Reciprocity-based applications of the TD contour integral method

by

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Presentation overview

- Time-domain reciprocity theorems and their application
- Time-domain contour integral method – formulation
- Time-domain mutual coupling between power-ground structures
- Time-domain radiated susceptibility of a power-ground structure
- Conclusions

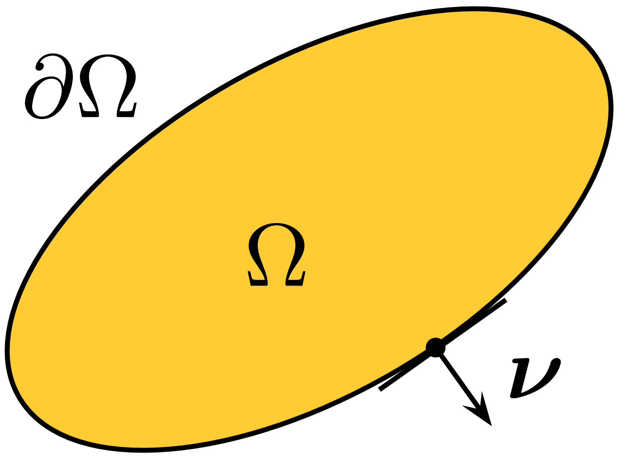
Reciprocity theorems and their application (1)

state A $\Longleftrightarrow$ **RECIPROCITY** $\Longleftrightarrow$ state B

$$\{E_r^A, H_k^A\} \qquad \{E_r^B, H_k^B\}$$

$$\{J_k^A, K_j^A\} \qquad \{J_k^B, K_j^B\}$$

$$\{\epsilon_{k,p}^A, \mu_{r,j}^A\} \qquad \{\epsilon_{k,p}^B, \mu_{r,j}^B\}$$



A. T. de Hoop, *Handbook of Radiation and Scattering of Waves*, Academic Press, London, UK, 1995.

Reciprocity theorems and their application (2)

GENERIC (GLOBAL) INTERACTION QUANTITY

$$\int_{\boldsymbol{x}' \in \partial\Omega} [\text{Interaction of the Field States}] \cdot \boldsymbol{\nu}(\boldsymbol{x}')\,\mathrm{dA}$$
$$= \int_{\boldsymbol{x}' \in \Omega} [\text{Interaction of the Field and Material States}]\,\mathrm{dV}$$
$$+ \int_{\boldsymbol{x}' \in \Omega} [\text{Interaction of the Field and Source States}]\,\mathrm{dV}$$

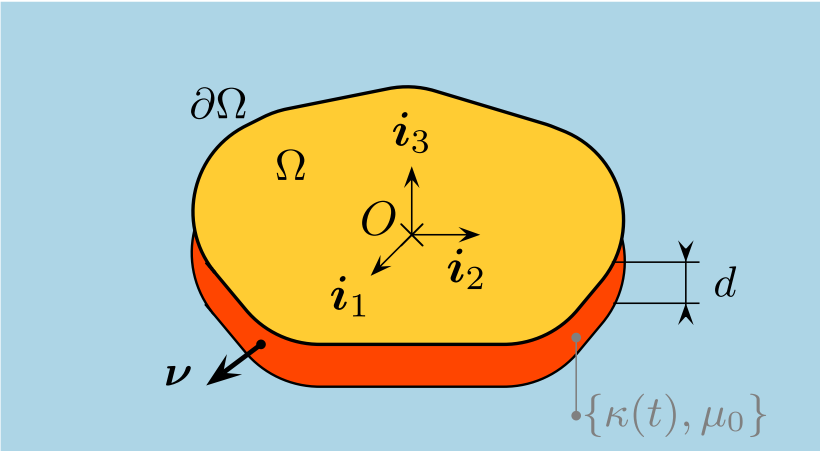
Temporal interactions

- time-convolution type
- time-correlation type

TIME-DOMAIN CONTOUR INTEGRAL METHOD

Time-domain analysis of a power-ground structure (1)

CONFIGURATION



RECIPROCITY

Domain $\Omega \subset \mathbb{R}^2$		
	Actual Field (A)	Testing Field (B)
Field State	$\{E_3, H_1, H_2\}$	$\{E_3^B, H_1^B, H_2^B\}$
Material State	$\{\kappa(t), \mu_0\delta(t)\}$	$\{\kappa(t), \mu_0\delta(t)\}$
Source State	J_3	∂J_3^B

Time-domain analysis of a power-ground structure (2)

Time-Domain Contour Integral Method*

$$\boxed{\begin{aligned}&\frac{1}{2}\int_{\boldsymbol{x}\in\partial\Omega} E_3(\boldsymbol{x},t) \overset{(t)}{*} \partial J_3^B(\boldsymbol{x}|\boldsymbol{x}^S,t)\mathrm{d}l(\boldsymbol{x}) - \int_{\boldsymbol{x}\in\partial\Omega} E_3(\boldsymbol{x},t) \overset{(t)}{*} \boldsymbol{\nu}(\boldsymbol{x})\cdot\partial\boldsymbol{J}^B(\boldsymbol{x}|\boldsymbol{x}^S,t)\mathrm{d}l(\boldsymbol{x})\\ &= \int_{\boldsymbol{x}\in\Omega} E_3^B(\boldsymbol{x}|\boldsymbol{x}^S,t) \overset{(t)}{*} J_3(\boldsymbol{x},t)\mathrm{d}A(\boldsymbol{x}) - \int_{\boldsymbol{x}\in\partial\Omega} E_3^B(\boldsymbol{x}|\boldsymbol{x}^S,t) \overset{(t)}{*} \boldsymbol{\nu}(\boldsymbol{x})\cdot\partial\boldsymbol{J}(\boldsymbol{x},t)\mathrm{d}l(\boldsymbol{x})\end{aligned}}$$

