An Introduction to TLM

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TLM Basics:

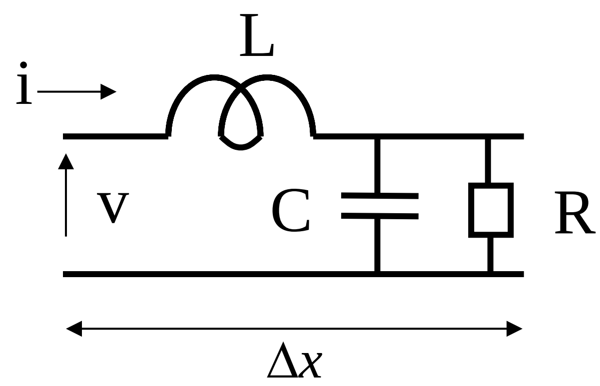
- Field and Network Concepts
- Huyghens Principle
- The basic computational molecule in TLM-SCN node
- Scattering
- Connection
- Boundaries
- Input/output
- Simple boundaries/thin panels/cables

TLM: generic aspects

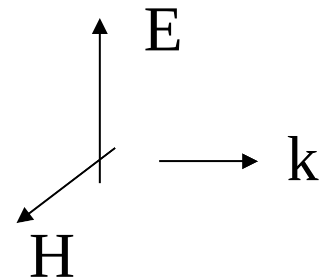
- Time-domain (frequency-domain also available)
- Differential equation based method
- Explicit, unconditionally stable method
- Discrete implementation of Huyghen's principle
- Algorithm-scattering and connection based on conservation laws

The relationship between voltage/current and electric/magnetic field is very close one. If one looks at the equations for voltage and current on a lossy line and the corresponding one-dimensional field equations one gets:

$$\frac{\partial^2 i}{\partial x^2} = \frac{LC}{(\Delta x)^2}\frac{\partial^2 i}{\partial t^2} + \frac{L}{(\Delta x)^2 R}\frac{\partial i}{\partial t}$$



$$\frac{\partial^2 j}{\partial x^2} = \mu\varepsilon\frac{\partial^2 j}{\partial t^2} + \mu\sigma\frac{\partial j}{\partial t}$$



Equations are isomorphic!

Equivalence between circuits and fields!!

$$i \Leftrightarrow j$$

circuit quantities

$$\frac{L}{\Delta x} \Leftrightarrow \mu$$

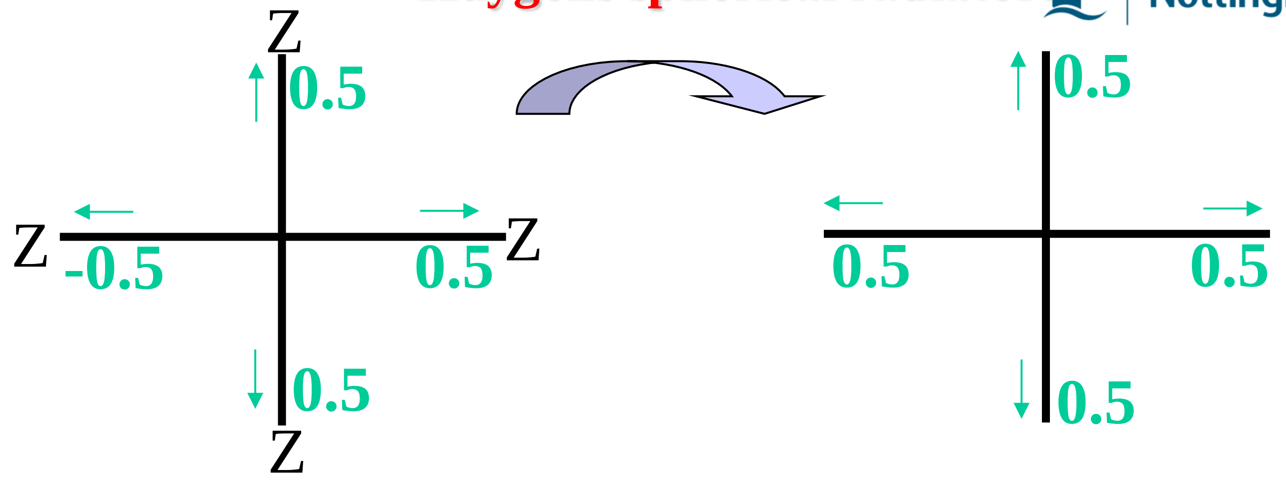
field quantities

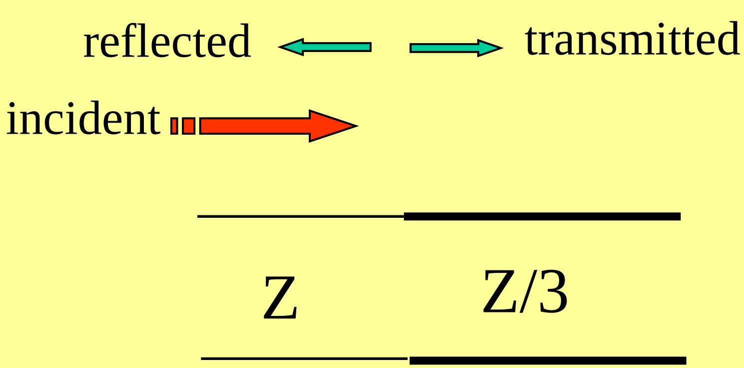
$$\frac{C}{\Delta x} \Leftrightarrow \varepsilon$$

$$\frac{1}{R\Delta x} \Leftrightarrow \sigma$$

Huygens spherical radiator!

