Random Coupling Model: Statistical Predictions of Interference in Complex Enclosures

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Outline

• The Problem: Electromagnetic Interference

• Our Approach – A Wave Chaos Statistical Description

• The Random Coupling Model (RCM)

• Examples of the RCM in Practice

• Conclusions
Electromagnetic Interference and High-Power Microwave Effects on Electronics

How to defend electronics from electromagnetic interference?

Electromagnetic Interference (EMI)
Electromagnetic Compatibility (EMC)
Electromagnetic Compatibility Issues in Automobiles

How to defend electronics from electromagnetic interference?
What Happens to Electronics Housed in Electrically-Large Enclosures?

Examples: Aircraft cockpit, Rooms, Automobile engine electronics, Computer, etc. System size >> Wavelength

Empirical evidence suggests that some electronics are susceptible under some conditions …

Many failure modes have been identified:

Internal circuit signal disruption – spurious signal generation
Front-end diodes produce baseband + harmonic signal input
Burnout of traces, contacts, components
Electromagnetic Coupling to Enclosures and Circuits
A Complicated Problem

Arbitrary Enclosure (with losses “1/Q”)

- Coupling of external radiation to computer chips is a complex process:
  - Apertures
  - Resonant cavities
  - Transmission Lines
  - Circuit Elements

- System Size >> Wavelength

What can we say about the nature of fields and induced voltages inside such a cavity?

- Statistical Description using Wave Chaos!!

Rather than make predictions for a specific configuration, determine a probability distribution function (PDF) of the relevant quantities.
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Wave Chaos?

1) Waves do not have trajectories

It makes no sense to talk about “diverging trajectories” for waves

2) Linear wave systems can’t be chaotic

Maxwell’s equations, Schrödinger’s equation are linear

3) However in the semiclassical limit, you can think about rays

In the ray-limit it is possible to define chaos

“ray chaos”

Wave Chaos concerns solutions of linear wave equations which, in the semiclassical limit, can be described by chaotic ray trajectories
From Classical to Wave Chaos

wave

Quantum

Semiclassical limit
(quantum chaos)

ray trajectory

Classical
(chaos)
Random Matrix Theory (RMT) and Wave Chaos
Wigner; Dyson; Mehta; Bohigas …

The RMT Approach:
Complicated Hamiltonian: e.g. Nucleus: Solve $H\Psi = E\Psi$

Replace with a Hamiltonian with matrix elements chosen randomly from a Gaussian distribution
Examine the statistical properties of the resulting Hamiltonians

$$H = \begin{pmatrix}
- & - & - & \cdots \\
- & - & - & \\
\vdots & & & \\
\end{pmatrix}$$

Universality Classes of RMT:
Orthogonal (real matrix elements, $\beta = 1$)
Unitary (complex matrix elements, $\beta = 2$)
Symplectic (quaternion matrix elements, $\beta = 4$)

Hypothesis: Complicated Quantum/Wave systems that have chaotic classical/ray counterparts possess universal statistical properties described by Random Matrix Theory (RMT) “BGS Conjecture”

This hypothesis has been tested in many systems:
Nuclei, atoms, molecules, quantum dots, acoustics (room, solid body, seismic), optical resonators, random lasers,…

Some Questions:
Is this hypothesis supported by data in other systems?
What new applications are enabled by wave chaos?
Can losses / decoherence be included?
What causes deviations from RMT predictions?

Cassati, 1980
Bohigas, 1984
Where is Wave Chaos Found?

Nuclear scattering:
Ericson fluctuations

Transport in 2D quantum dots:
Universal Conductance Fluctuations

Electromagnetic Cavities:
Complicated $S_{11}$, $S_{22}$, $S_{21}$ versus frequency

$\lambda_{\text{deBroglie}} \ll \text{nucleus size}$

$\lambda_{\text{deBroglie}} \ll \text{billiard size}$

$|S_{xx}|$ vs frequency

$|S_{11}|$, $|S_{22}|$, $|S_{21}|$ vs B (T)

Proton energy
$\frac{d\sigma}{d\omega}$

$\lambda_{\text{deBroglie}} \ll \text{billiard size}$
The extreme sensitivity of the properties of wave chaotic systems to small perturbations suggests a statistical approach (RMT).