$$\boxed{\begin{aligned}E_3^B(\boldsymbol{x}|\boldsymbol{x}^S,t) &= -\mu_0\partial_t\int_{\boldsymbol{x}^T\in\partial\Omega} G_\infty[r(\boldsymbol{x}|\boldsymbol{x}^T),t] \overset{(t)}{*} \partial J_3^B(\boldsymbol{x}^T|\boldsymbol{x}^S,t)\mathrm{d}l(\boldsymbol{x}^T)\\ \partial J_\kappa^B(\boldsymbol{x}|\boldsymbol{x}^S,t) &= -\partial_\kappa\int_{\boldsymbol{x}^T\in\partial\Omega} G_\infty[r(\boldsymbol{x}|\boldsymbol{x}^T),t] \overset{(t)}{*} \partial J_3^B(\boldsymbol{x}^T|\boldsymbol{x}^S,t)\mathrm{d}l(\boldsymbol{x}^T)\end{aligned}}$$

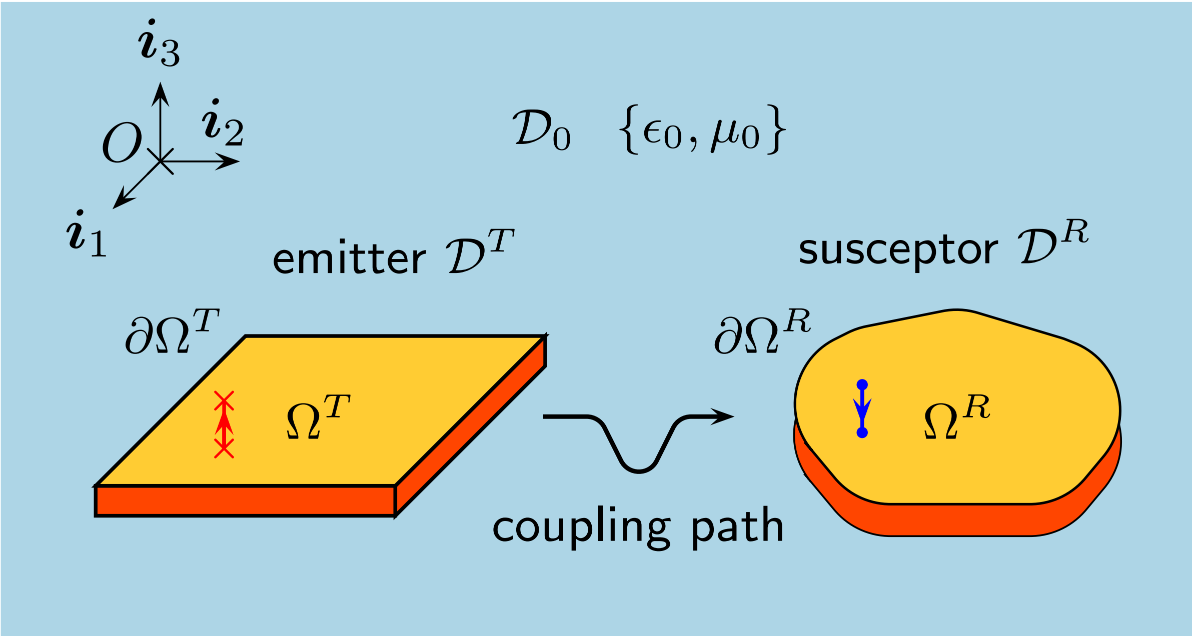
*M. Štumpf, "The time-domain contour-integral method – an approach to the analysis of double plane circuits," *IEEE Trans. Electromagn. Compat.*, vol. 56, no. 2, April 2014.

SPACE-TIME EM MUTUAL COUPLING

BETWEEN TWO P/G STRUCTURES

Time-domain mutual coupling between power-ground structures (1)

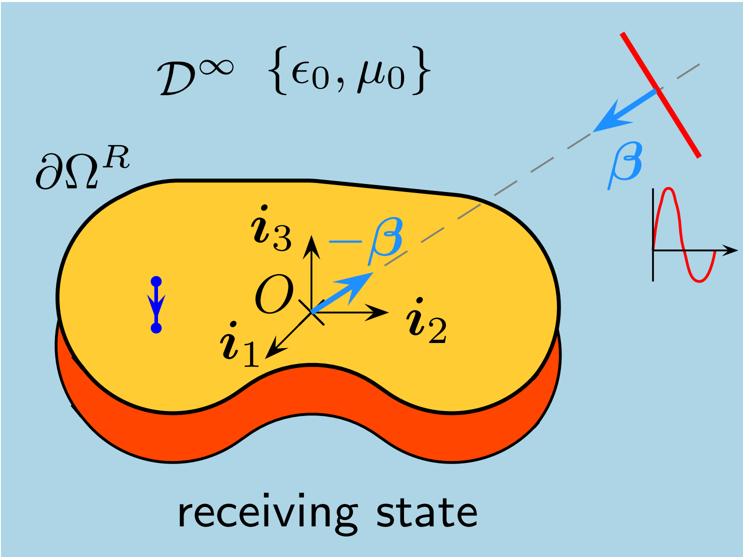
PROBLEM CONFIGURATION*



*M. Štumpf, "Time-domain mutual coupling between power-ground structures," In *Proc. 2014 IEEE Int. Symp. Electromagn. Compat.*, pp. 240–243, Raleigh, NC, USA, 04–08 August 2014.