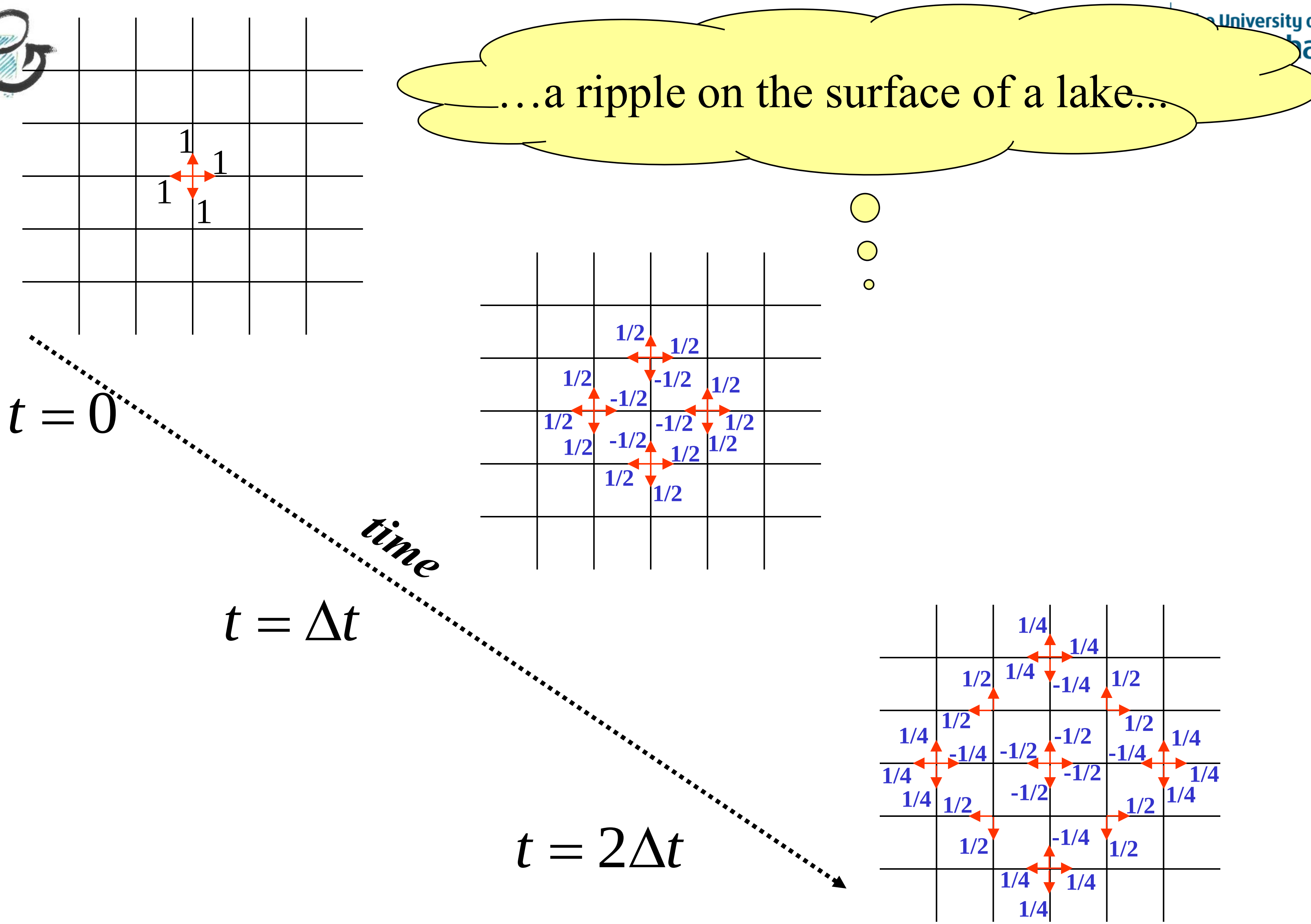
Incident pulse (1 volt)





Reflection coefficient: $$\frac{\frac{Z}{3}-Z}{\frac{Z}{3}+Z}=-0.5$$

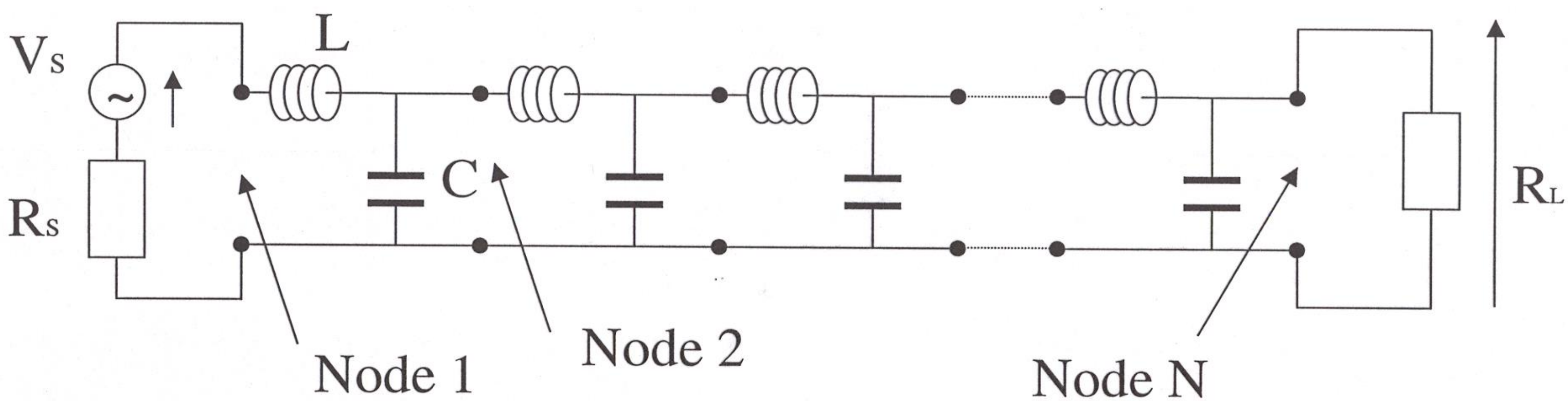
Transmission coefficient: $$\frac{2\frac{Z}{3}}{\frac{Z}{3}+Z}=0.5$$


…a ripple on the surface of a lake...
$t = 0$
time
$t = \Delta t$
$t = 2\Delta t$
1
1/2
-1/2
1/4
-1/4

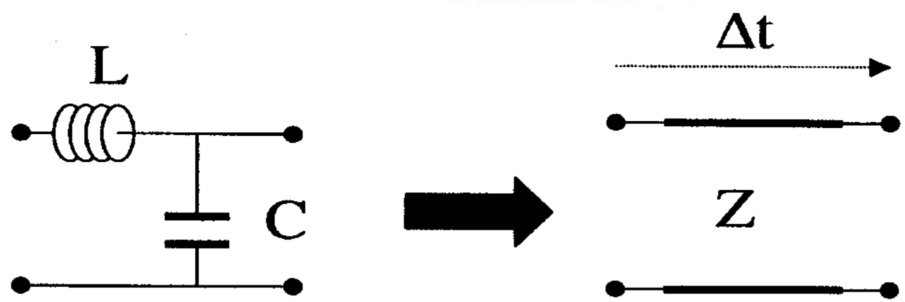
1D TLM Model (example)

