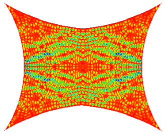

Random Coupling Model: Statistical Predictions of Interference in Complex Enclosures

Steven M. Anlage

with Bo Xiao, Edward Ott, Thomas Antonsen

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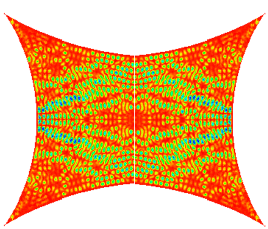
ELECTRICAL & COMPUTER ENGINEERING
A. JAMES CLARK SCHOOL of ENGINEERING

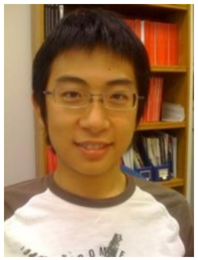


COST Action IC1407 (ACCREDIT)
University of Nottingham
4 April, 2016

Research funded by AFOSR and ONR

The Maryland Wave Chaos Group

Graduate Students (current + former)



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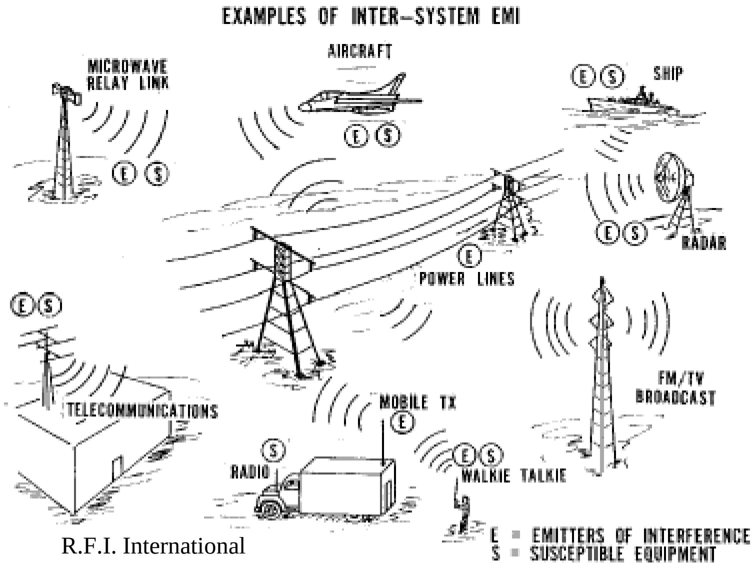
Funding: ONR, AFOSR, DURIP

Outline

- The Problem: Electromagnetic Interference
- Our Approach – A Wave Chaos Statistical Description
- The Random Coupling Model (RCM)
- Examples of the RCM in Practice
- Conclusions

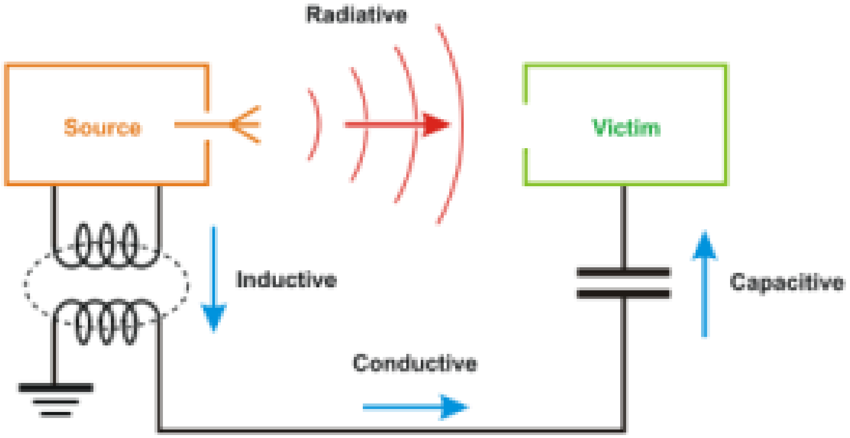
Electromagnetic Interference and High-Power Microwave Effects on Electronics

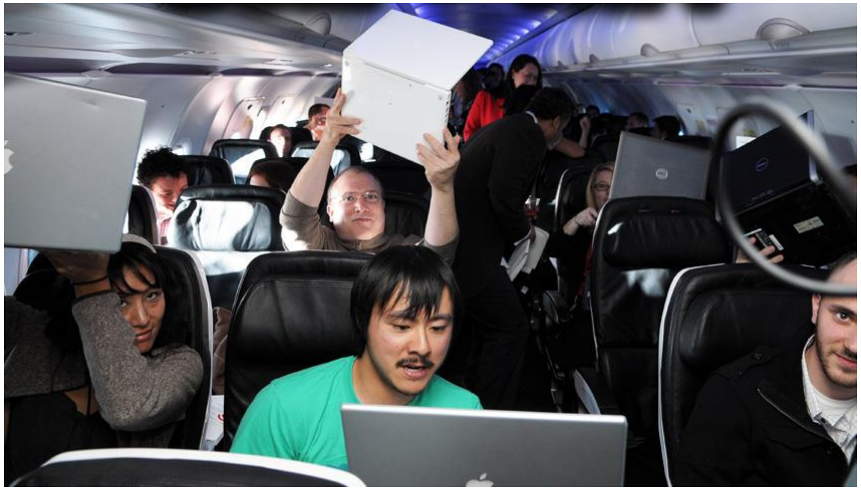
How to defend electronics from electromagnetic interference?



R.F.I. International

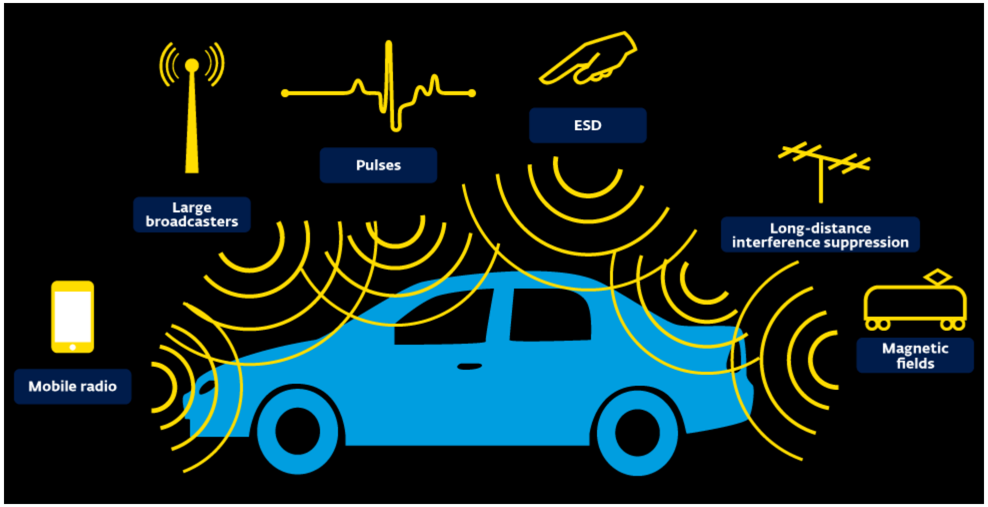
Electromagnetic Interference (EMI)
Electromagnetic Compatibility (EMC)

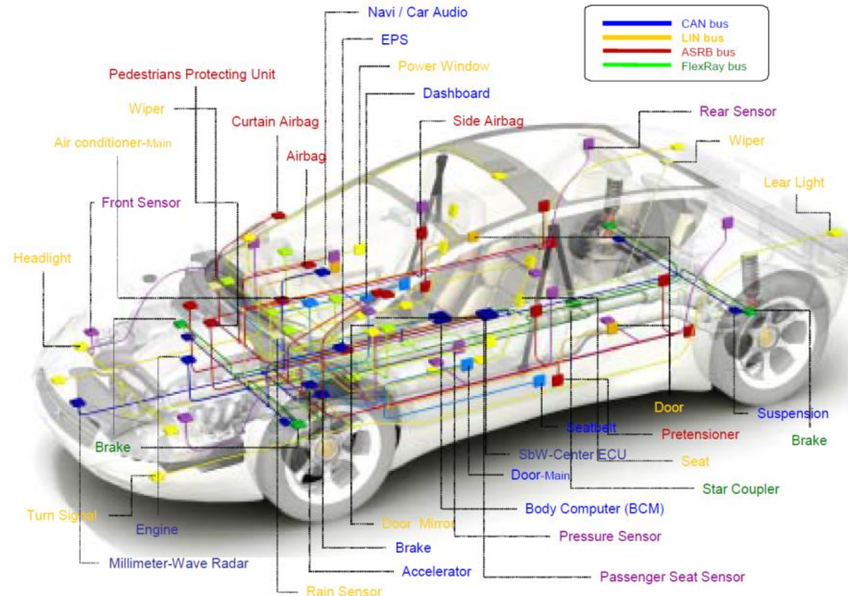




Electromagnetic Compatibility Issues in Automobiles

How to defend electronics from electromagnetic interference?

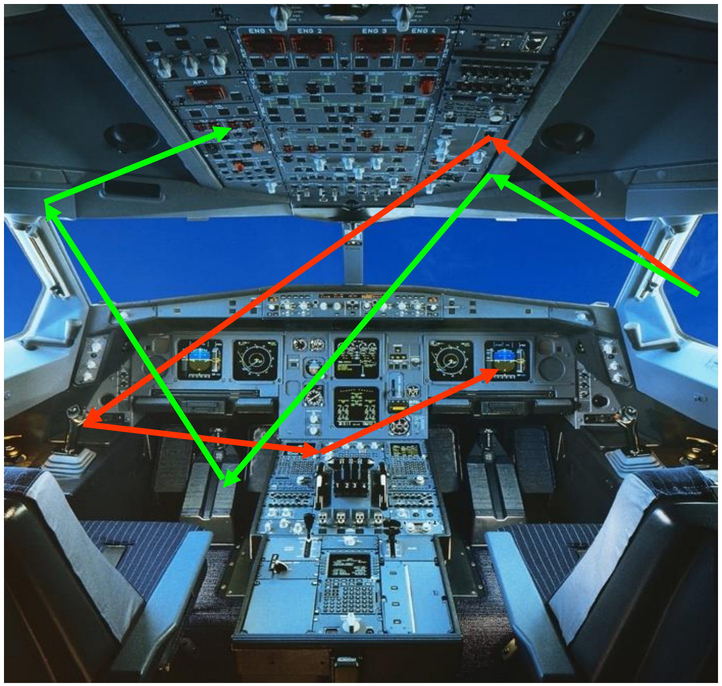


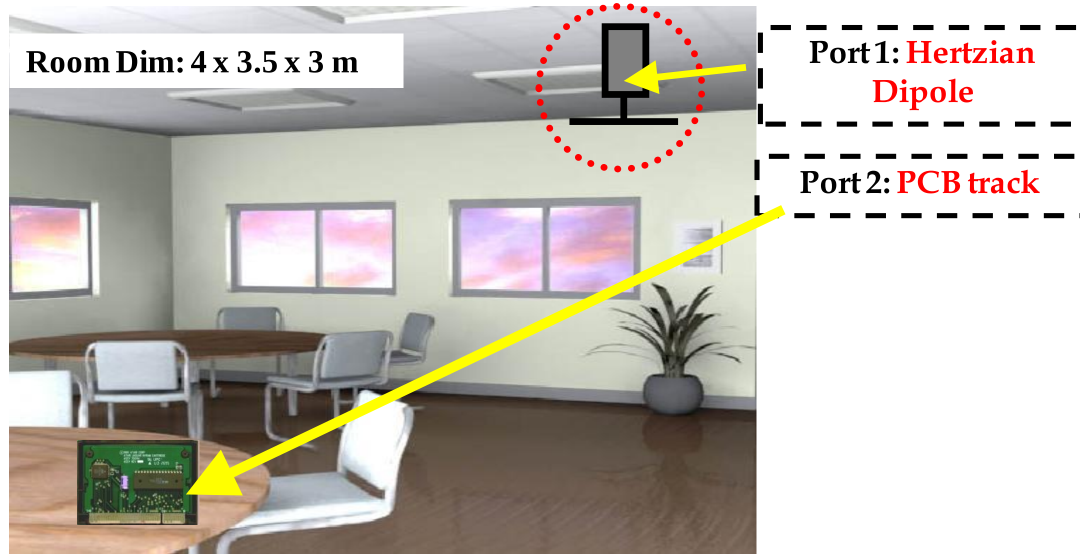


What Happens to Electronics Housed in Electrically-Large Enclosures?