Time-domain mutual coupling between power-ground structures (2)

RECEIVING SITUATION



$$\boxed{\{\boldsymbol{E}^s, \boldsymbol{H}^s\} \triangleq \{\boldsymbol{E}^R - \boldsymbol{E}^i, \boldsymbol{H}^R - \boldsymbol{H}^i\}}$$

- s = scattered field
- R = total field
- i = incident field

Time-domain mutual coupling between power-ground structures (3)

RECEIVING SITUATION - RECIPROCITY #1

Domain $\mathbb{R}^3 \setminus (\mathcal{D}^R \cup \partial\mathcal{D}^R)$		
	Scattered Field (s)	Testing Field (B)
Field State	$\{\boldsymbol{E}^s, \boldsymbol{H}^s\}$	$\{\boldsymbol{E}^B, \boldsymbol{H}^B\}$
Material State	$\{\epsilon_0, \mu_0\}\delta(t)$	$\{\epsilon_0, \mu_0\}\delta(t)$
Source State	0	0

- $+$ causality (radiation) condition

$$\boxed{\int_{\boldsymbol{x}\in\partial\mathcal{D}^R} \left[\boldsymbol{E}^B(\boldsymbol{x},t) \overset{*}{\times} \boldsymbol{H}^s(\boldsymbol{x},t) - \boldsymbol{E}^s(\boldsymbol{x},t) \overset{*}{\times} \boldsymbol{H}^B(\boldsymbol{x},t)\right] \cdot \boldsymbol{\nu}^R(\boldsymbol{x})\mathrm{d}A(\boldsymbol{x}) = 0}$$

Time-domain mutual coupling between power-ground structures (4)

RECEIVING SITUATION - RECIPROCITY #2

Domain $\mathcal{D}^R \subset \mathbb{R}^3$		
	Total Field (R)	Testing Field (B)
Field State	$\{\boldsymbol{E}^R, \boldsymbol{H}^R\}$	$\{\boldsymbol{E}^B, \boldsymbol{H}^B\}$
Material State	$\{\kappa(t), \mu_0\delta(t)\}$	$\{\kappa(t), \mu_0\delta(t)\}$
Source State	0	$\boldsymbol{J}^B = j^B(t)\delta(\boldsymbol{x} - \boldsymbol{x}^S)\boldsymbol{i}_3$

$$\boxed{\begin{aligned}\int_{\boldsymbol{x}\in\partial\mathcal{D}^R} \left[\boldsymbol{E}^B(\boldsymbol{x},t) \overset{*}{\times} \boldsymbol{H}^R(\boldsymbol{x},t) - \boldsymbol{E}^R(\boldsymbol{x},t) \overset{*}{\times} \boldsymbol{H}^B(\boldsymbol{x},t)\right] \cdot \boldsymbol{\nu}^R(\boldsymbol{x})\mathrm{d}A(\boldsymbol{x}) \\ = \int_{\boldsymbol{x}\in\mathcal{D}^R} \boldsymbol{J}^B(\boldsymbol{x},t) \overset{*}{\cdot} \boldsymbol{E}^R(\boldsymbol{x},t)\mathrm{d}V(\boldsymbol{x})\end{aligned}}$$

Time-domain mutual coupling between power-ground structures (5)

RECEIVING SITUATION - RECIPROCITY #1 & #2

$$\boxed{\int_{\boldsymbol{x}\in\mathcal{D}^R} \boldsymbol{J}^B(\boldsymbol{x},t) \overset{*}{\cdot} \boldsymbol{E}^R(\boldsymbol{x},t)\mathrm{d}V(\boldsymbol{x}) = \int_{\boldsymbol{x}\in\partial\mathcal{D}^R} \left[\boldsymbol{E}^B(\boldsymbol{x},t) \overset{*}{\times} \boldsymbol{H}^i(\boldsymbol{x},t)\right] \cdot \boldsymbol{\nu}^R(\boldsymbol{x})\mathrm{d}A(\boldsymbol{x})}$$

- magnetic-wall boundary condition

$$\boxed{\left[\boldsymbol{i}_3 \times \boldsymbol{\nu}^R(\boldsymbol{x})\right] \cdot \boldsymbol{H}^B(\boldsymbol{x},t) = 0 \quad \text{for all } \boldsymbol{x}\in\partial\Omega^R, t>0}$$

- thin-slab approximation with $\mathcal{V}(\boldsymbol{x},t) = -dE_3(\boldsymbol{x},t)$ leads to

$$\mathcal{V}^R(\boldsymbol{x}^S,t) \overset{(t)}{*} j^B(t) \simeq -\int_{\boldsymbol{x}\in\partial\Omega^R} \mathcal{V}^B(\boldsymbol{x}|\boldsymbol{x}^S,t) \overset{(t)}{*} \boldsymbol{\tau}^R(\boldsymbol{x}) \cdot \boldsymbol{H}^i(\boldsymbol{x},t)\mathrm{d}l(\boldsymbol{x})$$
$$\text{for all } \boldsymbol{x}^S\in\Omega^R, t>0$$

?