Study the propagation of a pulse along the line shown below:

V_L

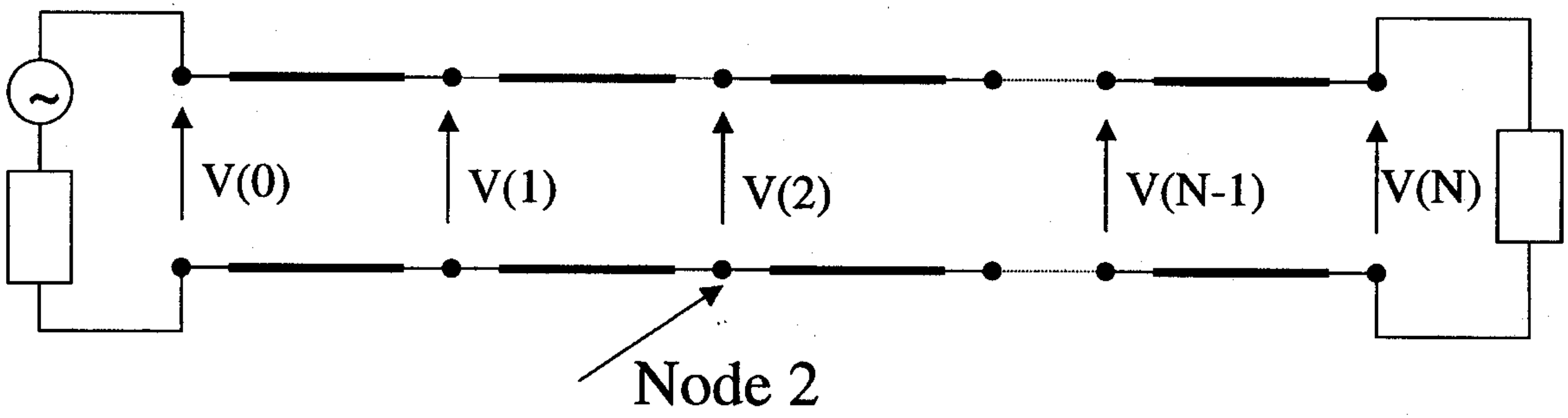


STEP1:
Replace each section with a TL segment



$$\textit{where} \quad \Delta t = \sqrt{LC}, \quad \textit{and} \quad Z = \sqrt{L/C}$$

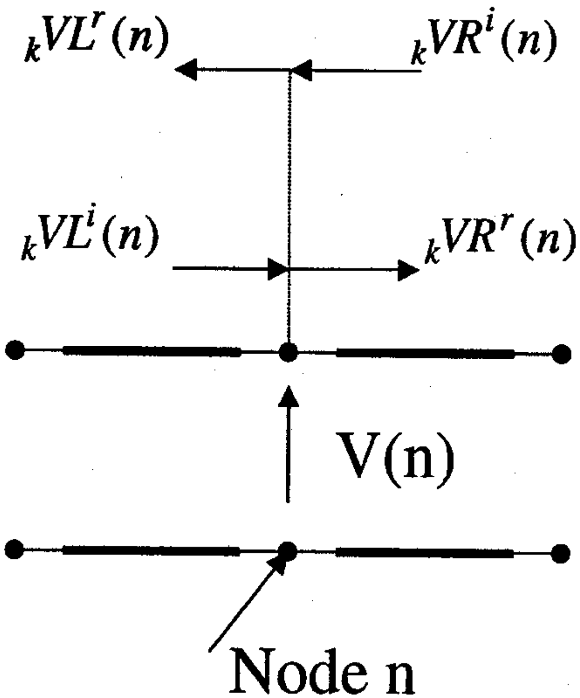
The circuit now looks as shown below:



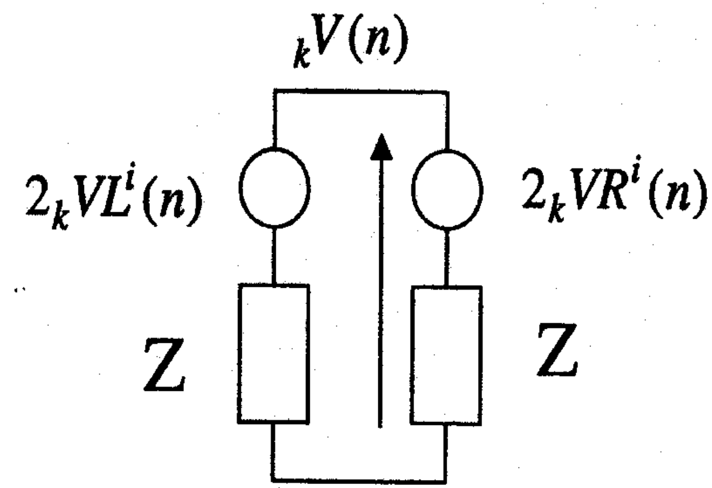
This circuit will be analysed in three parts:

- node 0 (the first node by the source)
- node n (a node away from the extremities of the line)
- node N (the last node by the load)

Scattering at a general node n:



→ Thevenin equivalent:



$$V(n)=\frac{\dfrac{2\,_kVL^i(n)}{Z}+\dfrac{2\,_kVR^i(n)}{Z}}{\dfrac{1}{Z_s}+\dfrac{1}{Z}}={}_kVL^i(n)+{}_kVR^i(n)$$

$$_kVR^r={}_kV(n)-{}_kVR^i(n) \qquad _kVL^r(n)={}_kV(n)-{}_kVL^i(n)$$

Connection is very simple.

It involves only an exchange of information between nearest neighbours!