Examples: Aircraft cockpit, Rooms, Automobile engine electronics, Computer, etc. **System size >> Wavelength**

Empirical evidence suggests that some electronics are susceptible under some conditions …



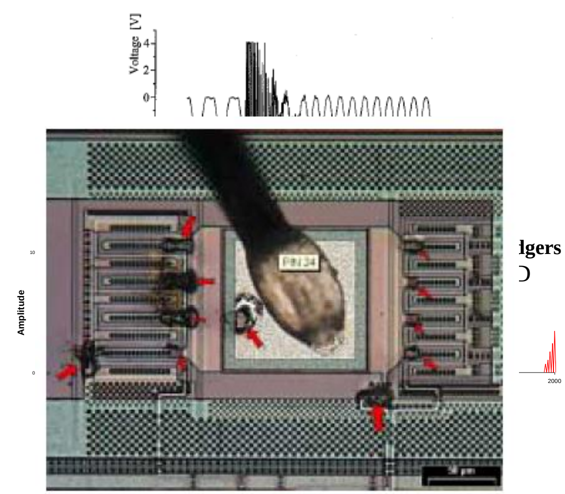


Many failure modes have been identified:

Internal circuit signal disruption – spurious signal generation

Front-end diodes produce baseband + harmonic signal input

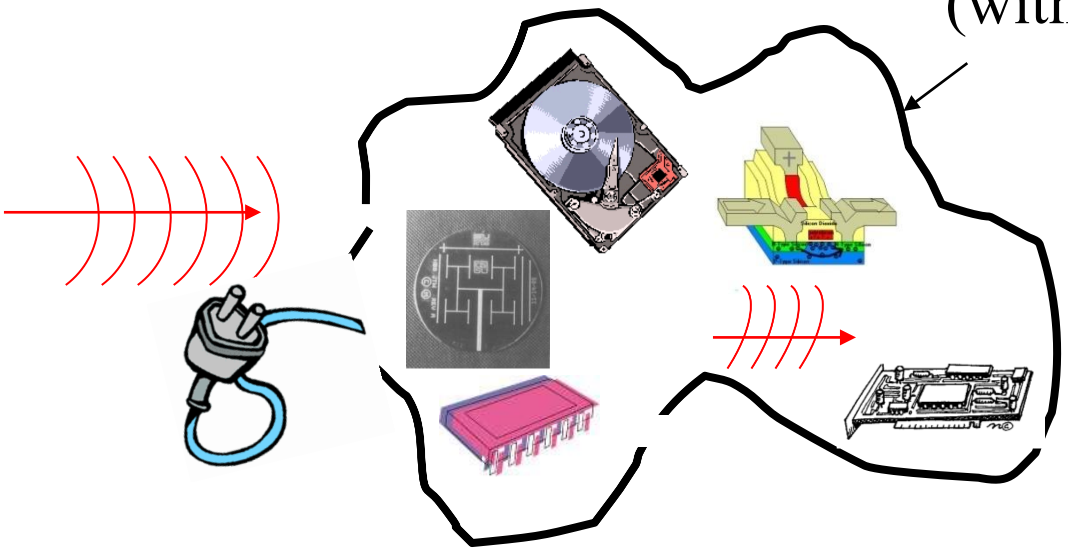
Burnout of traces, contacts, components



Electromagnetic Coupling to Enclosures and Circuits

A Complicated Problem

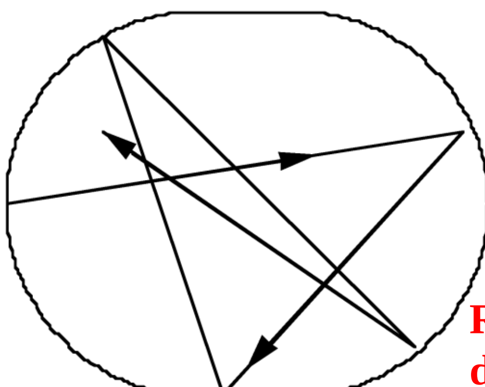
Arbitrary Enclosure
(with losses “1/Q”)



- Coupling of external radiation to computer chips is a complex process:
  - Apertures
  - Resonant cavities
  - Transmission Lines
  - Circuit Elements
- **System Size >> Wavelength**

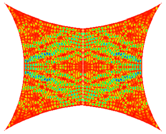
What can we say about the nature of fields and induced voltages inside such a cavity?

Chaotic Ray Trajectories



- Statistical Description using Wave Chaos!!

Rather than make predictions for a specific configuration, determine a probability distribution function (PDF) of the relevant quantities



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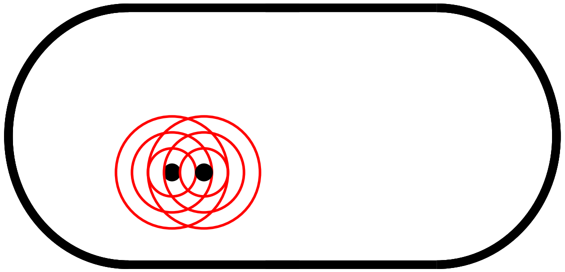
Wave Chaos?

1) Waves do not have trajectories



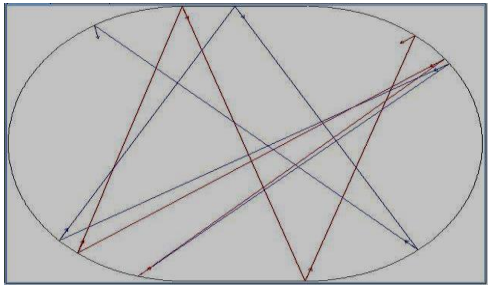
It makes no sense to talk about “diverging trajectories” for waves

2) Linear wave systems can’t be chaotic

Maxwell’s equations, Schrödinger’s equation are linear

3) However in the semiclassical limit, you can think about <u>rays</u>

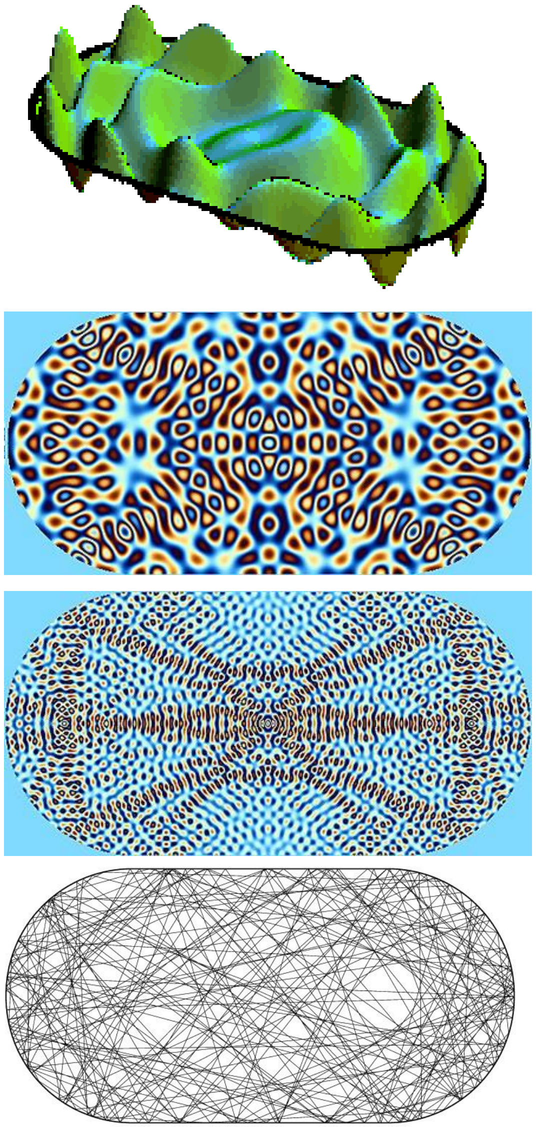
In the ray-limit it is possible to define chaos



“ray chaos”

Wave Chaos concerns solutions of linear wave equations which, in the semiclassical limit, can be described by chaotic ray trajectories

From Classical to Wave Chaos



wave

Quantum

Semiclassical limit

(quantum chaos)

ray
trajectory

Classical

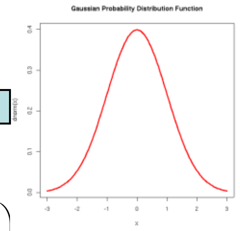
(chaos)

Random Matrix Theory (RMT) and Wave Chaos

Wigner; Dyson; Mehta; Bohigas …

The RMT Approach:

Complicated Hamiltonian: e.g. Nucleus: Solve $H\Psi = E\Psi$



Replace with a Hamiltonian with matrix elements chosen randomly from a Gaussian distribution

Examine the <u>statistical</u> properties of the resulting Hamiltonians

$$H = \begin{pmatrix} - & - & - & - & \\ - & - & - & - & \cdots \\ - & - & - & - & \\ - & - & - & - & \\ & & \vdots & & \end{pmatrix}$$

Universality Classes of RMT:
- Orthogonal (real matrix elements, $\beta = 1$)
- Unitary (complex matrix elements, $\beta = 2$)
- Symplectic (quaternion matrix elements, $\beta = 4$)

<u>Hypothesis</u>: Complicated Quantum/Wave systems that have chaotic classical/ray counterparts possess <u>universal statistical properties</u> described by Random Matrix Theory (RMT) "BGS Conjecture"

Cassati, 1980
Bohigas, 1984

This hypothesis has been tested in many systems:

Nuclei, atoms, molecules, quantum dots, acoustics (room, solid body, seismic), optical resonators, random lasers,…

Some Questions:

Is this hypothesis supported by data in other systems?

What new applications are enabled by wave chaos?

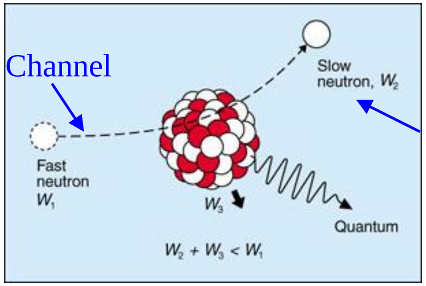
Can losses / decoherence be included?

What causes deviations from RMT predictions?

Where is Wave Chaos Found?

Nuclear scattering: Ericson fluctuations

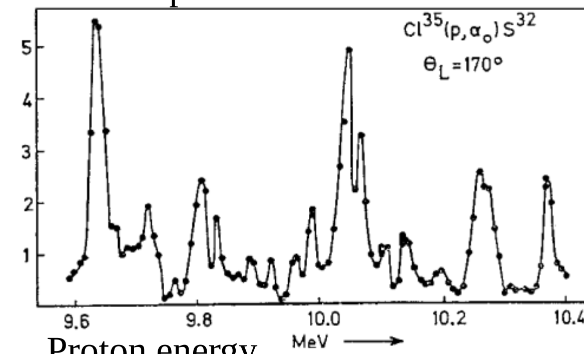
Incoming Channel



Outgoing Channel

Compound nuclear reaction

$$\frac{d\sigma}{d\omega}$$



Proton energy

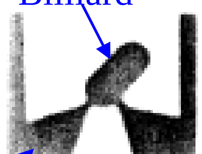
$\lambda_{deBroglie}$ << nucleus size

Transport in 2D quantum dots: Universal Conductance Fluctuations

$\lambda_{deBroglie}$ << billiard size

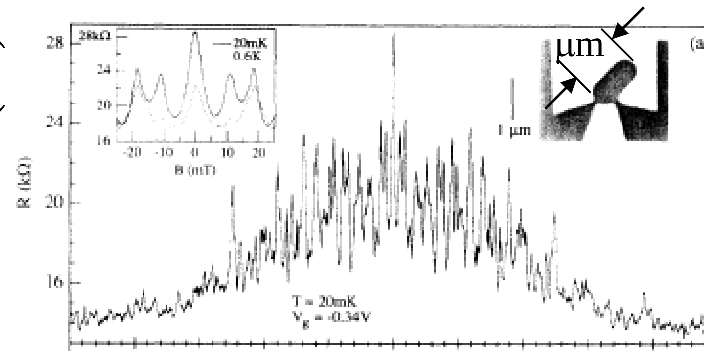
Billiard

Incoming Channel



Outgoing Channel

Resistance (kΩ)



B (T)