Time-domain mutual coupling between power-ground structures (6)

TRANSMITTING SITUATION - **Huygens-Kirchhoff representations**

$$\boxed{\boldsymbol{H}^{i} = \boldsymbol{H}^{i;\mathrm{NF}} + \boldsymbol{H}^{i;\mathrm{IF}} + \boldsymbol{H}^{i;\mathrm{FF}}}$$

- FF = Far Field
- IF = Intermediate Field
- NF = Near Field

- FF constituent, for example

$$\boldsymbol{H}^{i;\mathrm{FF}}(\boldsymbol{x}^R, t) = -\epsilon_0 \int_{\boldsymbol{x}\in\partial\Omega^T} \frac{\partial_t \mathcal{V}^T(\boldsymbol{x}, t - |\boldsymbol{x}^R - \boldsymbol{x}|/c_0)}{4\pi|\boldsymbol{x}^R - \boldsymbol{x}|} \left[\boldsymbol{\xi}(\boldsymbol{x}^R - \boldsymbol{x})\boldsymbol{\xi}^{\mathrm{T}}(\boldsymbol{x}^R - \boldsymbol{x}) - \mathbb{I}\right] \cdot \boldsymbol{\tau}^T(\boldsymbol{x})\mathrm{d}l(\boldsymbol{x})$$

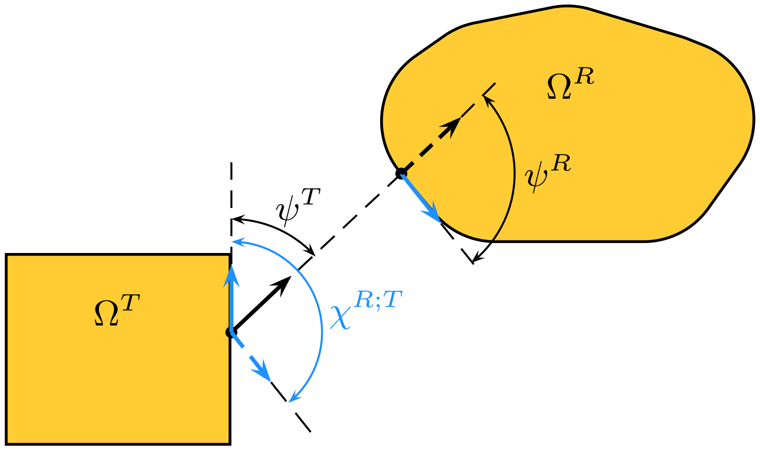
Time-domain mutual coupling between power-ground structures (7)

RIM-to-RIM RELATIONS - coupling-strength optimization

- NF/IF directional pattern, for example

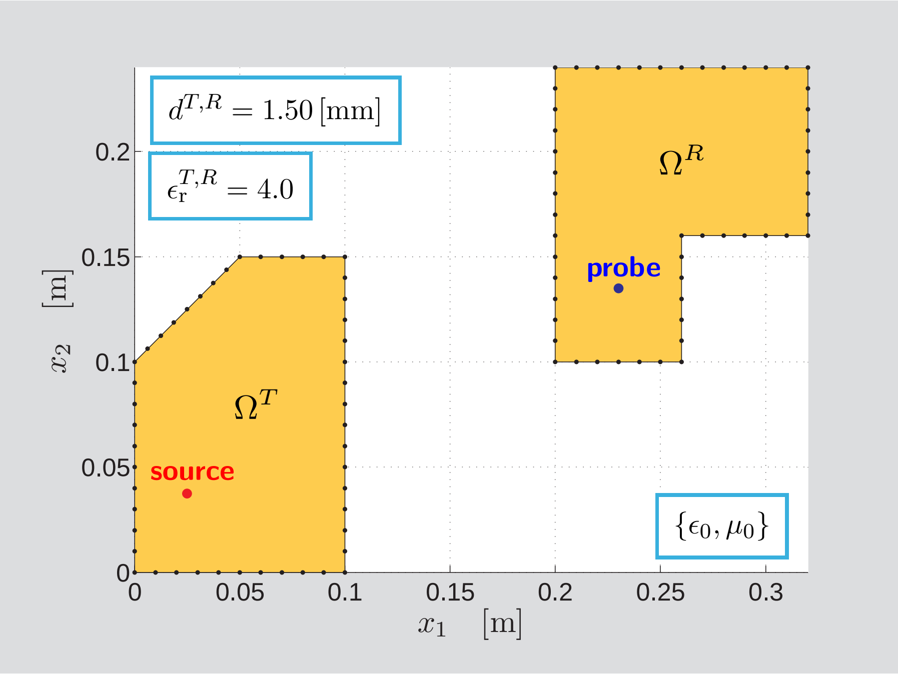
$$\boxed{\begin{aligned}&\boldsymbol{\tau}^R(\boldsymbol{x}^R)\cdot\left[3\boldsymbol{\xi}(\boldsymbol{x}^R-\boldsymbol{x})\boldsymbol{\xi}^{\mathrm{T}}(\boldsymbol{x}^R-\boldsymbol{x})-\mathbb{I}\right]\cdot\boldsymbol{\tau}^T(\boldsymbol{x})\\&=3\cos(\psi^R)\cos(\psi^T)-\cos(\chi^{R;T})\end{aligned}}$$

for all $\boldsymbol{x}^R \in \partial\Omega^R, \boldsymbol{x} \in \partial\Omega^T$