The rule for connection is that the new incident voltage at say time-step k+1 is the reflected voltage at the previous time-step k from an adjacent node…

$$ {}_{k+1}VL^{i}(n) = {}_{k}VR^{r}(n-1) $$

$$ {}_{k+1}VR^{i}(n) = {}_{k}VL^{r}(n-1) $$

Thus the **TLM algorithm** is simply the following:

1. From the initial conditions, obtain the incident voltage at the first time-step

2. Obtain all the reflected voltages (scattering)

3. Obtain all incident voltages at the next time-step (connection)

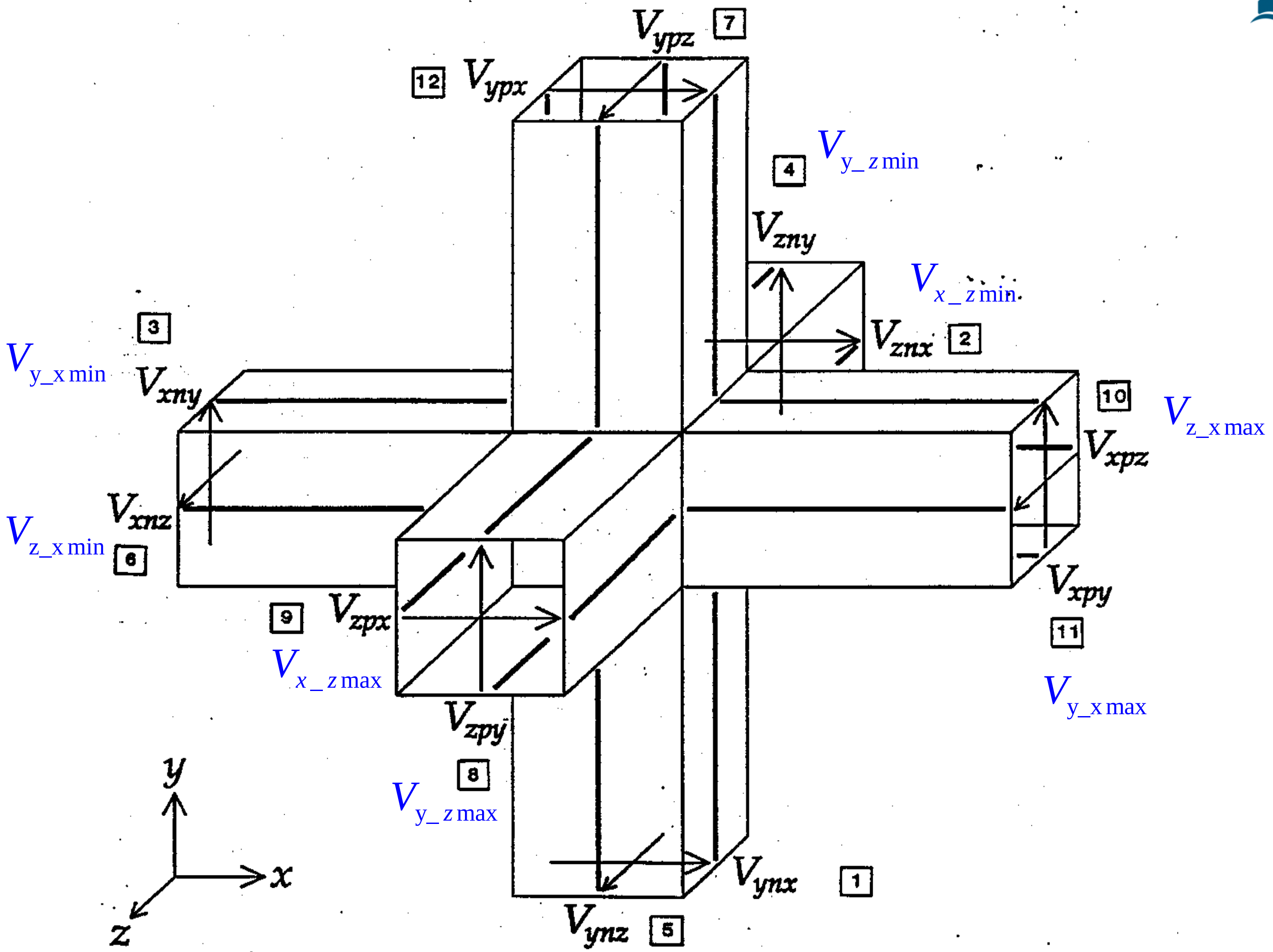
4. Go back to Step 2 for a many time-steps as required

5. Obtain outputs as required

What we have done so far is to show how to model fields using networks for the one-dimensional (1D) case. We can carry on along similar lines to do the same for two-dimensional (2D) fields. But, to speed things along we go straight to three-dimensional (3D) fields since most interesting practical problems are 3D.

Thinking about it, for 3D fields, we need a network of TLs such as that we have ports available for each coordinate direction (looking towards the positive and negative axes). Since there are 3 axes and 2 directions per axis this implies 6 ports. But since we do not know the polarization in each direction we need two orthogonal ports per coordinate direction-in total 12 ports!

The most successful such network is the so-called Symmetrical Condensed Node (SCN).



Symmetrical Condensed Node (SCN)

Notation!

We are aware from the 1D example that operating the TLM algorithm requires a clear and meticulous notation for all pulse propagating through the system. For pulses related to a node, there are several notations based on numbering, or the alternative scheme shown in the previous slide.

$$V^{i}_{xpy}$$

Superscript, i-incident towards the centre of the node, or, r-reflected travelling away from the centre of the node

Polarization direction, x,y, or, z

Segment along positive (p) or negative (n) coordinate axis

Direction of propagation, x,y, or, z

There will also be labels to indicate the position of the node in space and also the time-step at which the pulse is calculated… Yet another notation used in the GGI_TLM is also partially shown in blue on the last slide,