Outgoing Voltage waves

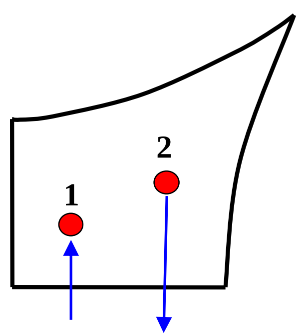
Scattering matrix

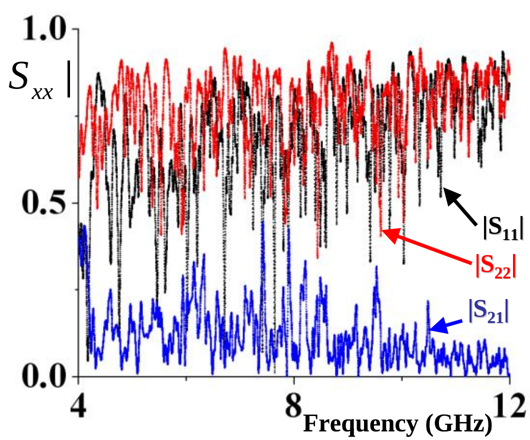
$$\begin{bmatrix} V^-_1 \\ V^-_2 \\ \bullet \\ \bullet \\ \bullet \\ V^-_N \end{bmatrix} = [S] \begin{bmatrix} V^+_1 \\ V^+_2 \\ \bullet \\ \bullet \\ \bullet \\ V^+_N \end{bmatrix}$$

Incoming Voltage waves

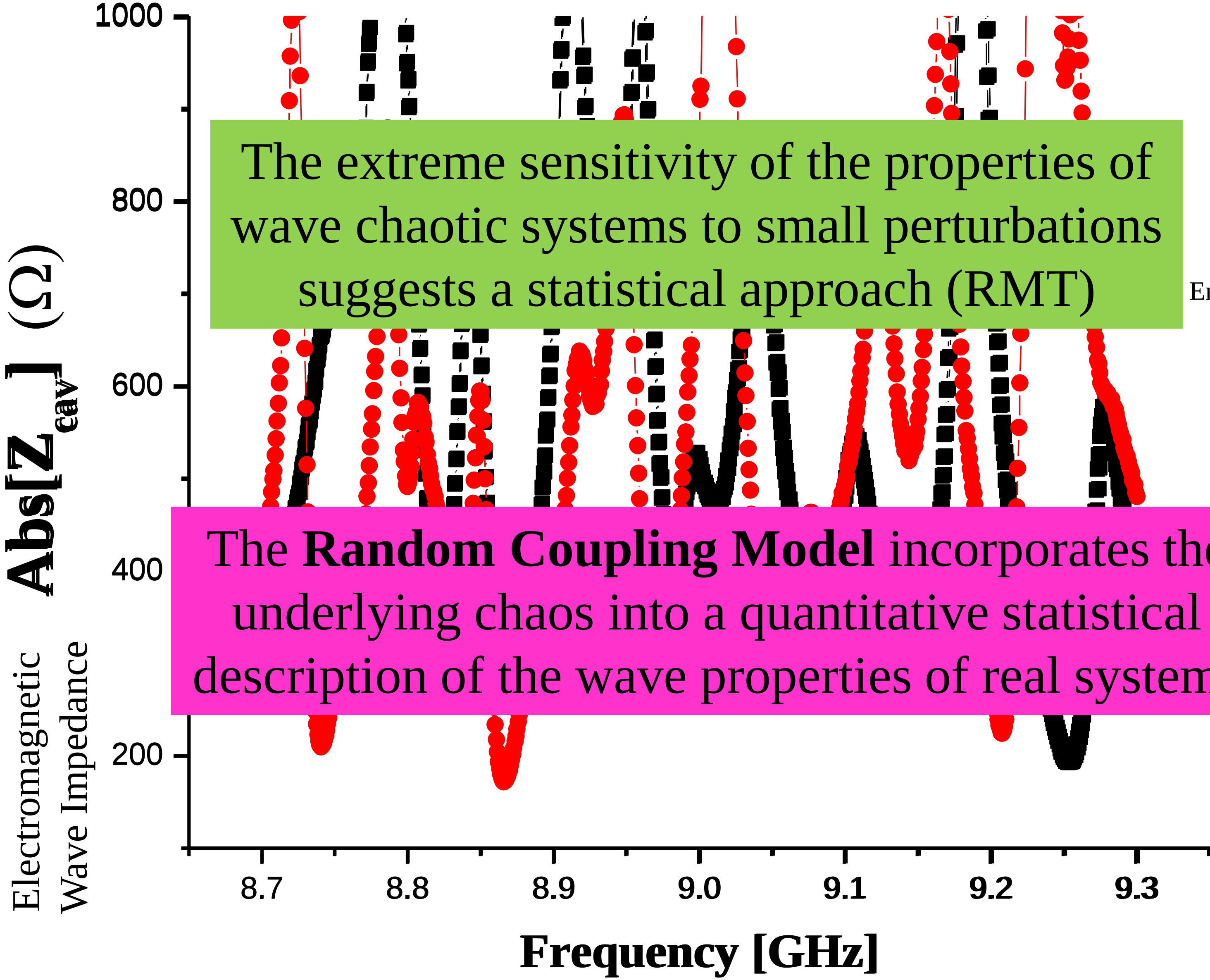
Electromagnetic Cavities: Complicated S_{11}, S_{22}, S_{21} versus frequency

λ << billiard size



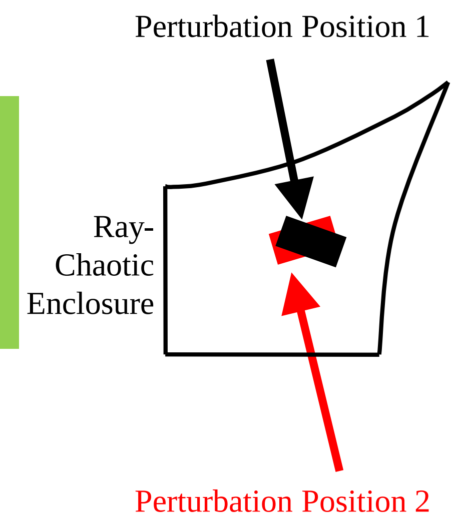


The Difficulty in Making Predictions in Wave Chaotic Systems…

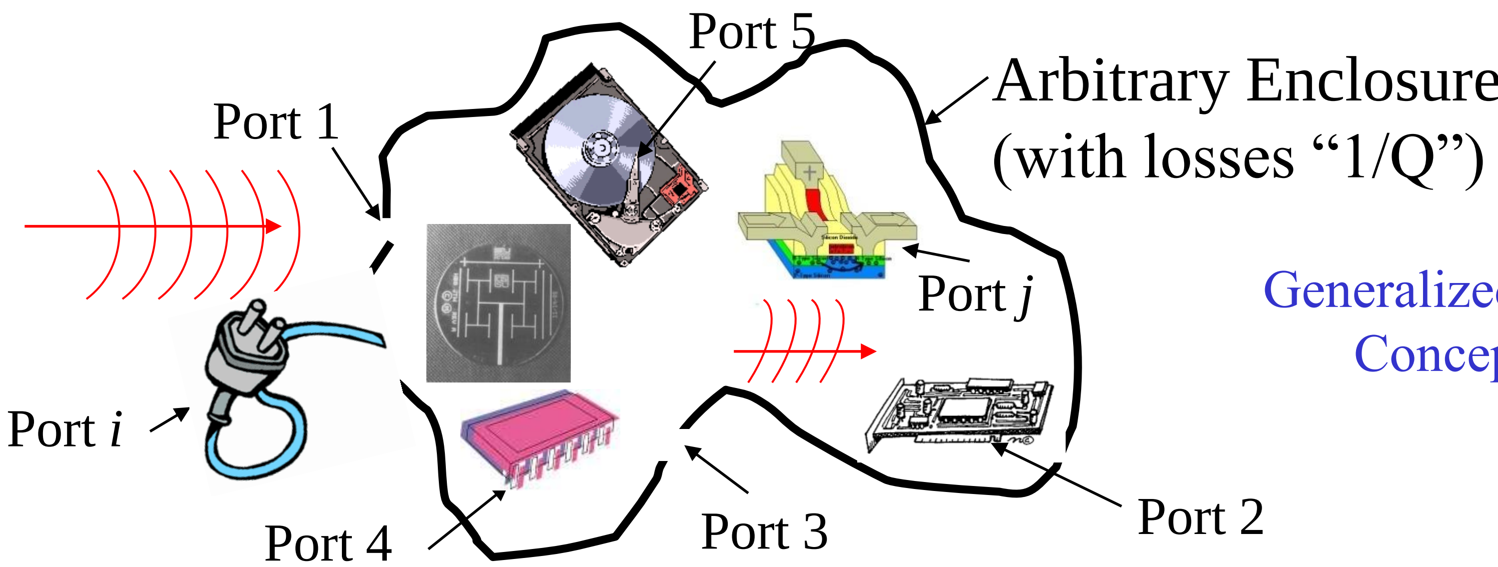


The extreme sensitivity of the properties of wave chaotic systems to small perturbations suggests a statistical approach (RMT)

The **Random Coupling Model** incorporates the underlying chaos into a quantitative statistical description of the wave properties of real systems



Induced Voltage Statistical Distributions for Objects in an Arbitrary Enclosure



Generalized Port Concept

Our approach treats all objects of interest as "ports"

Incident rf energy enters the enclosure through one or more ports

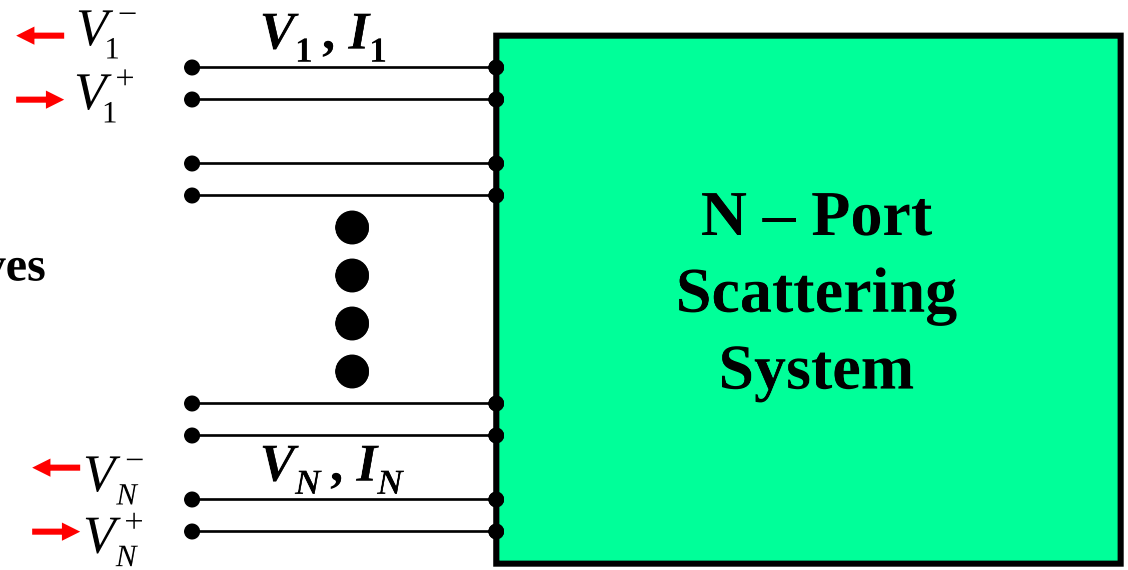
The energy reverberates and is absorbed by one or more ports inside the enclosure

Formulate a quantitative statistical theory of absorbed energy

N-Port Description of an Arbitrary Scattering System

N Ports

- **Voltages and Currents,**
- **Incoming and Outgoing Waves**



S matrix

$$\begin{bmatrix} V^-{}_1 \\ V^-{}_2 \\ \bullet \\ \bullet \\ \bullet \\ V^-{}_N \end{bmatrix} = [S] \cdot \begin{bmatrix} V^+{}_1 \\ V^+{}_2 \\ \bullet \\ \bullet \\ \bullet \\ V^+{}_N \end{bmatrix}$$