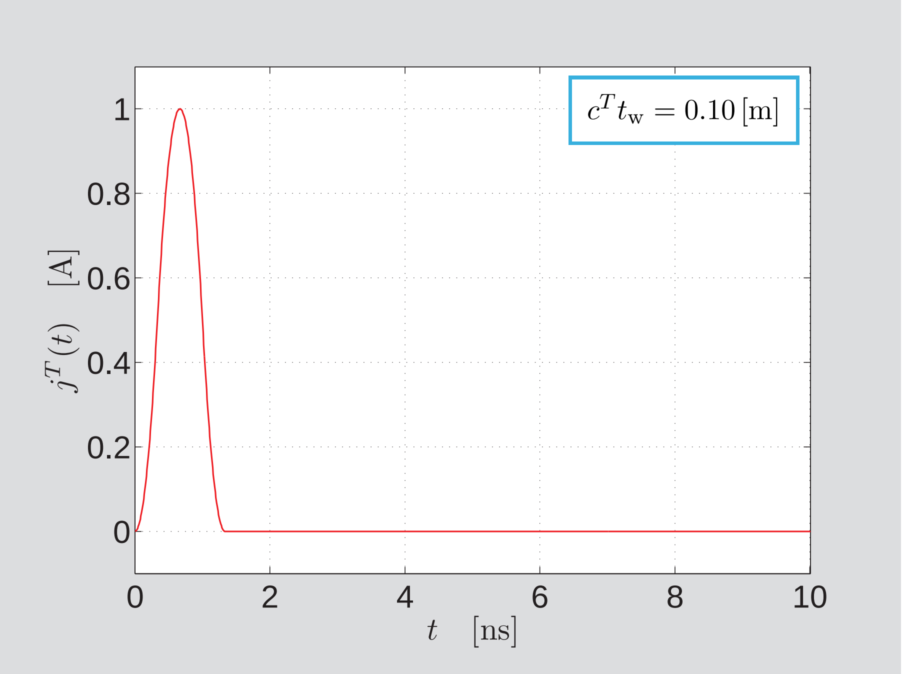


EM coupling: Numerical results (1)

PROBLEM CONFIGURATION

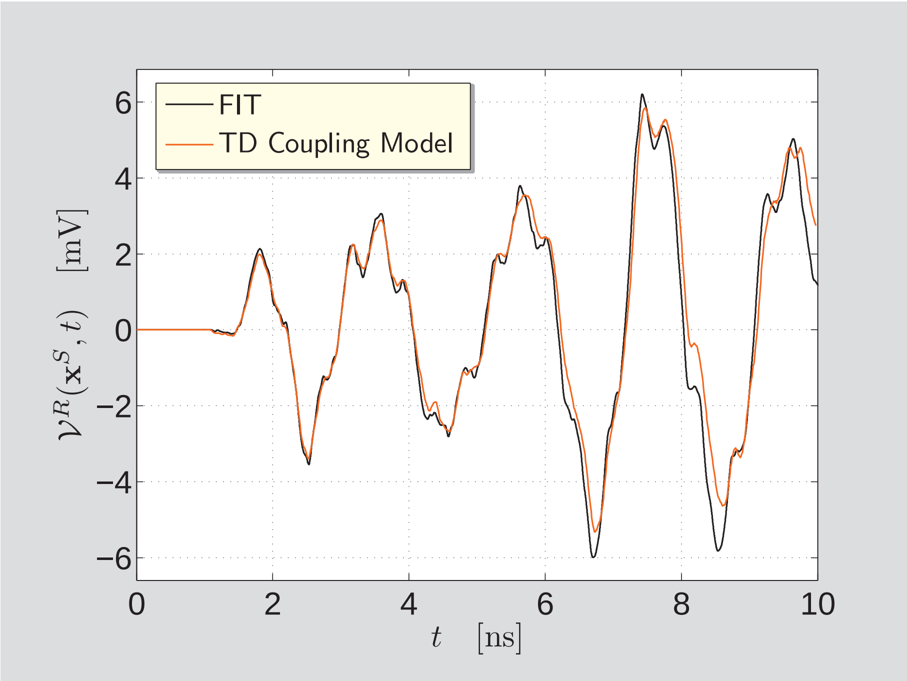


EXCITATION PULSE SHAPE



EM coupling: Numerical results (2)

OBSERVED PULSED RESPONSES



RIM-to-RIM relations (MATLAB®)

- $(48 + 52)$ line segments

3D-FIT (CST MWS®)

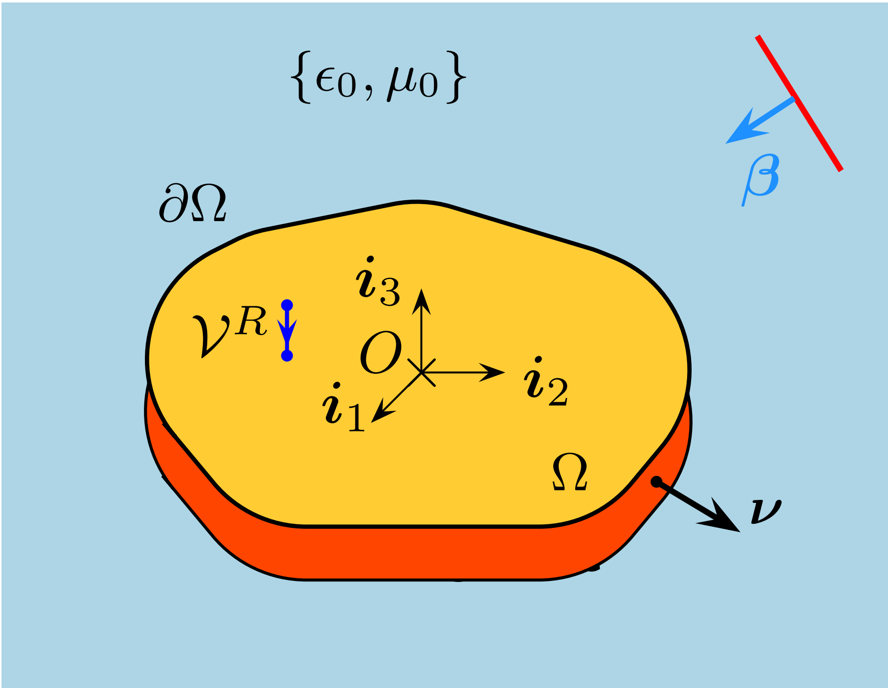
- $\sim 620\,000$ hexahedral meshcells

RADIATED TIME-DOMAIN SUSCEPTIBILITY

OF A P/G STRUCTURE

Time-domain radiated susceptibility of a power-ground structure (1)