$$V_{pol\_face}$$

Pol, the polarization of the port, x,y, or z

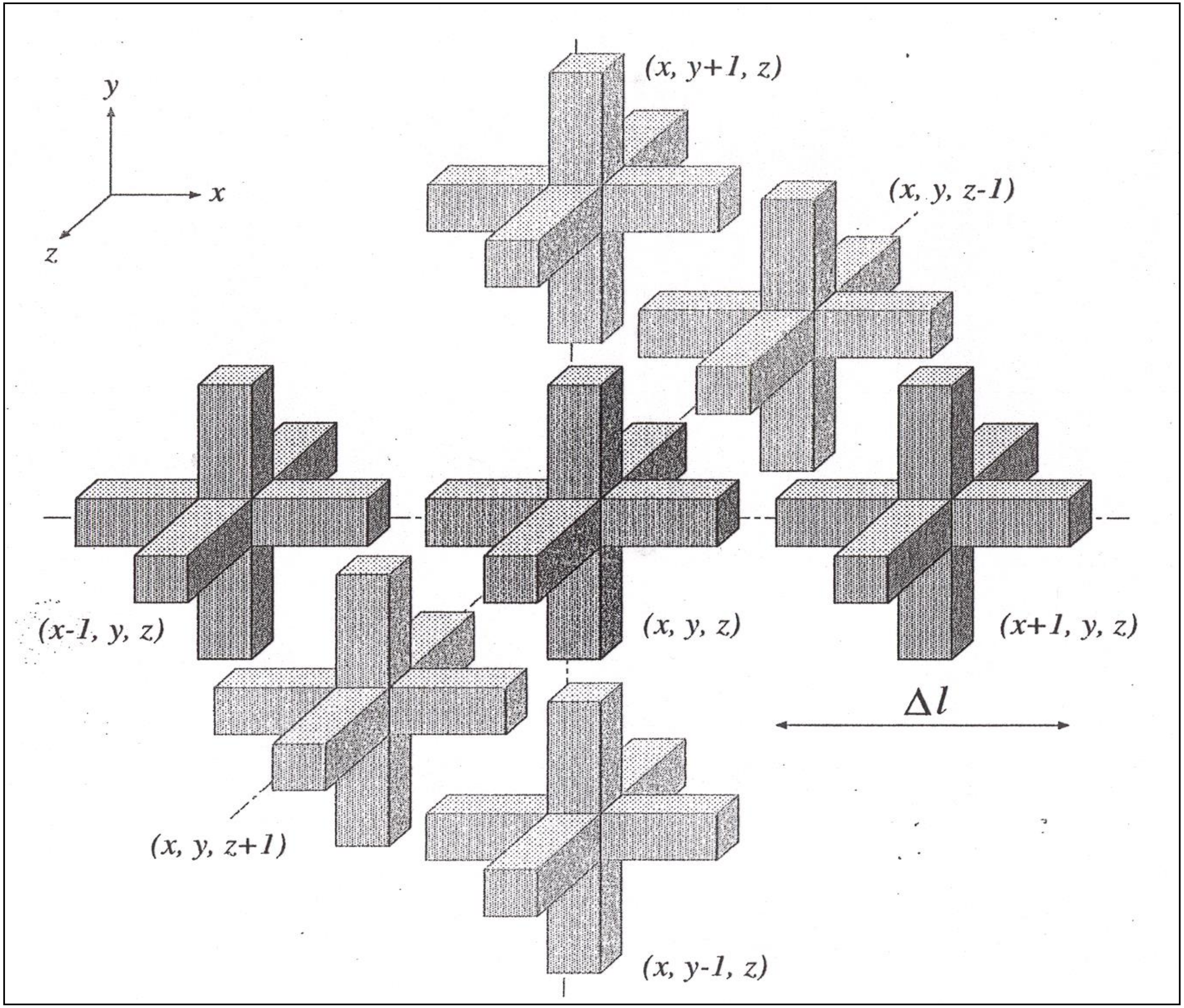
face, where the port is situated, xmin, xmax, ymin,ymax,zmin,zmax

There are also other numbering or labelling schemes developed over the years and it is worth checking carefully the notation for each paper or piece of software!

To set up a TLM model of a problem we need to select the spatial discretization length and then populate the entire problem space with a cluster of nodes as shown schematically on the next slide. We also need to do the following things:

1. Select the parameters of the TLs comprising each node to represent the electrical parameters of the medium at that location
2. Determine and implement the scattering procedure at each node
3. Determine and implement the connection procedure at each node
4. Define and implement input/output procedures to obtain fields from voltage pulses
5. Map boundaries, wires and other features onto the mesh of nodes

Populate problem space with TLM nodes:

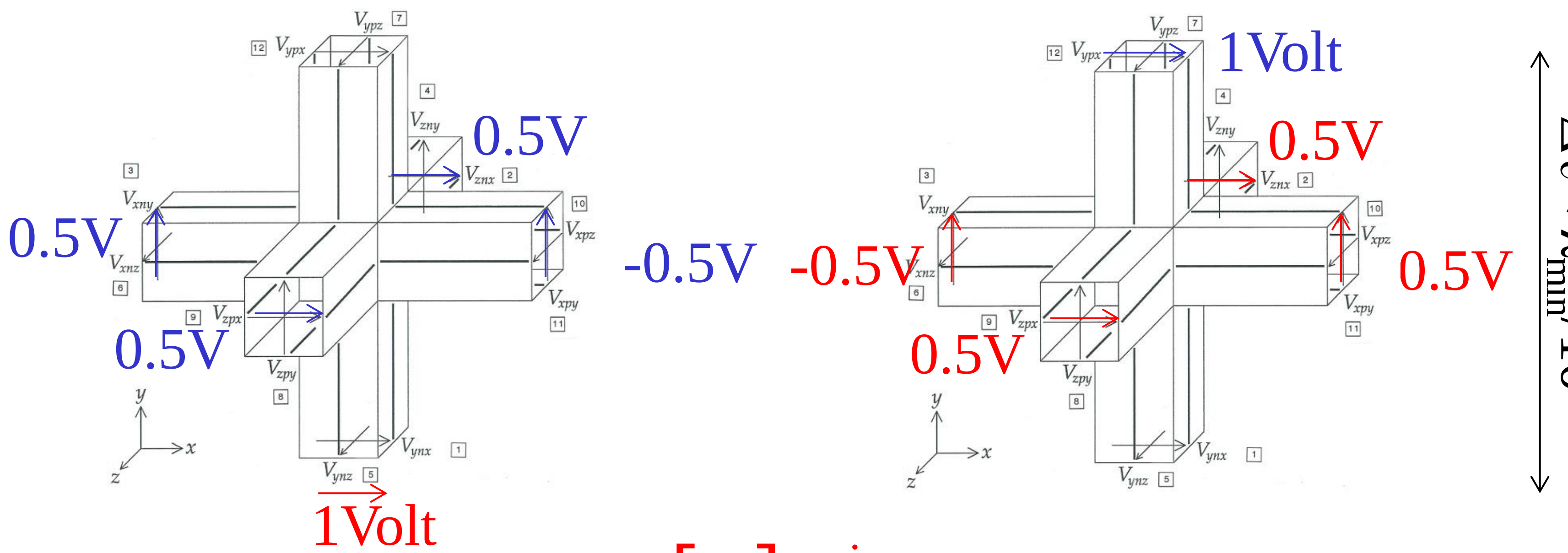


In TLM, to control dispersion we choose $\Delta\ell < \lambda_{min}/10$ where the wavelength is calculated at the highest frequency and $\Delta t = \Delta\ell/(2c)$ where c is the velocity of propagation in the medium (see explanation below)

Can we exceed f_{max} ?