Z matrix

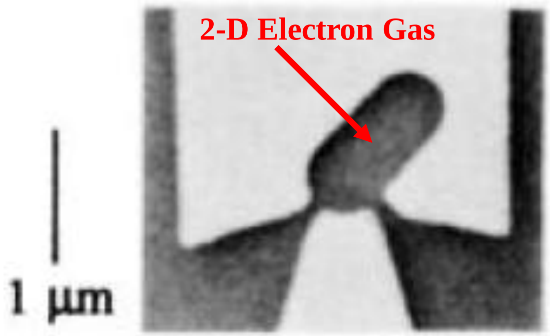
$$\begin{bmatrix} V_1 \\ V_2 \\ \bullet \\ \bullet \\ \bullet \\ V_N \end{bmatrix} = [Z] \cdot \begin{bmatrix} I_1 \\ I_2 \\ \bullet \\ \bullet \\ \bullet \\ I_N \end{bmatrix}$$

$$S = (Z + Z_0)^{-1}(Z - Z_0)$$

$$Z(\omega), S(\omega)$$

- Complicated Functions of frequency
- Detail Specific (Non-Universal)

Traditional Approach to Describing Wave Chaotic Scattering Systems – the Scattering Matrix



C. M. Marcus (1992)

electron mean free path >> system size
electron wavelength << system size

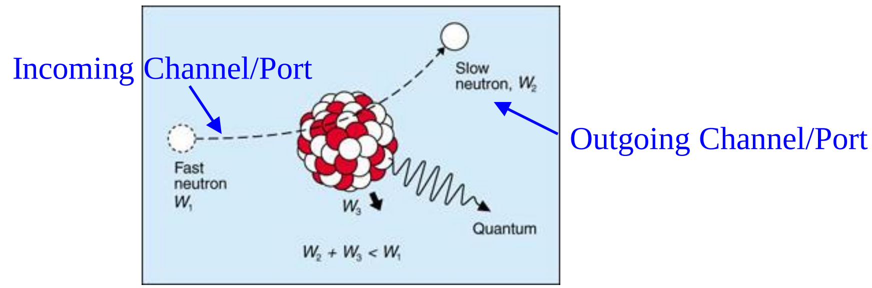
Ballistic Quantum Transport

$$G = \frac{2e^2}{h} \sum_{n=1}^{N_1} \sum_{m=1}^{N_2} \left| S_{nm} \right|^2$$

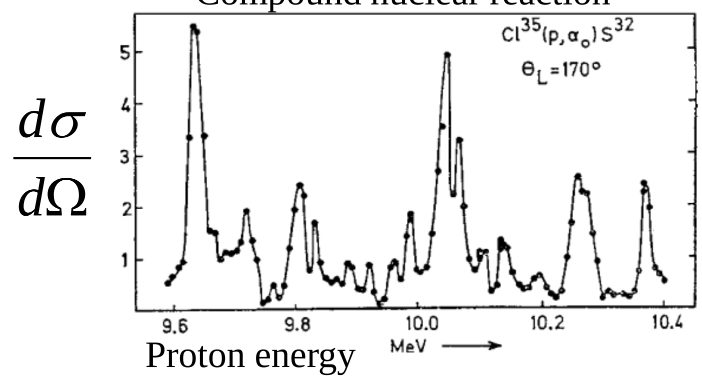
Landauer-Büttiker

Quantum interference → Fluctuations in $G \sim e^2/h$
"Universal Conductance Fluctuations"

Nuclear scattering:
Ericson fluctuations



Compound nuclear reaction



In contrast, our approach uses the Impedance (Z) description of wave scattering

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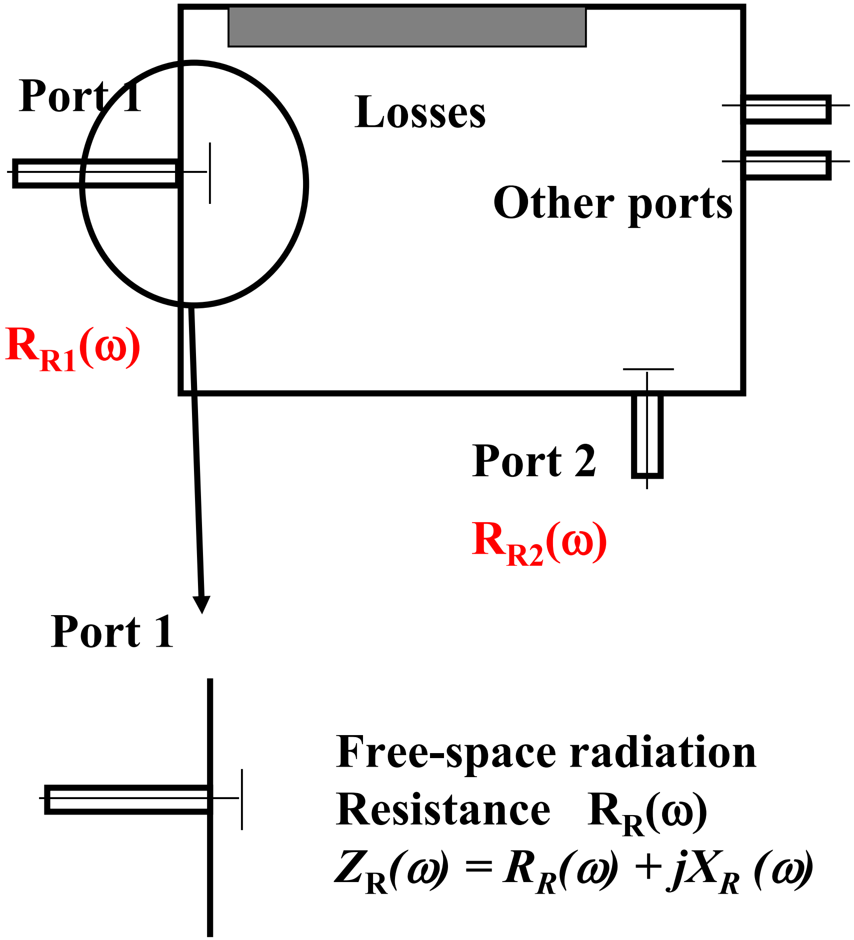
Statistical Model of Impedance (Z) Matrix

S. Hemmady, *et al.*, Phys. Rev. Lett. 94, 014102 (2005)
L. K. Warne, *et al.*, IEEE Trans. on Anten. and Prop. 51, 978 (2003)
X. Zheng, *et al.*, Electromagnetics 26, 3 (2006); Electromagnetics 26, 37 (2006)



Free-space radiation Resistance $R_R(\omega)$

$Z_R(\omega) = R_R(\omega) + jX_R(\omega)$

Statistical Model Impedance

$$Z_{ij}(\omega) = -\frac{j}{\pi}\sum_{\text{modes } n} R_{Ri}^{1/2}(\omega_n) R_{Rj}^{1/2}(\omega_n) \frac{\Delta\omega_n^2 \, w_{in} w_{jn}}{\omega^2(1+jQ^{-1}) - \omega_n^2}$$

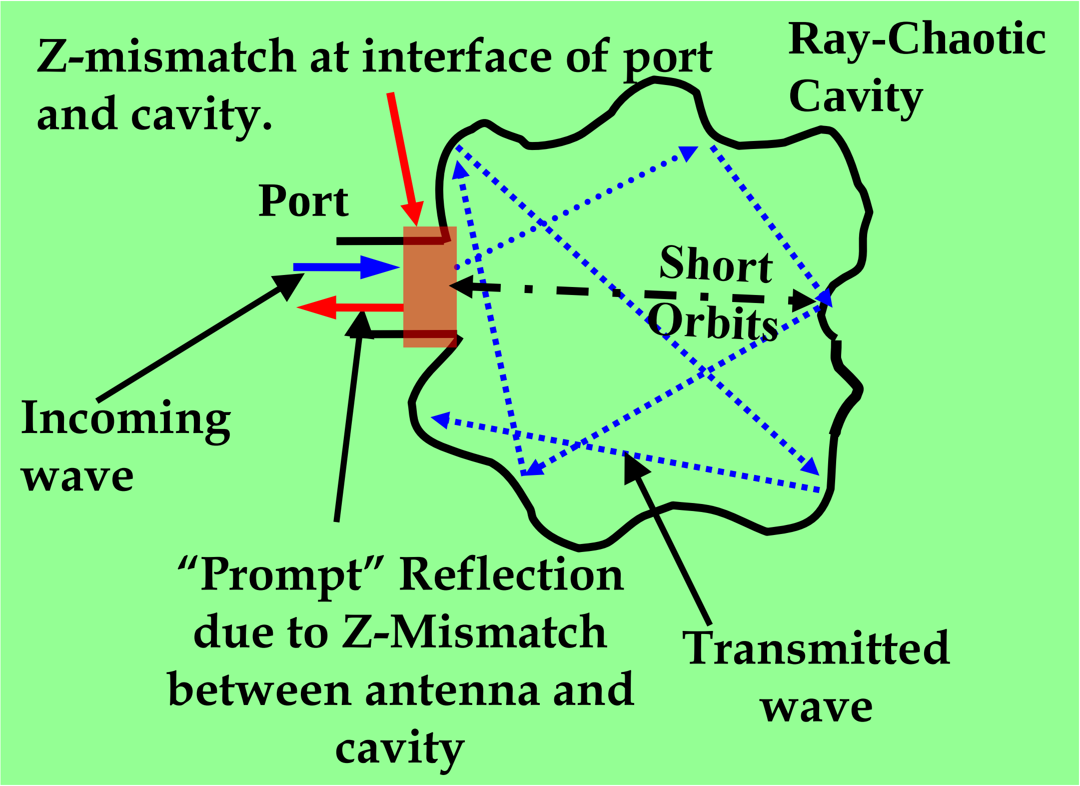
System parameters
- **Radiation Resistance $R_{Ri}(\omega)$**
- **$\Delta\omega_n^2$ - mean spectral spacing**
- **Q -quality factor**

Statistical parameters
- **ω_n - random spectrum**
- **w_{in}- Guassian Random variables**

Universal Fluctuations are Usually Obscured by Non-Universal System-Specific Details

The Most Common Non-Universal Effects:

1) **Non-Ideal Coupling between external scattering states and internal modes (i.e. Antenna/Port properties)**
2) **Short-Orbits between the antenna and fixed walls of the billiards**



The Random Coupling Model
Divide and Conquer!

Coupling Problem

Solution: Radiation Impedance Matrix $\mathbf{Z}_{rad}$
+ Short Orbits

Enclosure Problem

Solution: Random Matrix Theory;
Electromagnetic statistical properties are
governed by Loss Parameter $\boldsymbol{\alpha} = k^2/(\Delta k_n^2\, Q) = \delta f_{3dB}/\Delta f_{spacing}$

$$Z = \overline{Z} + \widetilde{Z} = iX_{Rad} + i\xi\, R_{Rad}$$

Mean part

Fluctuating Part (depends on $\boldsymbol{\alpha}$)

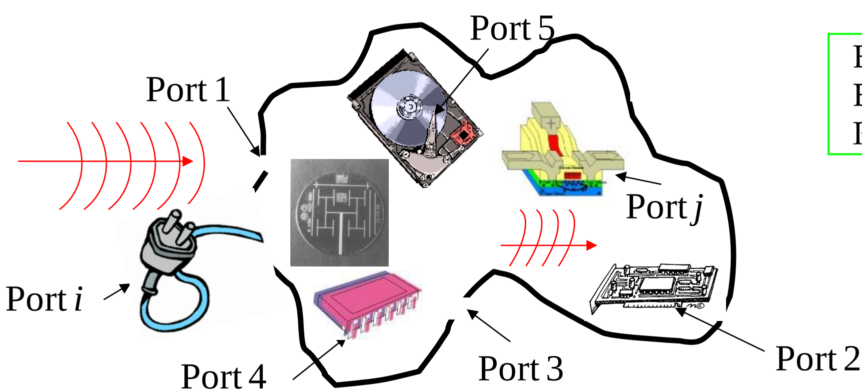
$\langle \mathrm{Im}\xi \rangle = 1$

$\langle \mathrm{Re}\xi \rangle = 0$

Z matrix

$$\begin{bmatrix} V_1 \\ V_2 \\ \bullet \\ \bullet \\ \bullet \\ V_N \end{bmatrix} = [Z] \cdot \begin{bmatrix} I_1 \\ I_2 \\ \bullet \\ \bullet \\ \bullet \\ I_N \end{bmatrix}$$

IEEE Trans. EMC 54, 758 (2012)



Electromagnetics 26, 3 (2006)
Electromagnetics 26, 37 (2006)
Phys. Rev. Lett. 94, 014102 (2005)