PROBLEM CONFIGURATION



$$\boxed{\{\boldsymbol{E}^s, \boldsymbol{H}^s\} \triangleq \{\boldsymbol{E}^R - \boldsymbol{E}^i, \boldsymbol{H}^R - \boldsymbol{H}^i\}}$$

- s = scattered field
- R = total field
- i = incident field

M. Štumpf, "The pulsed EM plane-wave response of a thin planar antenna," *Journal EM Waves Appl.*, vol. 30, no. 9, 2016.

Time-domain radiated susceptibility of a power-ground structure (2)

SUSCEPTIBILITY ANALYSIS - RECIPROCITY #1

Domain $\mathbb{R}^3 \setminus (\mathcal{D} \cup \partial\mathcal{D})$		
	Scattered Field (s)	Testing Field (B)
Field State	$\{\boldsymbol{E}^s, \boldsymbol{H}^s\}$	$\{\boldsymbol{E}^B, \boldsymbol{H}^B\}$
Material State	$\{\epsilon_0, \mu_0\}\delta(t)$	$\{\epsilon_0, \mu_0\}\delta(t)$
Source State	0	0

- + causality (radiation) condition

$$\boxed{\begin{aligned}&\int_{\boldsymbol{x}\in\partial\mathcal{D}} \left[\boldsymbol{E}^B(\boldsymbol{x},t) \overset{*}{\times} \boldsymbol{H}^R(\boldsymbol{x},t) - \boldsymbol{E}^R(\boldsymbol{x},t) \overset{*}{\times} \boldsymbol{H}^B(\boldsymbol{x},t)\right] \cdot \boldsymbol{\nu}(\boldsymbol{x})\mathrm{d}A(\boldsymbol{x}) \\ =&\int_{\boldsymbol{x}\in\partial\mathcal{D}} \left[\boldsymbol{E}^B(\boldsymbol{x},t) \overset{*}{\times} \boldsymbol{H}^i(\boldsymbol{x},t) - \boldsymbol{E}^i(\boldsymbol{x},t) \overset{*}{\times} \boldsymbol{H}^B(\boldsymbol{x},t)\right] \cdot \boldsymbol{\nu}(\boldsymbol{x})\mathrm{d}A(\boldsymbol{x})\end{aligned}}$$

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Time-domain radiated susceptibility of a power-ground structure (3)

SUSCEPTIBILITY ANALYSIS - RECIPROCITY #1

$$\begin{aligned}&\int_{\boldsymbol{x}\in\partial\mathcal{D}}\left[\boldsymbol{E}^B(\boldsymbol{x},t)\overset{*}{\times}\boldsymbol{H}^R(\boldsymbol{x},t)-\boldsymbol{E}^R(\boldsymbol{x},t)\overset{*}{\times}\boldsymbol{H}^B(\boldsymbol{x},t)\right]\cdot\boldsymbol{\nu}(\boldsymbol{x})\mathrm{d}A(\boldsymbol{x})\\ =&\int_{\boldsymbol{x}\in\partial\mathcal{D}}\left[\boldsymbol{E}^B(\boldsymbol{x},t)\overset{*}{\times}\boldsymbol{H}^i(\boldsymbol{x},t)-\boldsymbol{E}^i(\boldsymbol{x},t)\overset{*}{\times}\boldsymbol{H}^B(\boldsymbol{x},t)\right]\cdot\boldsymbol{\nu}(\boldsymbol{x})\mathrm{d}A(\boldsymbol{x})\end{aligned}$$

- magnetic-wall boundary condition $[\boldsymbol{i}_3\times\boldsymbol{\nu}(\boldsymbol{x})]\cdot\boldsymbol{H}^B(\boldsymbol{x},t)=0\quad\text{for all }\boldsymbol{x}\in\partial\Omega, t>0$
- thin-slab approximation ($d\ll ct_\mathrm{w}$)

$$\int_{\boldsymbol{x}'\in\partial\Omega}\mathcal{V}^B(\boldsymbol{x}'|\boldsymbol{x}^S,t)\overset{(t)}{*}\boldsymbol{\tau}(\boldsymbol{x}')\cdot\boldsymbol{H}^R(\boldsymbol{x}',t)\mathrm{d}l(\boldsymbol{x}')=\int_{\boldsymbol{x}'\in\partial\Omega}\mathcal{V}^B(\boldsymbol{x}'|\boldsymbol{x}^S,t)\overset{(t)}{*}\boldsymbol{\tau}(\boldsymbol{x}')\cdot\boldsymbol{H}^i(\boldsymbol{x}',t)\mathrm{d}l(\boldsymbol{x}')$$

Time-domain radiated susceptibility of a power-ground structure (4)