The Random Coupling Model incorporates the underlying chaos into a quantitative statistical description of the wave properties of real systems.
Our approach treats all objects of interest as “ports”

*Incident rf energy enters the enclosure through one or more ports*

*The energy reverberates and is absorbed by one or more ports inside the enclosure*

*Formulate a quantitative statistical theory of absorbed energy*
N-Port Description of an Arbitrary Scattering System

N Ports
- Voltages and Currents,
- Incoming and Outgoing Waves

\[ S = (Z + Z_0)^{-1}(Z - Z_0) \]

\[ Z(\omega), S(\omega) \]
- Complicated Functions of frequency
- Detail Specific (Non-Universal)

**S matrix**
\[
\begin{bmatrix}
V_{-1}^- \\
V_{-2}^- \\
\vdots \\
V_{-N}^-
\end{bmatrix}
= [S] \cdot
\begin{bmatrix}
V_{+1}^+ \\
V_{+2}^+ \\
\vdots \\
V_{+N}^+
\end{bmatrix}
\]

**Z matrix**
\[
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_N
\end{bmatrix}
= [Z] \cdot
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_N
\end{bmatrix}
\]
Traditional Approach to Describing Wave Chaotic Scattering Systems – the Scattering Matrix

C. M. Marcus (1992)

2-D Electron Gas

2-D Electron Gas

electron mean free path >> system size
electron wavelength << system size

**Ballistic Quantum Transport**

\[ G = \frac{2e^2}{h} \sum_{n=1}^{N_1} \sum_{m=1}^{N_2} |S_{nm}|^2 \]

Landauer-Büttiker

Quantum interference → Fluctuations in \( G \sim e^2/h \)

“Universal Conductance Fluctuations”

In contrast, our approach uses the Impedance (Z) description of wave scattering
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Statistical Model of Impedance (Z) Matrix


\[ Z_{ij}(\omega) = -\frac{j}{\pi} \sum_{\text{modes } n} R_{Ri}^{1/2}(\omega_n) R_{Rj}^{1/2}(\omega_n) \frac{\Delta \omega_n^2 \omega_{in} \omega_{jn}}{\omega^2 (1 + jQ^{-1}) - \omega_n^2} \]

\[ Z_{R}(\omega) = R_{R}(\omega) + jX_{R}(\omega) \]

- **Radiation Resistance** \( R_{Ri}(\omega) \)
- **\( \Delta \omega^2 \) - mean spectral spacing**
- **\( Q \) -quality factor**
- **\( \omega_n \) - random spectrum**
- **\( w_{in} \) - Guassian Random variables**
Universal Fluctuations are Usually Obscured by Non-Universal System-Specific Details

The Most Common Non-Universal Effects:
1) Non-Ideal Coupling between external scattering states and internal modes (i.e. Antenna/Port properties)
2) Short-Orbits between the antenna and fixed walls of the billiards
The Random Coupling Model
Divide and Conquer!

**Coupling Problem**

Solution: Radiation Impedance Matrix $Z_{\text{rad}}$
+ Short Orbits

$$Z = \bar{Z} + \tilde{Z} = iX_{\text{Rad}} + i\xi R_{\text{Rad}}$$

- Mean part
- Fluctuating Part (depends on $\alpha$)
- $<\text{Im}\xi> = 1$
- $<\text{Re}\xi> = 0$

**Enclosure Problem**

Solution: Random Matrix Theory;
Electromagnetic statistical properties are governed by Loss Parameter $\alpha = k^2/(\Delta k_n^2 Q) = \delta f_{3\text{dB}}/\Delta f_{\text{spacing}}$

IEEE Trans. EMC 54, 758 (2012)
Electromagnetics 26, 3 (2006)
Electromagnetics 26, 37 (2006)
Theory of Non-Universal Wave Scattering Properties
Including Imperfect Coupling and Short Orbits

1-Port, Loss-less case: \( Z_{\text{cavity}} = iX_{\text{avg}} + R_{\text{avg}} \left( i\xi \right) \)

\[
Z_{\text{avg}} = Z_{\text{Rad}} + R_{\text{Rad}} \sum_{\text{orb}} \sqrt{D_{\text{orb}}} e^{ik(l+L_{\text{port}})-i\pi/4}
\]

Universally Fluctuating Complex Quantity with Mean 1 (0) for the Real (Imaginary) Part. Predicted by RMT

Complex Radiation Impedance (characterizes the non-universal coupling)

Semiclassical Expansion over Short Orbits

Index of ‘Short Orbit’ of length \( l \)

Stability of orbit

Action of orbit

Orbit Stability Factor:
- Segment length
- Angle of incidence
- Radius of curvature of wall
Assumes foci and caustics are absent!

Orbit Action:
- Segment length
- Wavenumber
- Number of Wall Bounces

Perfectly absorbing boundary

The waves do not return to the port

Inclusion of loss: $P_\alpha(Z)$

$\alpha = 3$-dB bandwidth / mean-spacing

Fyodorov+Savin JETP Lett. 80, 725 (2004)

Comparison of data (symbols) and RMT (solid lines)

Mean of $P_\alpha(z)$

\[ E\left\{ \text{Re} \left( \hat{\lambda}_z \right) \right\} = 1 \quad \text{Independent of } \alpha \]

Variance of $P_\alpha(z)$

\[ \sigma_{\text{Re}[\hat{\lambda}_z]}^2 = \sigma_{\text{Im}[\hat{\lambda}_z]}^2 = \frac{1}{\pi \alpha} \frac{1}{k^2/(\Delta k_n^2 Q)} = \frac{1}{\pi \alpha} \quad \alpha >> 1 \]
Inclusion of loss: $P_\alpha(Z)$