Scattered pulses after Δt

Scattered pulses after 2Δt



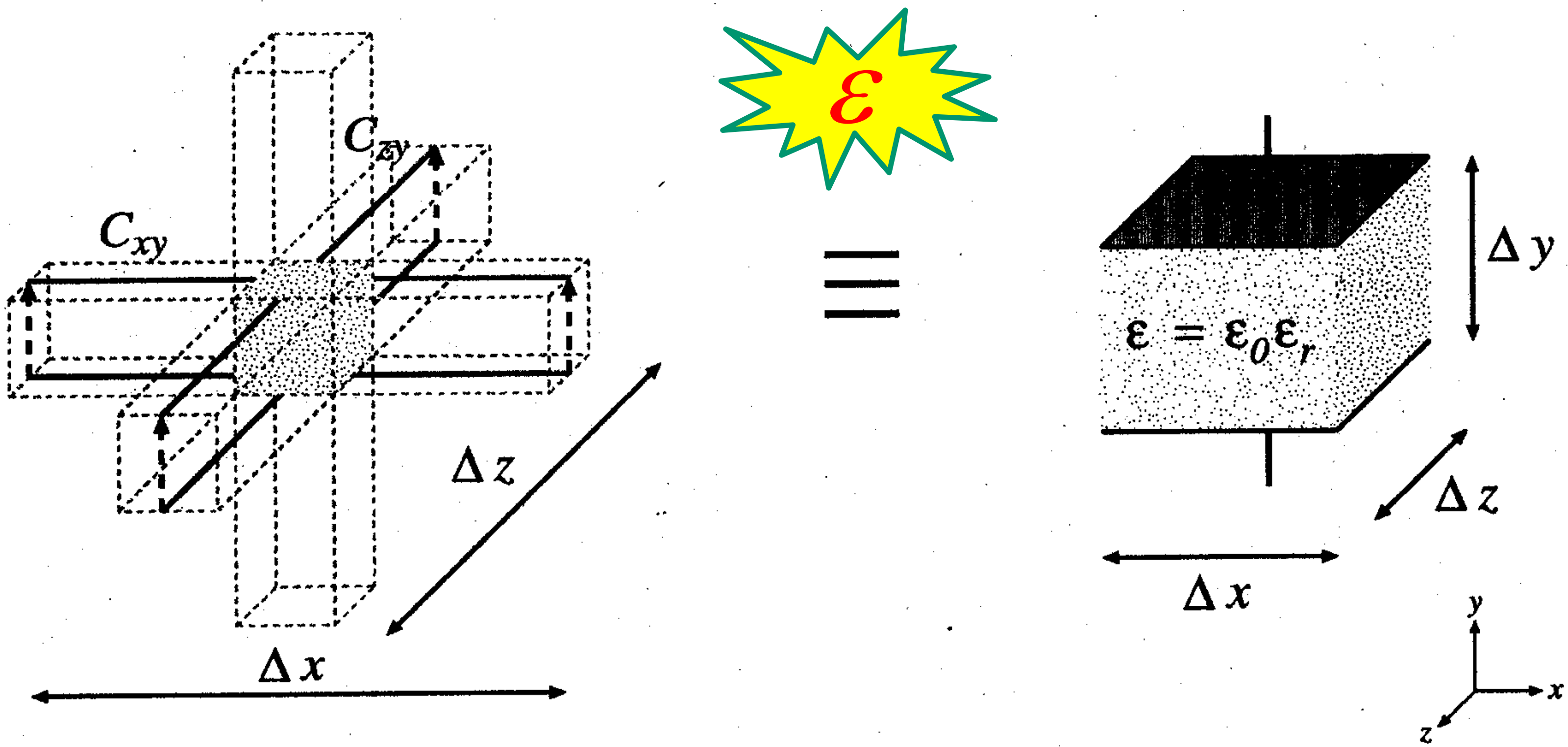
$V_1^i = 1, \quad V_{2\cdots12}^i = 0$

$$V^r = [S]V^i$$

$V_2^i = V_9^i = V_{11}^i = 0.5V, \quad V_3^i = -0.5V$, all else =0

Pulses injected on port 1on a cluster of nodes to launch a plane wave. It takes time 2Δt to travel distance Δℓ and emerge from port 12! Hence to get correct velocity of propagation, $\frac{\Delta\ell}{\Delta t} = 2c$

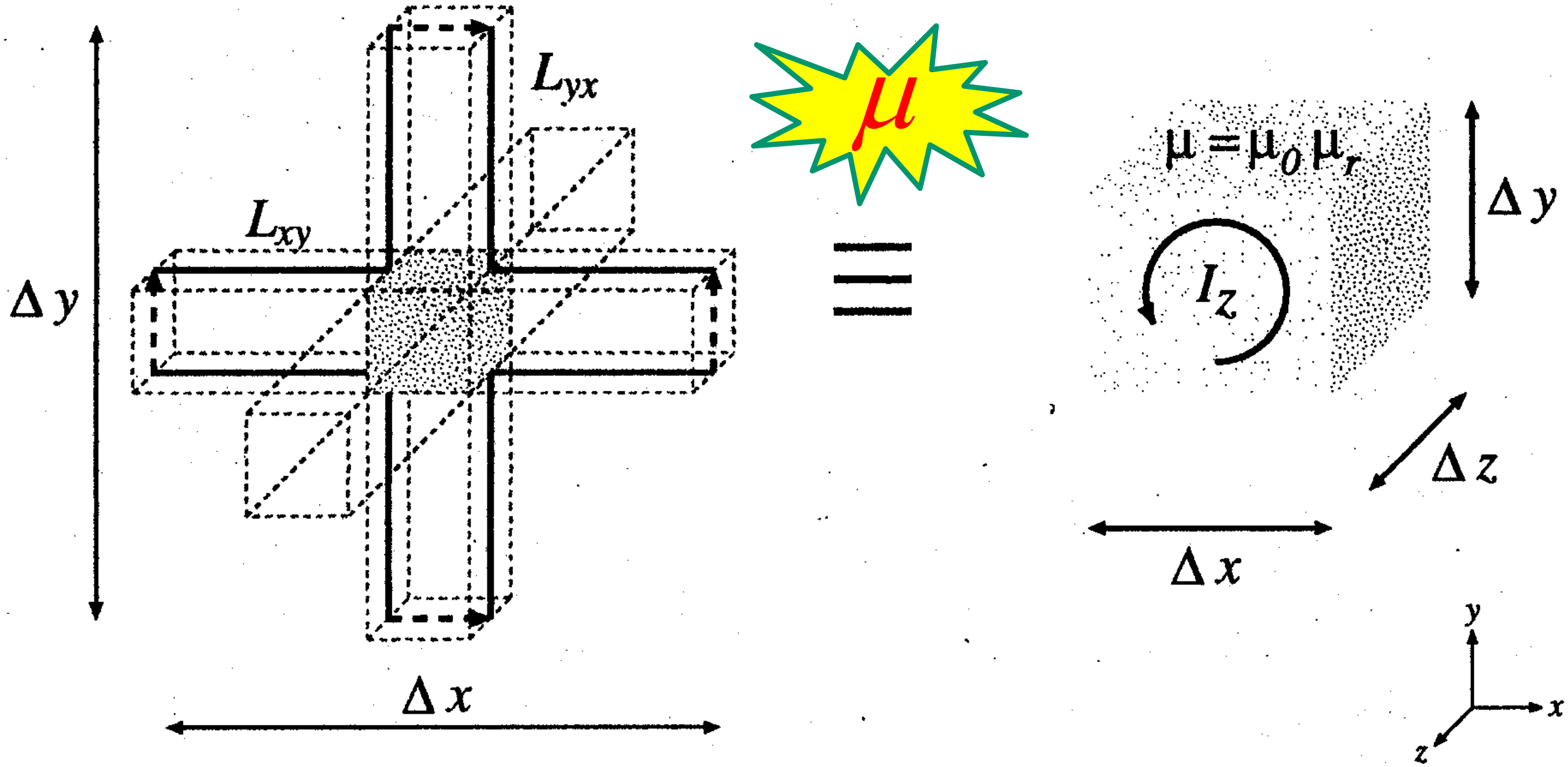
Select TL parameters to represent the properties of the medium:



$$C_y = C_{xy}\Delta x + C_{zy}\Delta z = \varepsilon \frac{\Delta x \Delta z}{\Delta y}$$

Direction of propagation

polarization



$$L_z = L_{xy}\Delta x + L_{yx}\Delta y = \mu \frac{\Delta x \Delta y}{\Delta z}$$

The previous scheme works well if we are dealing with a homogeneous medium (material properties the same throughout the problem space). It may seem that for an inhomogeneous medium all we have to do is to adjust the TL properties accordingly. This we can do but we will be confronted with one of two equally inconvenient options:

- Keep $\Delta\ell$ the same throughout. Since the velocity of propagation is different in each medium then the time-step Δt will be different…loss of synchronism!
- Keep Δt the same throughout. This then will result in different $\Delta\ell$ in each medium…loss of one-to-one correspondence between ports at interfaces between different materials!

Remember!

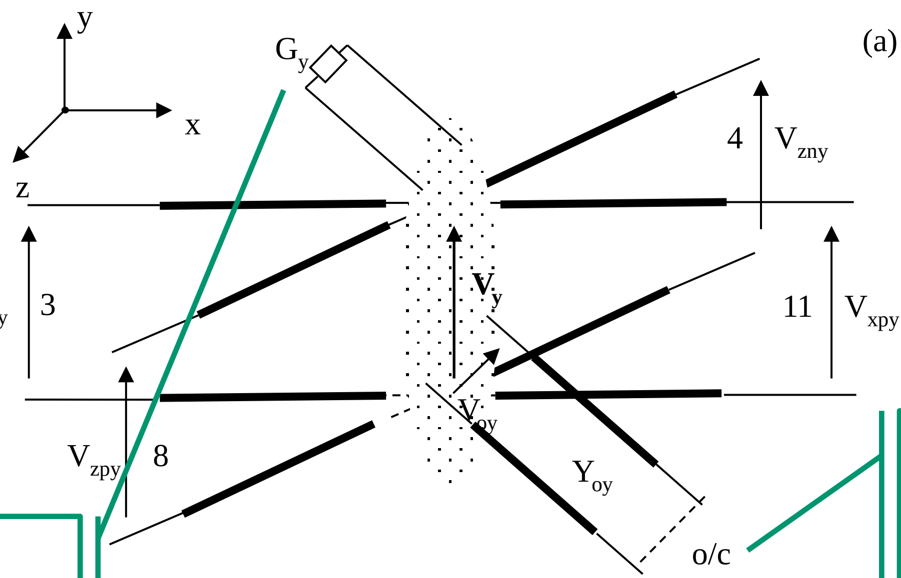
$$\frac{\Delta\ell}{\Delta t} = 2u$$

Loss of synchronism makes “connection” difficult since the exchange of information between nodes does not coincide in time. Similarly, lack of one-to-one correspondence across material interfaces mean that some kind of filtering must be devised. Neither of these options are therefore convenient. The most elegant solution is to employ, in addition to the 12 link lines of the 3D node, transmission line stubs.

For each coordinate direction we can have capacitive stubs (open-circuit one-port short line segments of round trip time Δt), inductive stubs (short-circuit segments) and infinitely long segments to represent loss-conductive stubs.

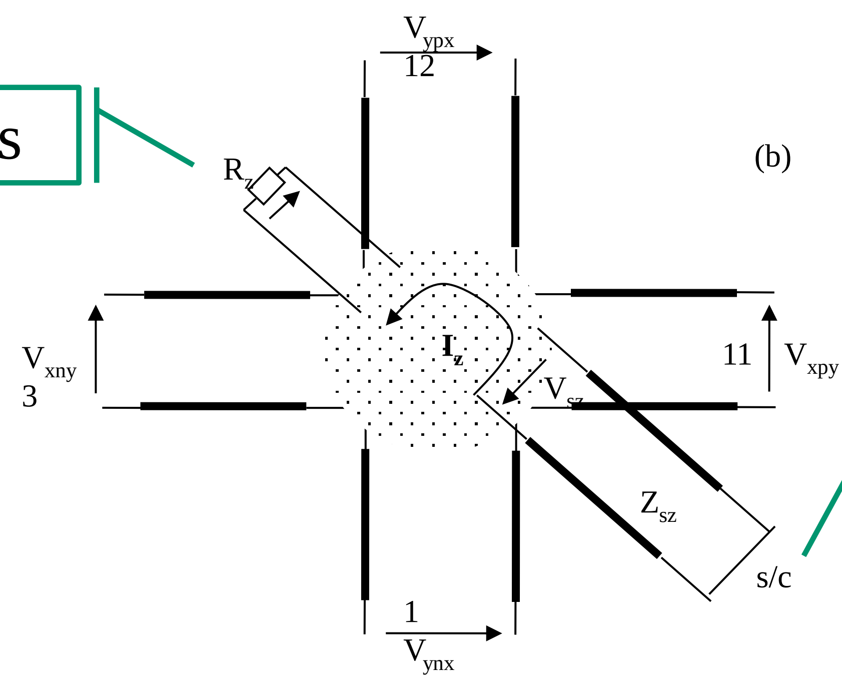
This arrangement is shown on the next slide for one coordinate direction.