SUSCEPTIBILITY ANALYSIS - RECIPROCITY #2

Domain $\mathcal{D} \subset \mathbb{R}^3$		
	Total Field (R)	Testing Field (B)
Field State	$\{\boldsymbol{E}^R, \boldsymbol{H}^R\}$	$\{\boldsymbol{E}^B, \boldsymbol{H}^B\}$
Material State	$\{\kappa(t), \mu_0\delta(t)\}$	$\{\kappa^B(t), \mu_0\delta(t)\}$
Source State	0	$\boldsymbol{J}^B = j^B(t)\delta(\boldsymbol{x} - \boldsymbol{x}^S)\boldsymbol{i}_3$

$$
\begin{aligned}
&\int_{\boldsymbol{x}' \in \partial\Omega} \mathcal{V}^B(\boldsymbol{x}'|\boldsymbol{x}^S, t) \overset{(t)}{*} \boldsymbol{\tau}(\boldsymbol{x}') \cdot \boldsymbol{H}^R(\boldsymbol{x}', t)\mathrm{d}l(\boldsymbol{x}') \\
&= -\mathcal{V}^R(\boldsymbol{x}^S, t) \overset{(t)}{*} j^B(t) + \partial_t \delta\kappa(t) \overset{(t)}{*} d^{-1} \int_{\boldsymbol{x}' \in \Omega} \mathcal{V}^B(\boldsymbol{x}'|\boldsymbol{x}^S, t) \overset{(t)}{*} \mathcal{V}^R(\boldsymbol{x}', t)\mathrm{d}A(\boldsymbol{x}')
\end{aligned}
$$

Time-domain radiated susceptibility of a power-ground structure (5)

SUSCEPTIBILITY ANALYSIS - RECIPROCITY #1 & #2

- CHOOSE (for simplicity): $\boxed{\kappa(t) = \kappa^B(t), \text{ i.e. } \delta\kappa(t) = 0}$

$$\Rightarrow \quad \boxed{\mathcal{V}^R(\boldsymbol{x}^S, t) \overset{(t)}{*} j^B(t) = -\int_{\boldsymbol{x}' \in \partial\Omega} \mathcal{V}^B(\boldsymbol{x}'|\boldsymbol{x}^S, t) \overset{(t)}{*} \boldsymbol{\tau}(\boldsymbol{x}') \cdot \boldsymbol{H}^i(\boldsymbol{x}', t)\mathrm{d}l(\boldsymbol{x}')}$$

- USE:

$$\boldsymbol{H}^i = (\epsilon_0/\mu_0)^{1/2}\boldsymbol{\beta} \times \boldsymbol{\alpha}\, e^i(t - \boldsymbol{\beta} \cdot \boldsymbol{x}/c_0)$$

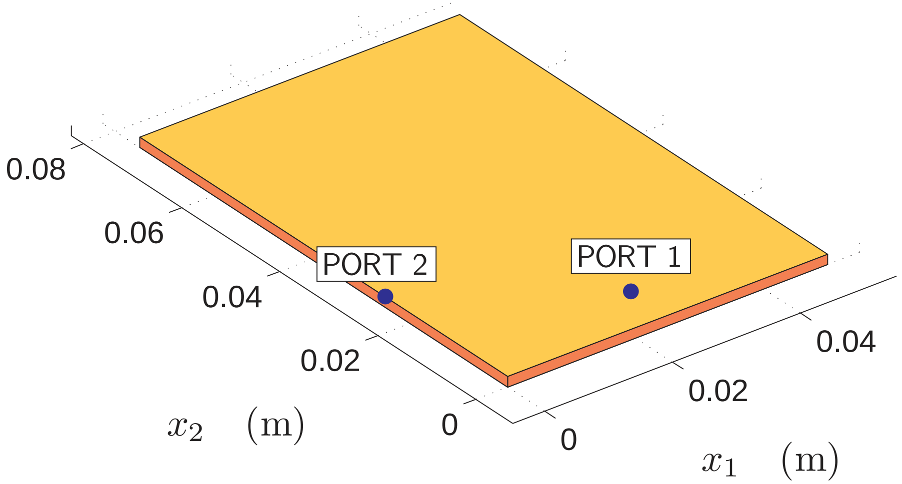
$$\mathcal{V}^B(\boldsymbol{x}|\boldsymbol{x}^S, t) = \mu_0 d\partial_t j^B(t) \overset{(t)}{*} G^B(\boldsymbol{x}|\boldsymbol{x}^S, t) \quad \Rightarrow$$

$$\boxed{\mathcal{V}^R(\boldsymbol{x}^S, t) = -d\, c_0^{-1}(\boldsymbol{\beta} \times \boldsymbol{\alpha}) \cdot \partial_t e^i(t) \overset{(t)}{*} \int_{\boldsymbol{x}' \in \partial\Omega} \overset{\star}{G}^B(\boldsymbol{x}'|\boldsymbol{x}^S, t - \boldsymbol{\beta} \cdot \boldsymbol{x}'/c_0)\boldsymbol{\tau}(\boldsymbol{x}')\mathrm{d}l(\boldsymbol{x}')}$$

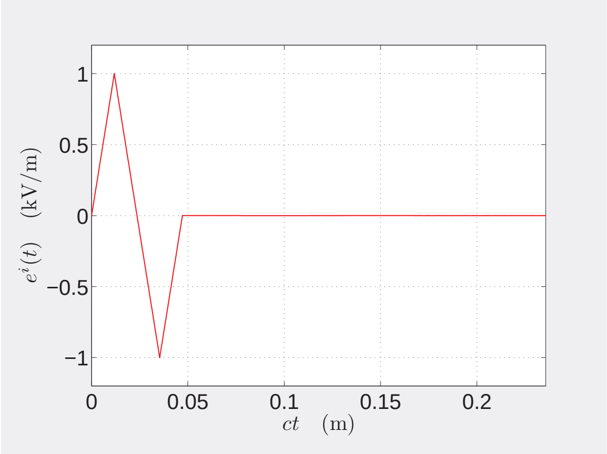
⋆M. Štumpf, "Time-domain analysis of rectangular power-ground structures with relaxation," *IEEE Trans. Electromagn. Compat.*, vol. 56, no. 5, October 2014.