Theory of Non-Universal Wave Scattering Properties

Including Imperfect Coupling and Short Orbits

1-Port, Loss-less case: $$Z_{cavity} = iX_{avg} + R_{avg}\, i\xi$$

Universally Fluctuating Complex Quantity with Mean 1 (0) for the Real (Imaginary) Part. Predicted by RMT

$$Z_{avg} = Z_{Rad} + R_{Rad} \sum_{b(l)} p_{b(l)} \sqrt{D_{b(l)}}\, e^{ik(l+L_{Port})-i\pi/4}$$

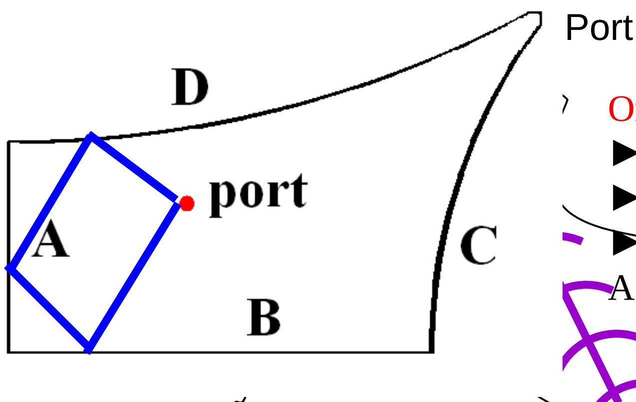
Semiclassical Expansion over Short Orbits

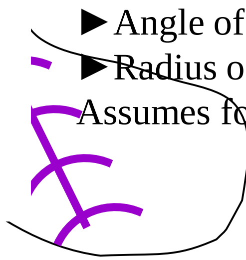
Complex Radiation Impedance (characterizes the non-universal coupling)

Index of 'Short Orbit' of length l

Stability of orbit

Action of orbit





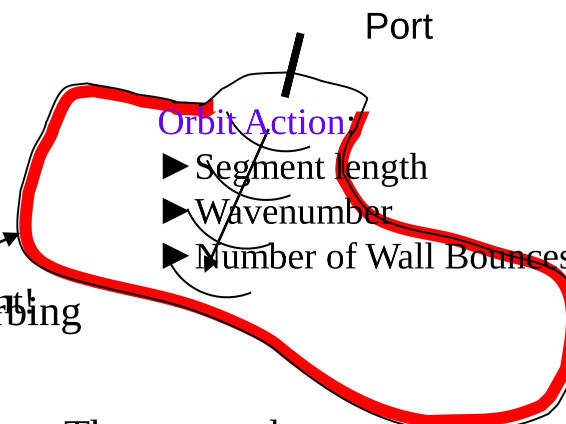
Orbit Stability Factor:

- ▶ Segment length
- ▶ Angle of incidence
- ▶ Radius of curvature of wall

Assumes foci and caustics are absent!

$\mathbf{Z_{Rad}}$

Perfectly absorbing boundary



Orbit Action:

- ▶ Segment length
- ▶ Wavenumber
- ▶ Number of Wall Bounces

The waves do not return to the port

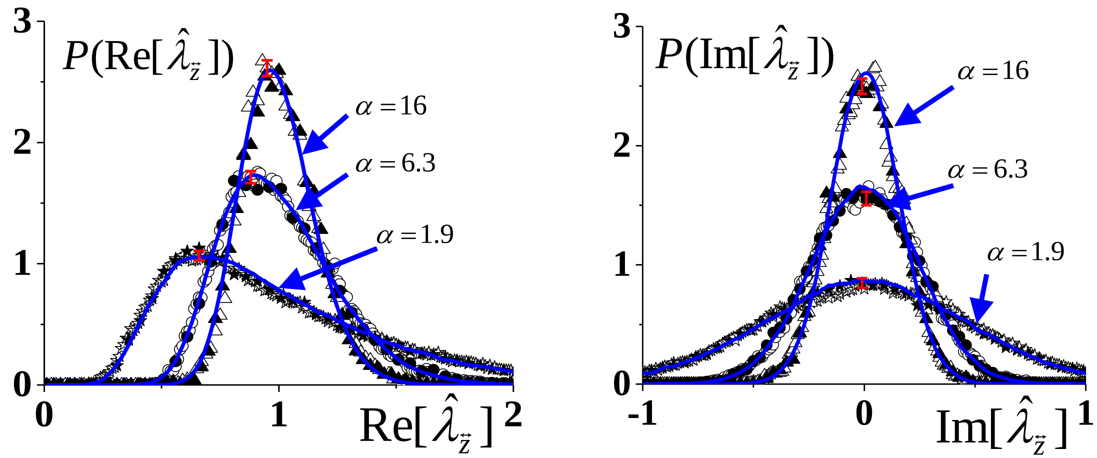
James Hart, T. Antonsen, E. Ott, Phys. Rev. E **80**, 041109 (2009)

Universal Impedance (Z) Statistics in the Presence of Loss

Inclusion of loss: $P_\alpha(Z)$

α = 3-dB bandwidth / mean-spacing

Fyodorov+Savin JETP Lett. **80**, 725 (2004)
Hemmady, *et al*., Phys. Rev. Lett. **94**, 014102 (2005)
Fyodorov+Savin+Sommers J Phys. A **38**, 10731 (2005)
Hemmady, *et al*., Phys. Rev. E **74** , 036213 (2006)



Comparison of data (symbols) and RMT (solid lines)

Mean of $P_\alpha(z)$ $\quad E\{\mathrm{Re}[\hat{\lambda}_{\tilde{z}}]\} = 1 \quad E\{\mathrm{Im}[\hat{\lambda}_{\tilde{z}}]\} = 0$ Independent of α

Variance of $P_\alpha(z)$ $\quad \sigma^2_{\mathrm{Re}[\lambda_{\tilde{z}}]} = \sigma^2_{\mathrm{Im}[\lambda_{\tilde{z}}]} = \frac{1}{\pi}\frac{1}{k^2/(\Delta k_n^2 Q)} = \frac{1}{\pi\alpha} \quad \alpha >> 1$

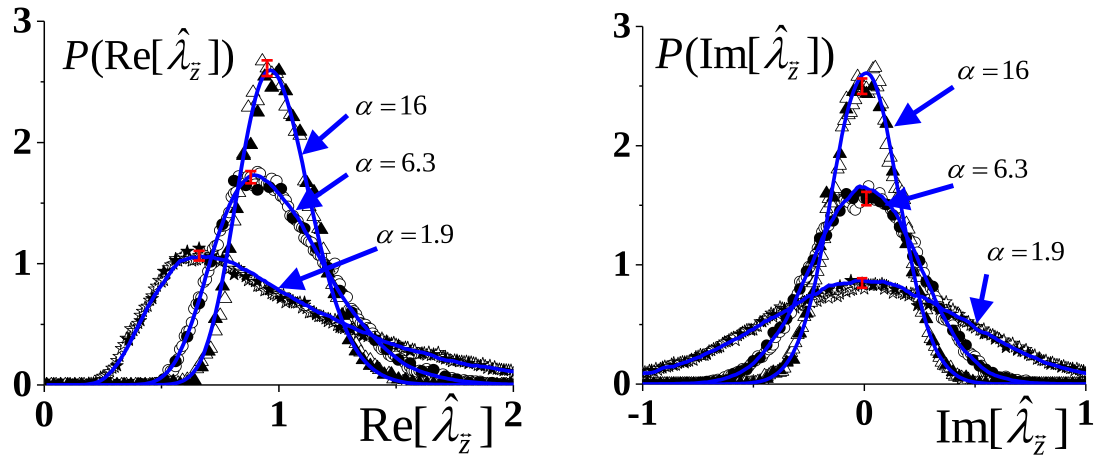
Universal Impedance (z) Statistics in the Presence of Loss

Inclusion of loss: $P_\alpha(Z)$

PDF of the eigenvalues λ_z of the universal impedance matrix (z)

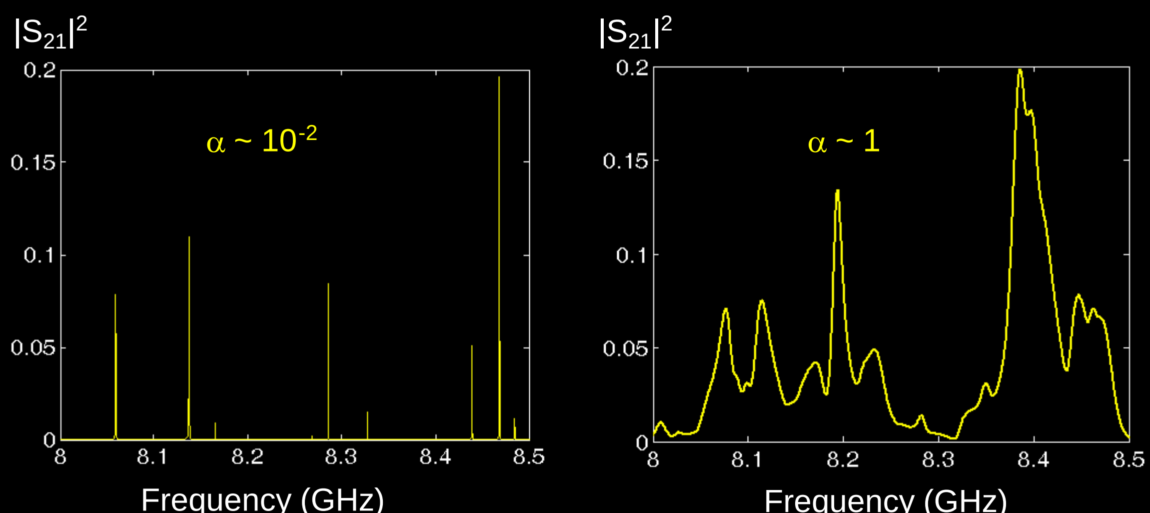
α = 3-dB bandwidth / mean-spacing

Fyodorov+Savin JETP Lett. **80**, 725 (2004)
Hemmady, *et al.*, Phys. Rev. Lett. **94**, 014102 (2005)
Fyodorov+Savin+Sommers J Phys. A **38**, 10731 (2005)
Hemmady, *et al.*, Phys. Rev. E **74** , 036213 (2006)



Comparison of data (symbols) and RMT (solid lines)

$$\text{Loss parameter } \alpha = \frac{\text{3 dB bandwidth of the closed cavity mode resonance}}{\text{mean spacing between cavity modes}}$$



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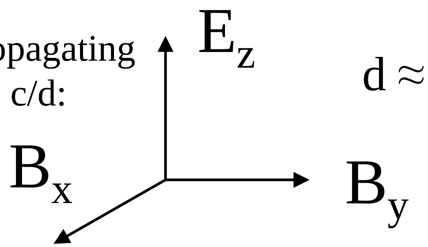
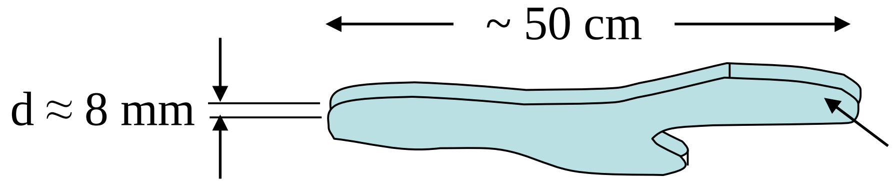
Microwave Cavity Analog of a 2D Quantum Infinite Square Well

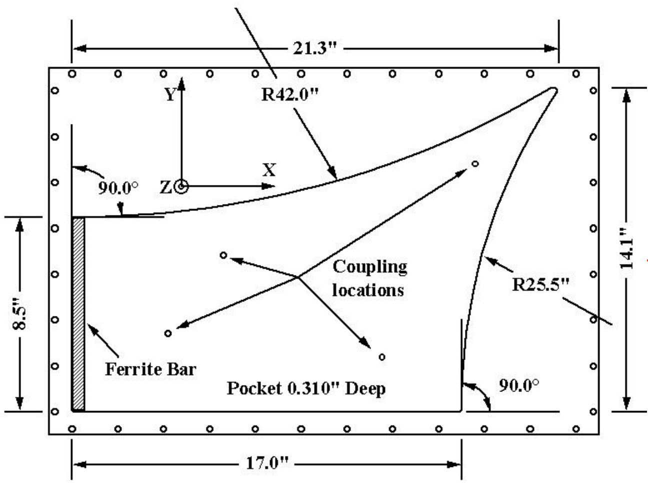
Table-top experiment!