PDF of the eigenvalues $\lambda_z$ of the universal impedance matrix ($z$)

$\alpha = 3$-dB bandwidth / mean-spacing

Fyodorov+Savin JETP Lett. 80, 725 (2004)

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Microwave Cavity Analog of a 2D Quantum Infinite Square Well

Table-top experiment!

\[ \nabla^2 \Psi_n + \frac{2m}{\hbar^2} (E_n - V) \Psi_n = 0 \]

with \( \Psi_n = 0 \) at boundaries

\[ \nabla^2 E_{z,n} + k_n^2 E_{z,n} = 0 \]

with \( E_{z,n} = 0 \) at boundaries

The only propagating mode for \( f < c/d \):

\( d \approx 8 \text{ mm} \)

An empty “two-dimensional” electromagnetic resonator

Metal walls

Schrödinger equation

Helmholtz equation

Bow-Tie Billiard

Stöckmann + Stein, 1990
Doron+Smilansky+Frenkel, 1990
Sridhar, 1991
Richter, 1992
The Experiment:
A simplified model of wave-chaotic scattering systems

ports

A thin hollow metal box

Coaxial cable

Side view

$\lambda$

$E_z$

21.6 cm

43.2 cm

0.8 cm
Microwave-Cavity Analog of a 2D Infinite Square Well with Coupling to Scattering States

Network Analyzer [measures Scattering (S)-matrix vs. frequency]

We measure from 500 MHz – 19 GHz, covering about 750 modes in the semi-classical limit
Testing Insensitivity to System Details

Cross Section View of Port

CAVITY LID

Diameter (2a)

CAVITY BASE

Coaxial Cable

Metallic Perturbations

Freq. Range: 9 to 9.75 GHz
Cavity Height: h = 7.87 mm
Statistics drawn from 100,125 pts.

Coaxial Cable

\[ Z_{\text{Cavity}} = iX_{\text{Rad}} + R_{\text{Rad}} i \xi \]

RAW Impedance PDF

\[ z = \frac{R_{\text{Cavity}}}{R_{\text{Rad}}} + i \frac{X_{\text{Cavity}} - X_{\text{Rad}}}{R_{\text{Rad}}} \]

NORMALIZED Impedance PDF

Probability Density

\[ \text{Im}(Z_{\text{Cavity}})(\Omega) \]

2a = 0.635 mm

2a = 1.27 mm

\[ \text{Im}(z) \]

2a = 0.635 mm

2a = 1.27 mm
Nonuniversal Properties Captured by the Extended RCM

Empty Cavity Data

Resistance (Ω)

Frequency (GHz)

Reactance (Ω)

Frequency (GHz)

Smoothed Resistance and Reactance,

\(\alpha \sim 1\)

Random Matrix Theory describes the fluctuations away from \(Z^{(L)}\)

Data and Theory smoothed with the same 125-cm (240 MHz window) low-pass filter

Theory includes all orbits to 200 cm length

The Random Coupling Model Applied to 3-Dimensional Enclosures

1. Inject microwaves at port 1 and measure induced voltage at port 2
2. Rotate mode-stirrer and repeat
3. Plot the PDF of the induced voltage and compare with RCM prediction

Uncovering Universal Impedance Statistics

Induced Voltage Statistics


US Naval Research Laboratory collaboration
RCM Predictions for Electrically-Large Apertures

G. Gradoni, et al.

\[ Y_{cav} = i \text{Im}\left( Y_{rad} \right) + G_{rad}^{1/2} i \xi G_{rad}^{1/2} \]

\[ Y_{rad}^{ss} (k_0 = \omega / c) = \sqrt{\frac{\varepsilon}{\mu}} \int \frac{d^3k}{(2\pi)^3} \frac{2ik_0}{k_0^2 - k^2} \bar{e}_s \cdot \Delta_{2} \cdot \bar{e}_{s'} \]

Aperture Modes

RCM for admittance

US Naval Research Laboratory collaboration
Induced Voltage Statistics for Enclosures with Mixed Regular and Chaotic Behavior

Random Coupling Model still works!

\[ Z_{ij} = Z_{ij,\text{Regular}} + Z_{ij,\text{Chaotic}} \]

Relevant variables:
- 3D enclosures with parallel walls
- Illuminate through regular and irregular apertures

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• Related Work

• Conclusions
Conclusions

The Random Coupling Model constitutes a comprehensive (statistical) description of the wave properties of wave-chaotic systems in the short wavelength limit.

We believe the RCM is of value to the EMC / EMI community for predicting the statistics of induced voltages on objects in complex enclosures, for example.

This description should apply to any wave system in the ‘mesoscopic’, ‘mid-frequency’, … limit.

Acoustics
Mechanical vibrations
Quantum mechanical
Electromagnetic
…

RCM Review articles:
G. Gradoni, et al., Wave Motion 51, 606 (2014)

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Students and Post-docs

• Present Students Wave Chaos

• Recent Post-docs

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