EM susceptibility: Numerical results (1)

PROBLEM CONFIGURATION



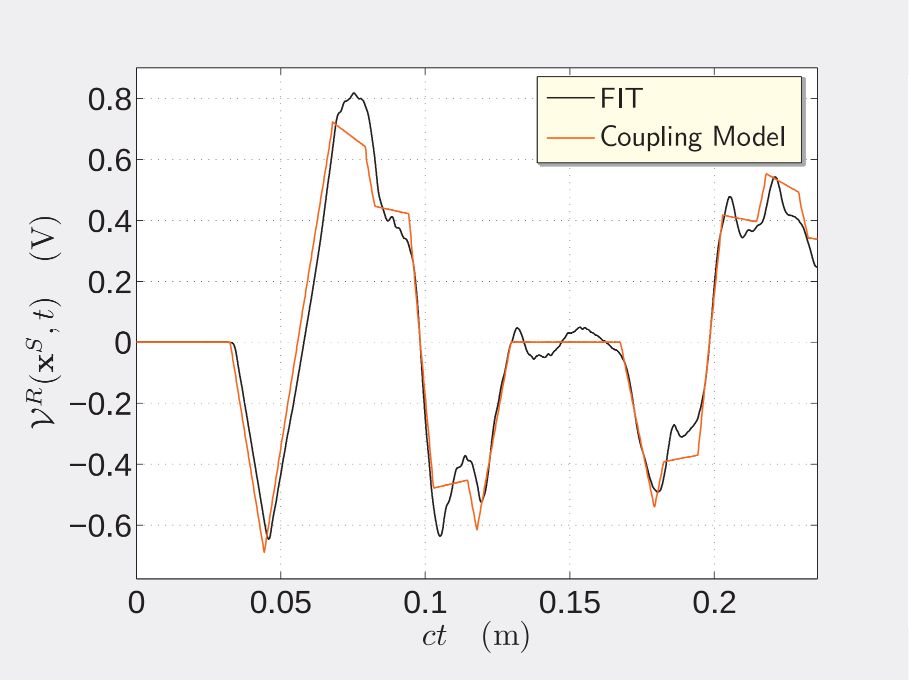
- Plane-wave signature



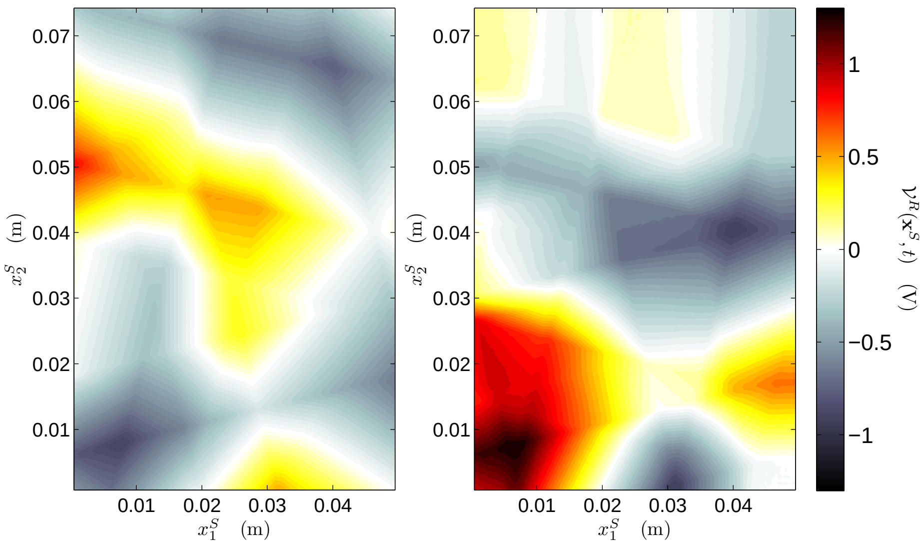
- PW: $\{\phi, \theta\} = \{\pi/4, \pi/4\}$
- $\epsilon_\mathrm{r} = 4.50$, $\sigma = 0.02\,(\mathrm{S/m})$
- $d = 1.50\,(\mathrm{mm})$

EM susceptibility: Numerical results (2)

OBSERVED PULSED RESPONSES



- $ct = \sqrt{2}/30\,(\mathrm{m})$
- $ct = \sqrt{2}/20\,(\mathrm{m})$



- $(x_1^S, x_2^S) = (50, 100)\,(\mathrm{mm})$

Conclusions

- Pulsed EM mutual coupling between two P/G structures was described via the novel rim-to-rim closed-form time-domain expressions (efficient, easy-to-implement, explicit)
  - The constructed expressions can be used for optimizing pulsed signal transfer and coupling strength between P/G structures
- Pulsed EM plane-wave response of a P/G structure described analytically
  - The constructed expressions can be used to determine the vulnerability of a P/G structure to an external EM pulsed disturbance
- Huge savings of computational resources with respect to purely numerical approaches such as FDTD and FIT
- Physical insights into EM mutual-coupling/radiated-susceptibility phenomena

THANKS FOR YOUR ATTENTION