The only propagating mode for f < c/d:





An empty "two-dimensional" electromagnetic resonator



$$\nabla^2 \Psi_n + \frac{2m}{\hbar^2}(E_n - V)\Psi_n = 0$$

$$\text{with } \Psi_n = 0 \text{ at boundaries}$$

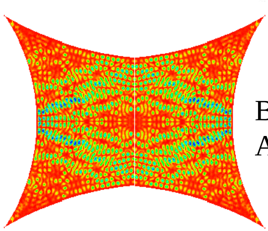
Schrödinger equation

$$\nabla^2 E_{z,n} + k_n^2 E_{z,n} = 0$$

$$\text{with } E_{z,n} = 0 \text{ at boundaries}$$

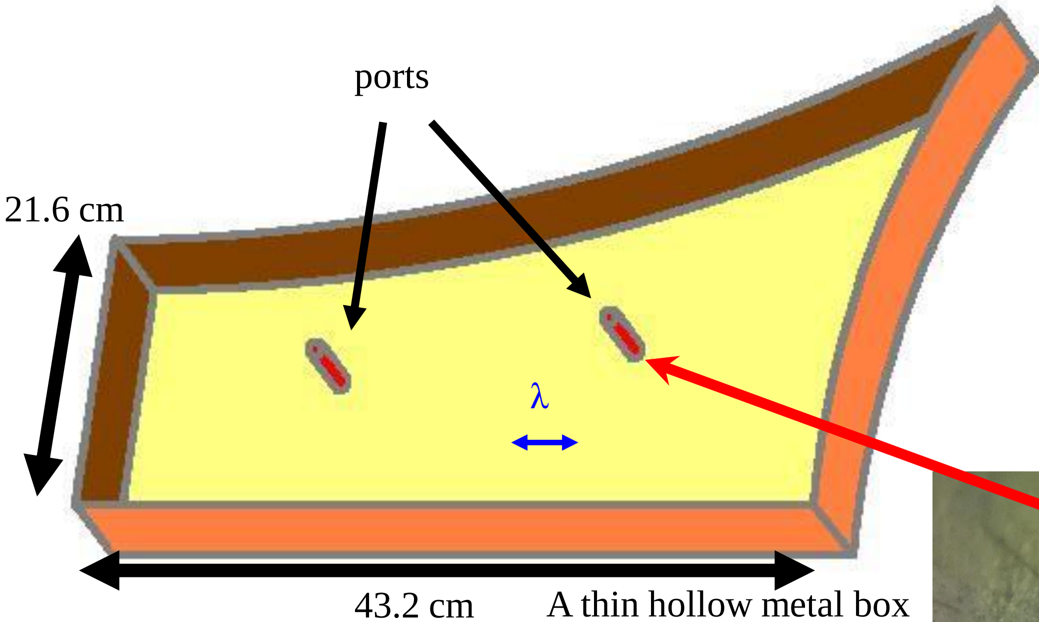
Helmholtz equation

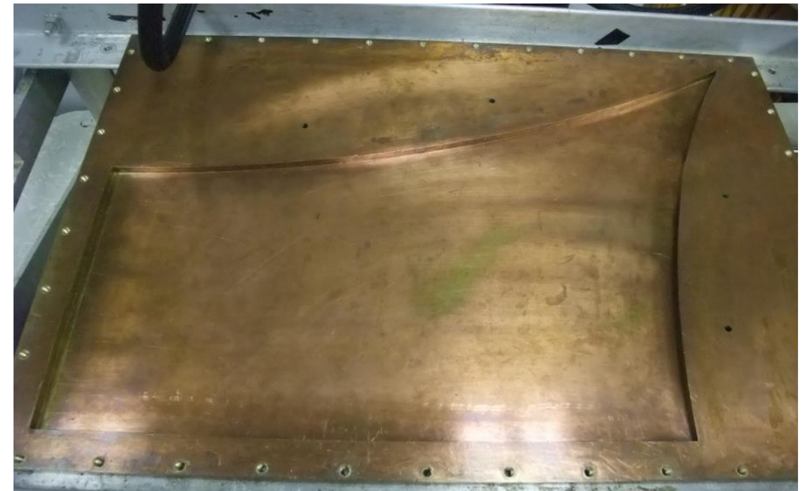
Stöckmann + Stein, 1990
Doron+Smilansky+Frenkel, 1990
Sridhar, 1991
Richter, 1992

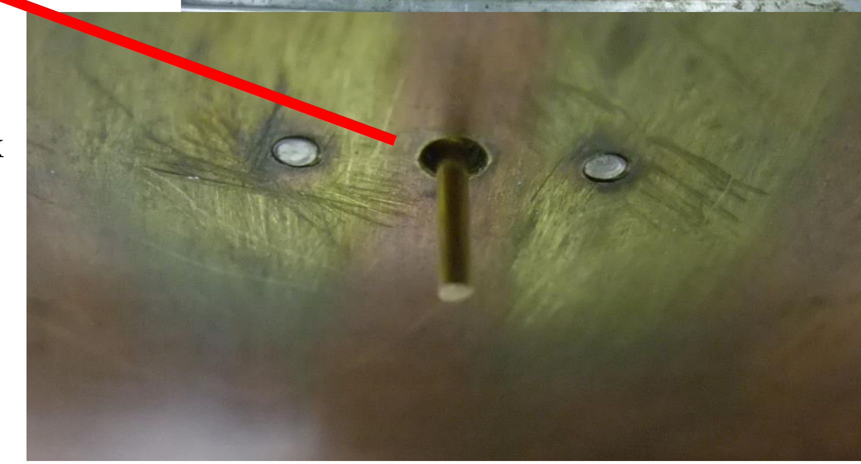


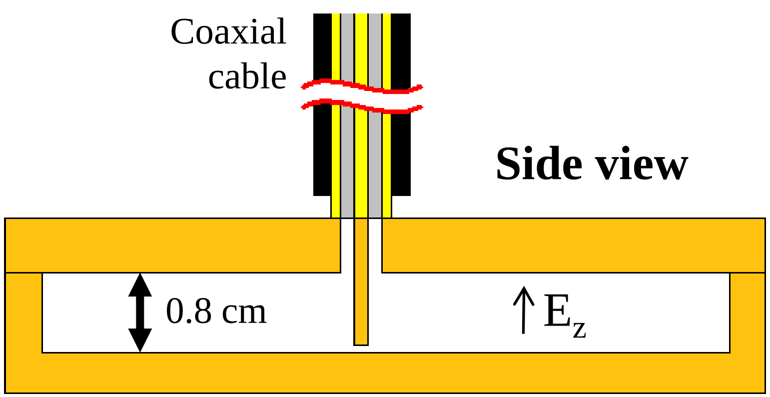
Bow-Tie Billiard
A. Gokirmak, *et al*. Rev. Sci. Instrum. **69**, 3410 (1998)

The Experiment:
A simplified model of wave-chaotic scattering systems





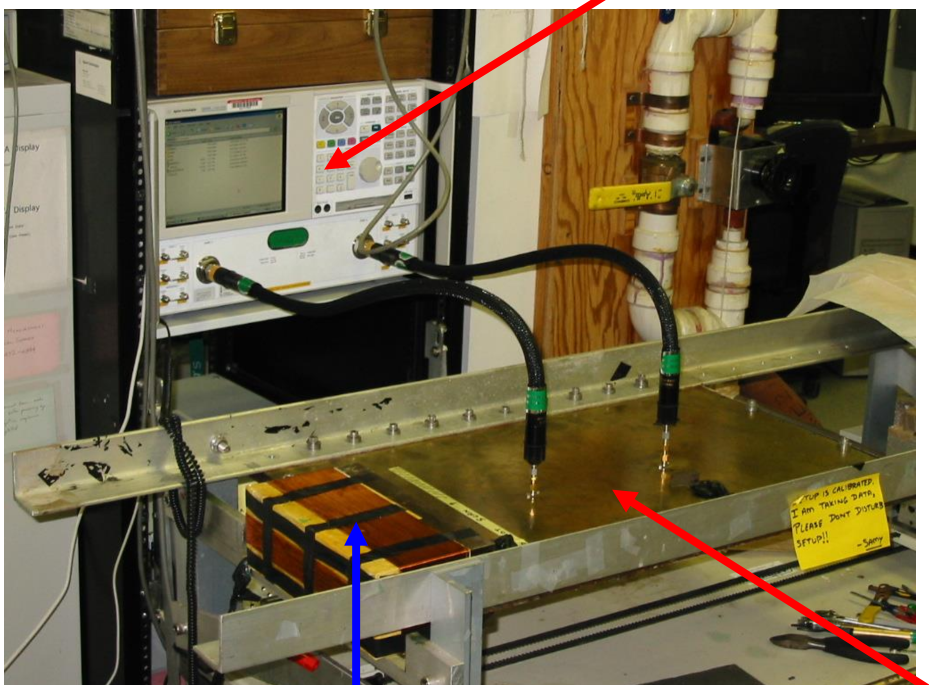


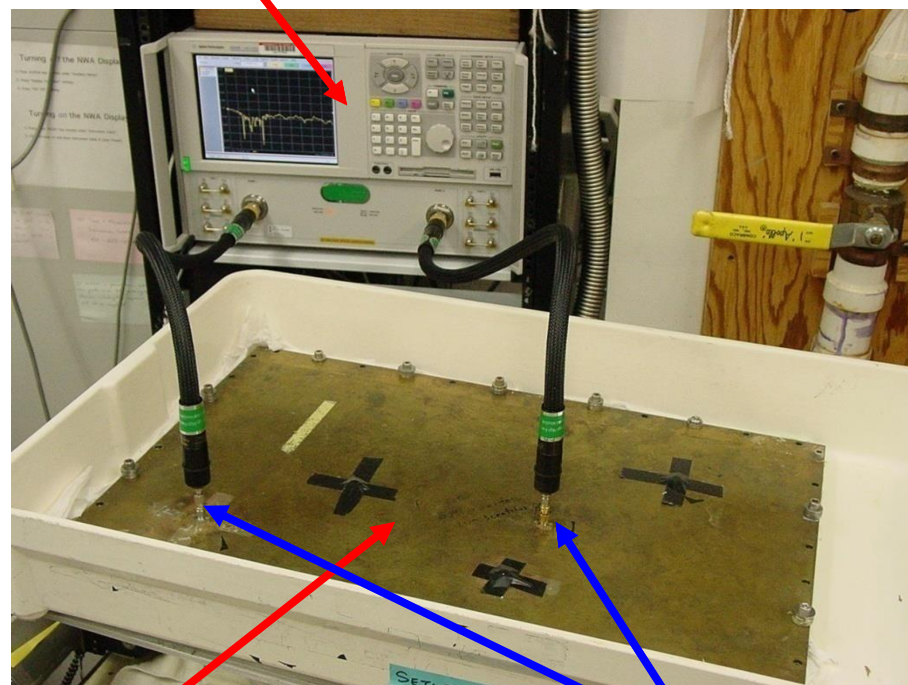


Microwave-Cavity Analog of a 2D Infinite Square Well

with Coupling to Scattering States

Network Analyzer [measures Scattering (S)-matrix vs. frequency]



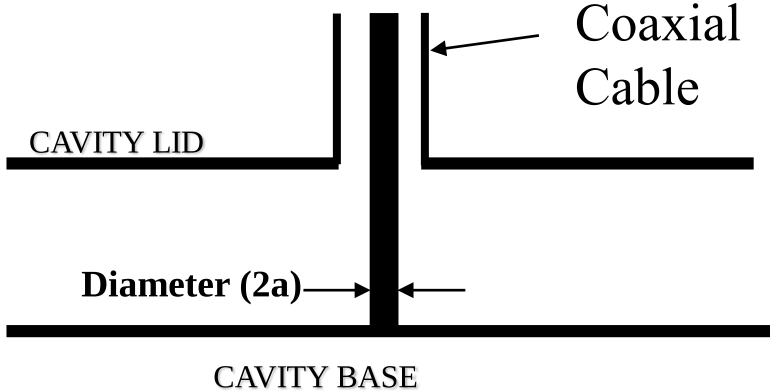


Electromagnet **Thin Microwave Cavity** **Ports**

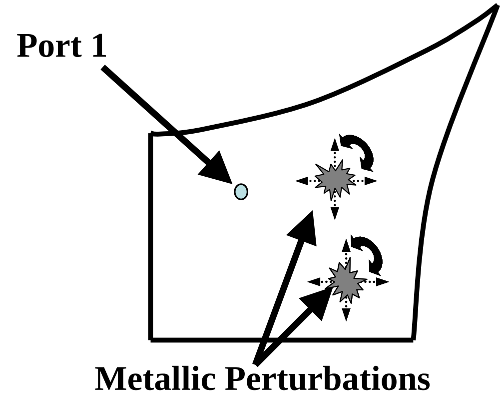
We measure from 500 MHz – 19 GHz, covering about 750 modes in the semi-classical limit

