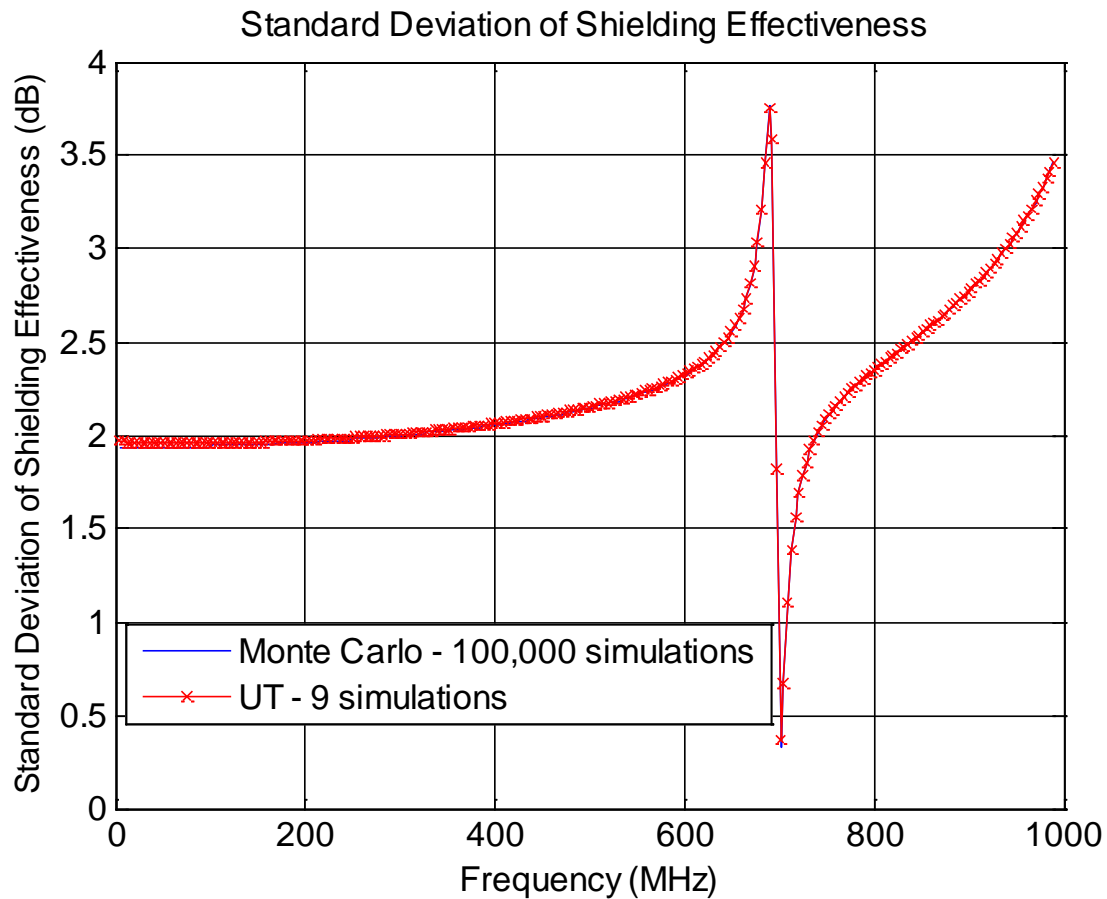
## Example : Shielding Effectiveness – Metal Box

- Analytical formulation – size of the aperture (2 RVs, width and length, Gaussian distribution, std deviation 1mm)



# Example : Shielding Effectiveness – Metal Box (cont.)

- Analytical formulation – size of the aperture (as in previous slide)



# Conclusions and Outlook:

- EMC and HF modelling in general makes severe demands on CEM
- Broadband responses, multi-scale problems, complexity and uncertainty require innovative approaches to simulation and not just a larger faster computer!
- Combining the best features of individual models into an efficient whole is a powerful approach which is capable of scaling up modelling capabilities to meet practical needs

- We will increasingly rely on numerical models BUT we must exploit fully synergies and complementarity between simulations and testing
- Need to characterise materials and sub-systems over a wide frequency range in a form suitable for embedding into whole system models
- De-featuring, complexity reduction, parametrised models and physically transparent models to aid creativity are needed!

# Contact

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