Testing Insensitivity to Sys

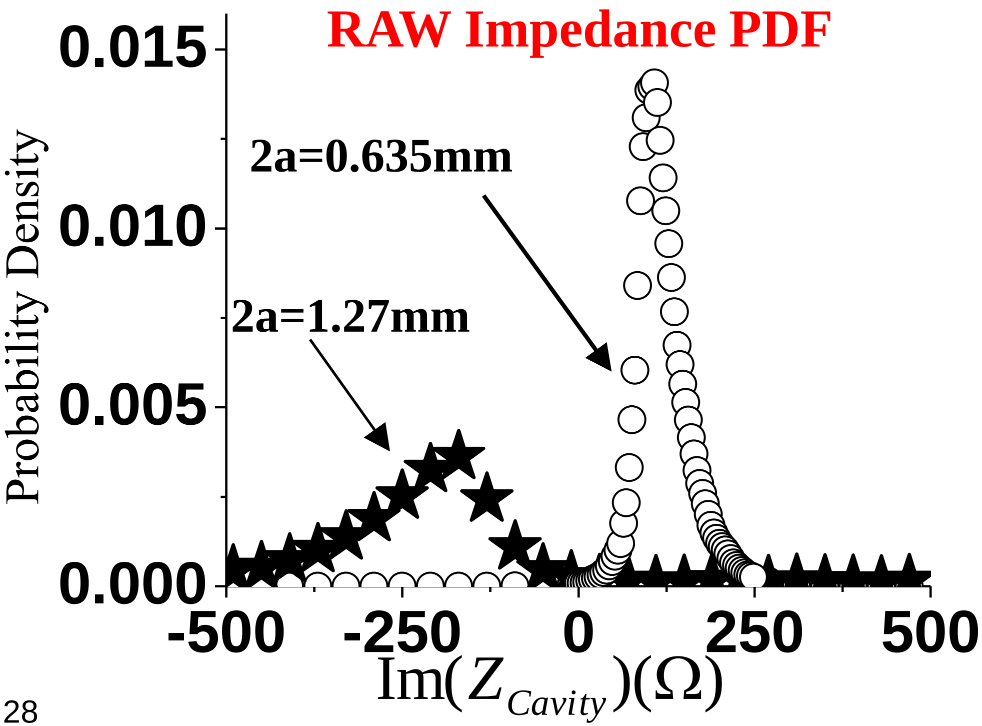
Cross Section View of Port

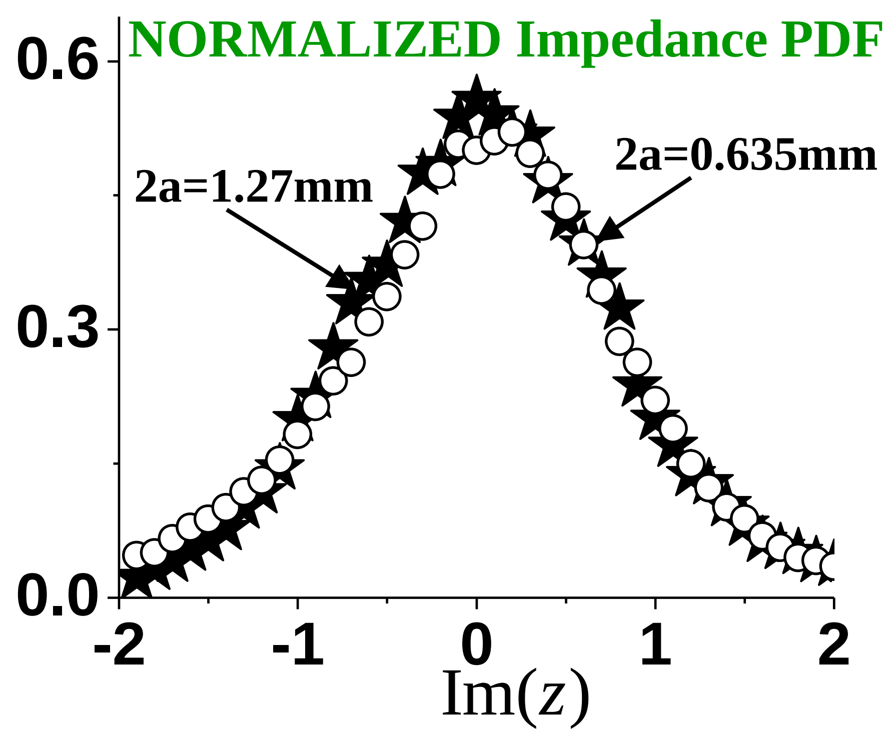


$$Z_{Cavity} = iX_{Rad} + R_{Rad} i\xi$$



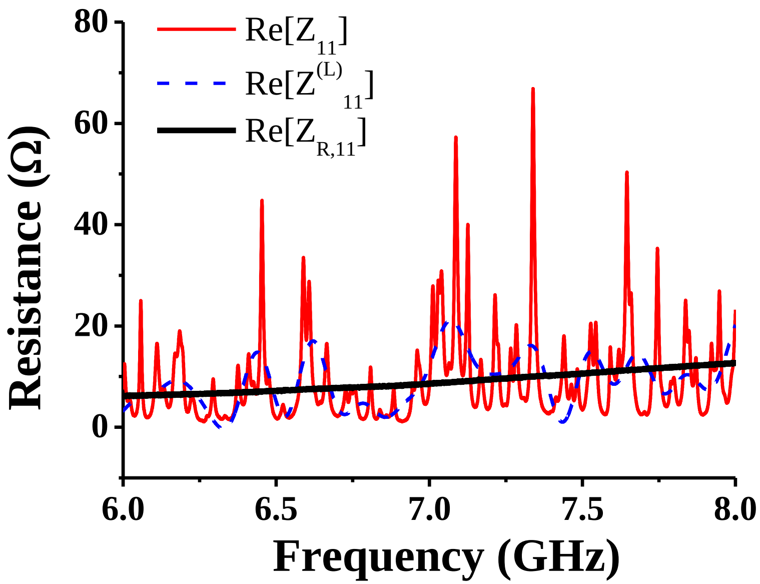
$$z = \frac{R_{Cavity}}{R_{Rad}} + i\frac{X_{Cavity} - X_{Rad}}{R_{Rad}}$$

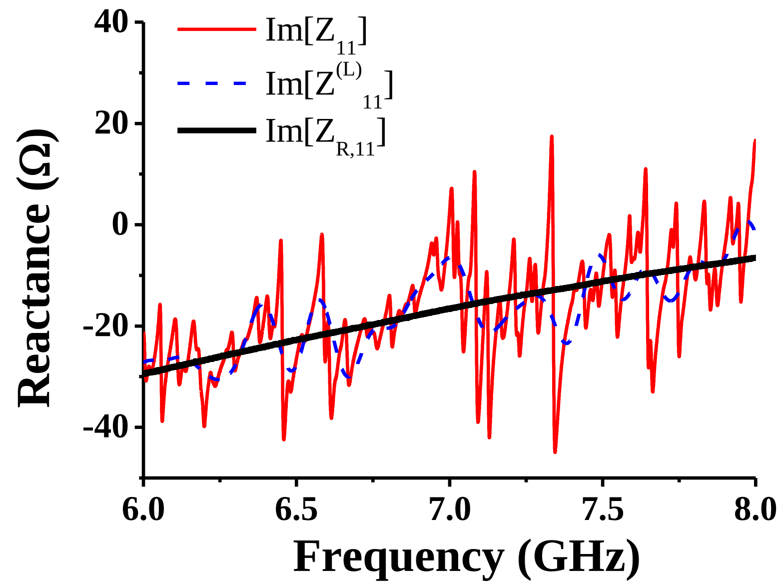


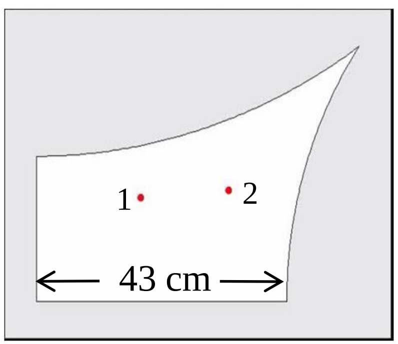


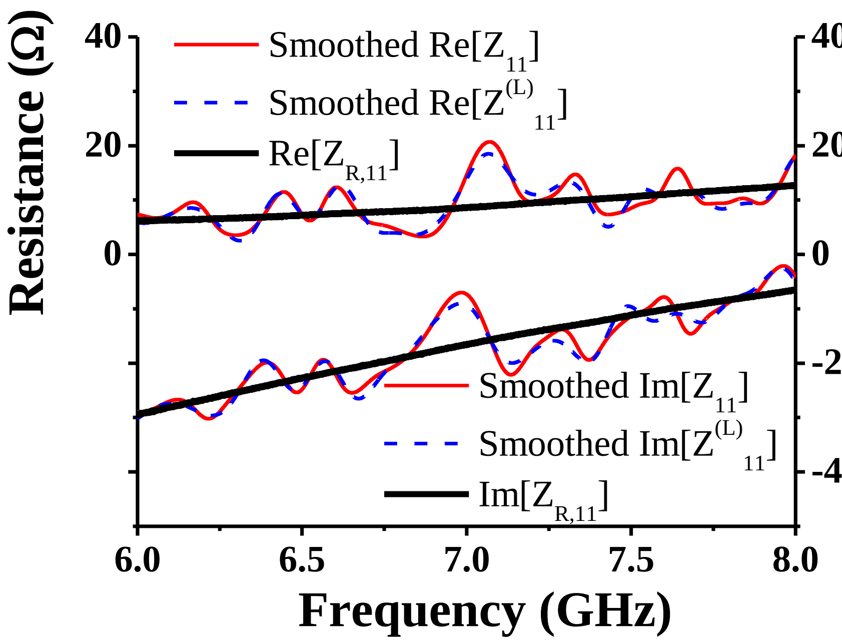
Nonuniversal Properties Captured by the Extended RCM

Empty Cavity Data









$\alpha \sim 1$

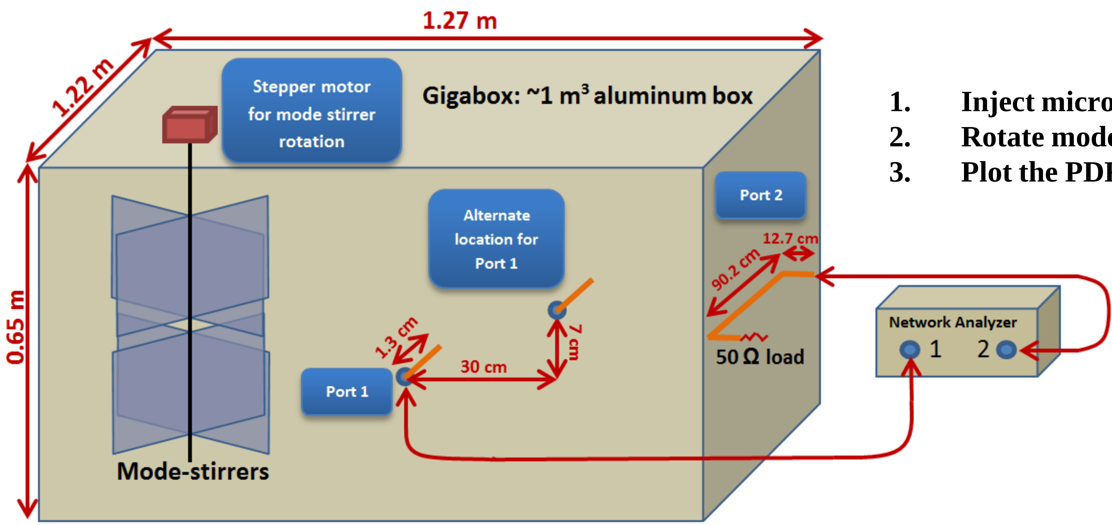
Data and Theory smoothed with the same 125-cm (240 MHz window) low-pass filter

Random Matrix Theory describes the fluctuations away from $Z^{(L)}$

Theory includes all orbits to 200 cm length

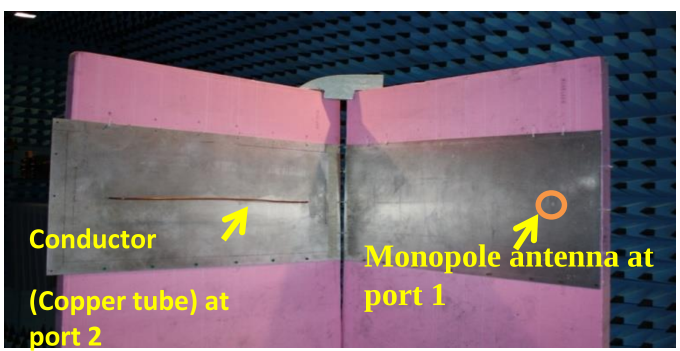
J.-H. Yeh, *et al.*, Phys. Rev. E **81**, 025201(R) (2010); J.-H. Yeh, *et al.*, Phys. Rev. E **82**, 041114 (2010).

The Random Coupling Model Applied to 3-Dimensional Enclosures

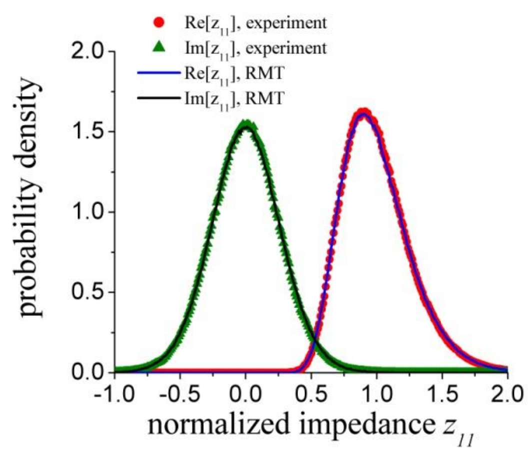


1. Inject microwaves at port 1 and measure induced voltage at port 2
2. Rotate mode-stirrer and repeat
3. Plot the PDF of the induced voltage and compare with RCM prediction

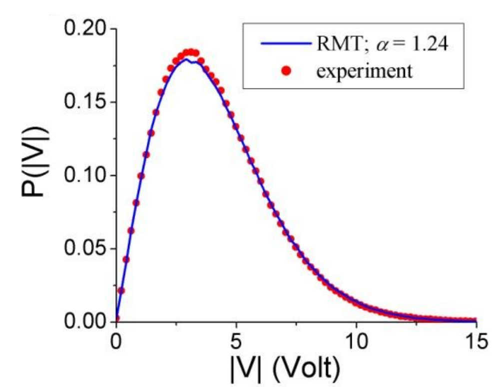
$\overleftrightarrow{Z}_{Rad}$ Measurement



Uncovering Universal Impedance Statistics



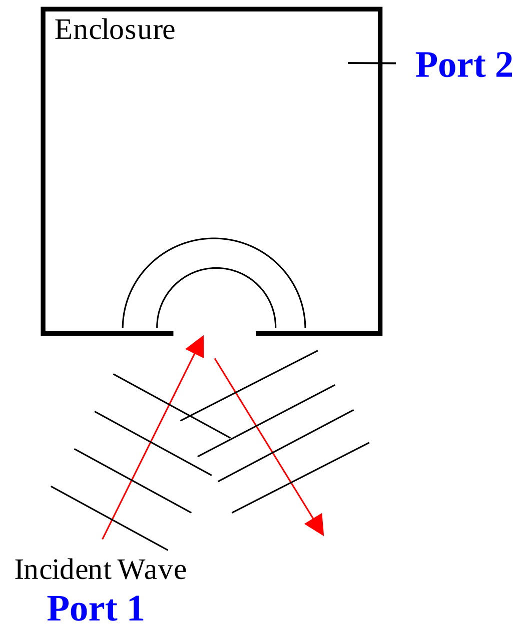
Induced Voltage Statistics



Z. B. Drikas, *et al*., IEEE Trans. Electromag. Compat. 56, 1480 (2014)
US Naval Research Laboratory collaboration

RCM Predictions for Electrically-Large Apertures

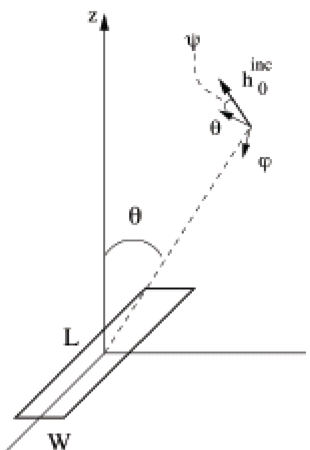
G. Gradoni, *et al.*



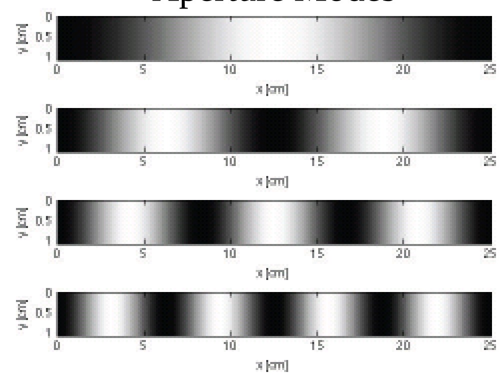
$$\underline{\underline{Y}}_{cav} = i\,\mathrm{Im}\left(\underline{\underline{Y}}_{rad}\right) + \underline{\underline{G}}_{rad}^{1/2}\, i\underline{\underline{\xi}}\, \underline{\underline{G}}_{rad}^{1/2}$$

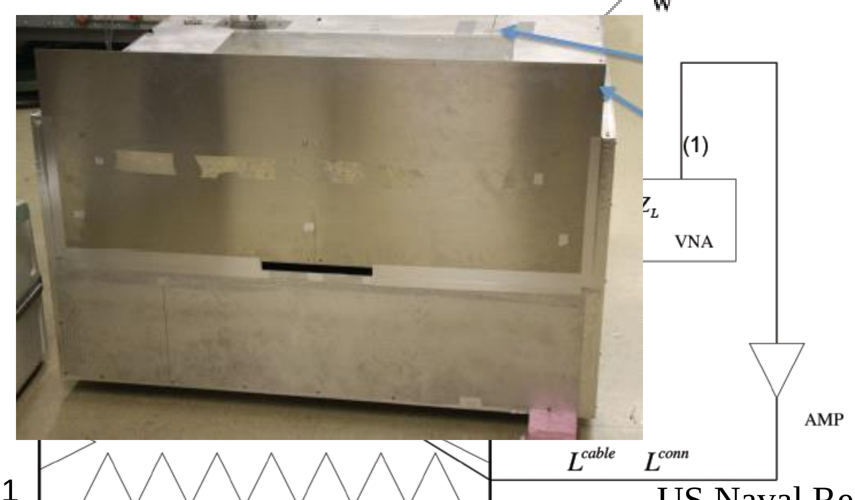
RCM for admittance

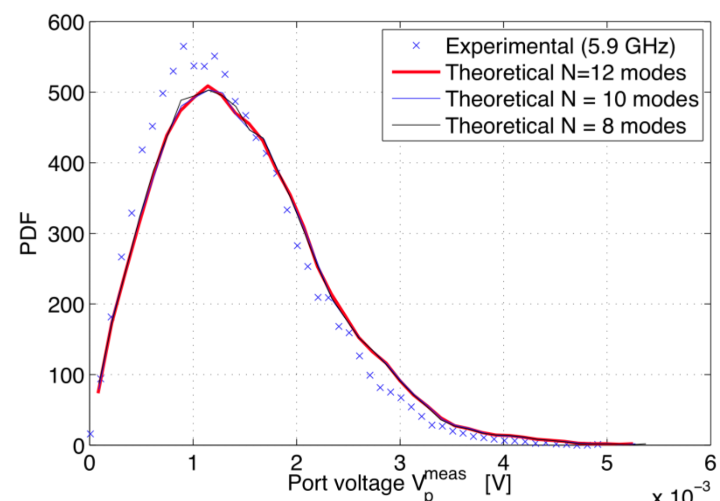
$$Y_{ss'}^{rad}(k_0 = \omega / c) = \sqrt{\frac{\varepsilon}{\mu}} \int \frac{d^3k}{(2\pi)^3} \frac{2ik_0}{k_0^2 - k^2}\; \bar{\mathbf{e}}_s \cdot \underline{\underline{\Delta}}_2 \cdot \bar{\mathbf{e}}_{s'}$$



Aperture Modes





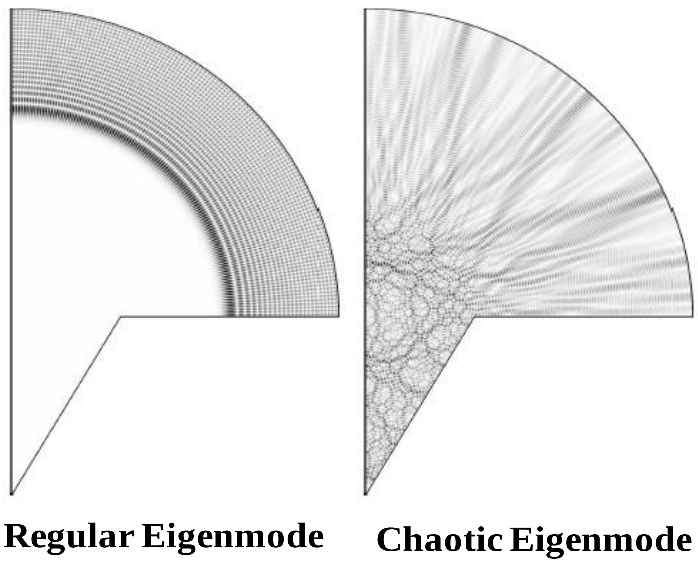


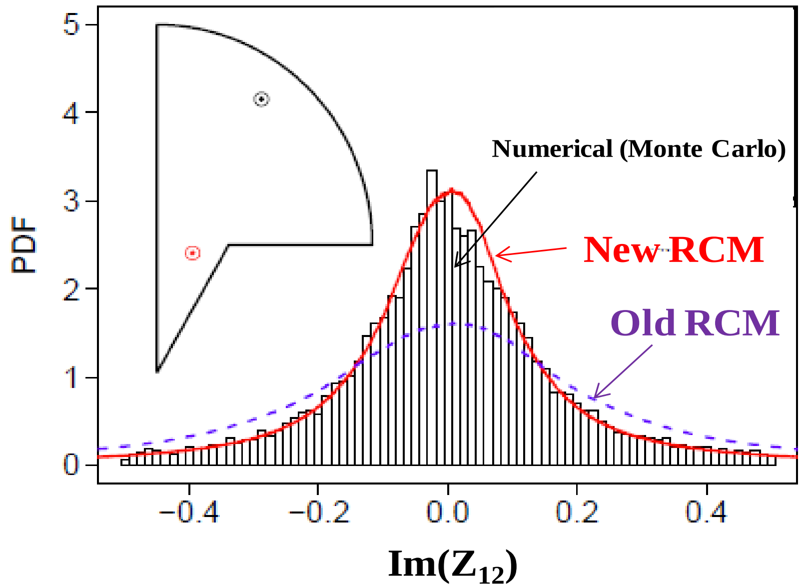
US Naval Research Laboratory collaboration

Induced Voltage Statistics for Enclosures with Mixed Regular and Chaotic Behavior

Random Coupling Model still works!

$$Z_{ij} = Z_{ij,Regular} + Z_{ij,Chaotic}$$





Relevant variables:

- 3D enclosures with parallel walls
- Illuminate through regular and irregular apertures

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Ming Jer Lee, *et al.*, Phys. Rev. E 87, 062906 (2013)

Outline

- The Problem: Electromagnetic Interference
- Our Approach – A Wave Chaos Statistical Description
- The Random Coupling Model (RCM)
- Examples of the RCM in Practice
- Related Work
- Conclusions

Conclusions

The Random Coupling Model constitutes a comprehensive (statistical) description of the wave properties of wave-chaotic systems in the short wavelength limit

We believe the RCM is of value to the EMC / EMI community for predicting the statistics of induced voltages on objects in complex enclosures, for example.

This description should apply to any wave system in the 'mesoscopic', 'mid-frequency', … limit

Acoustics
Mechanical vibrations
Quantum mechanical
Electromagnetic
…

RCM Review articles:
G. Gradoni, *et al.*, Wave Motion 51, 606 (2014)
Z. Drikas, *et al.*, IEEE Trans. EMC 56, 1480 (2014)

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Students and Post-docs

- Present Students Wave Chaos
- Recent Post-docs

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