A common choice is to select the link lines to represent free-space and use stubs to represent all other materials (then all stubs are positive…stability!). But there are more choices!...



Electric losses

o/c stub to account for extra ε



Magnetic losses

s/c stub to account for extra μ

GSCN

Six constitutive equations:

$$C_{ki}\Delta k + C_{ji}\Delta j + C_0^i = \varepsilon_0 \varepsilon_{ri} \frac{\Delta j \Delta k}{\Delta i}$$

$$L_{jk}\Delta j + L_{kj}\Delta k + L_s^i = \mu_0 \mu_{ri} \frac{\Delta j \Delta k}{\Delta i}$$

Six equations for synchronism:

$$\Delta t = \Delta i \sqrt{C_{ji} L_{ij}}$$

This leaves SIX degrees of freedom

We can devise many other types of node depending on the proportion of material properties which we model by link or stub lines!

Scattering matrix for 12-port SCN…

$$S = 0.5\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$V^r = [S]V^i$$

Scattering must be implemented at each node and for every time-step and involves as can be seen a lot of numerical operations. Ways can be found to make these calculations more efficient. One such way is the calculation of the equivalent voltage V_y and current I_z as indicated schematically on the slide with the stubs.

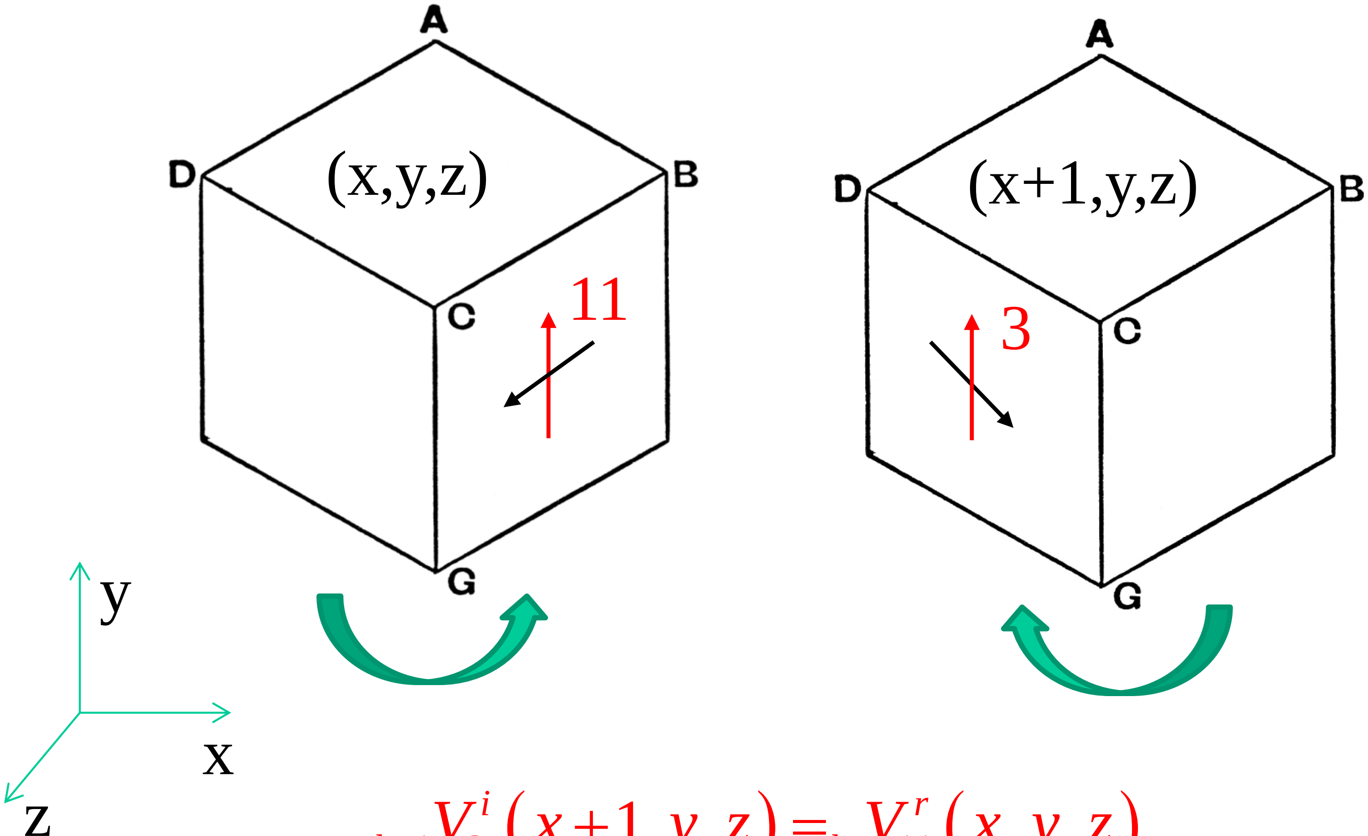
Assuming that all line have an impedance Z_{xy} scattering can be obtained from equations of the type,

$$V^r_{xny} = V_y + I_z Z_{xy} - V^i_{xpy}$$

More efficient scattering!

Three equivalent voltages and three equivalent currents must be calculated first from the incident voltages. Details may be found in references…

Connection:



$$ {}_{k+1}V_3^i(x+1,y,z) = {}_k V_{11}^r(x,y,z) $$

$$ {}_{k+1}V_{11}^i(x,y,z) = {}_k V_3^r(x+1,y,z) $$

Output:

Calculation of electric and magnetic fields

The electric field component E_x may be obtained from $E_x = -\frac{V_x}{\Delta l}$

The total x-directed voltage is the average of the total voltage on port 1,2,9 and 12, i.e.

$$V_x = \frac{1}{4}\left[\left(V_1^i + V_1^r\right) + \left(V_2^i + V_2^r\right) + \left(V_9^i + V_9^r\right) + \left(V_{12}^i + V_{12}^r\right)\right]$$

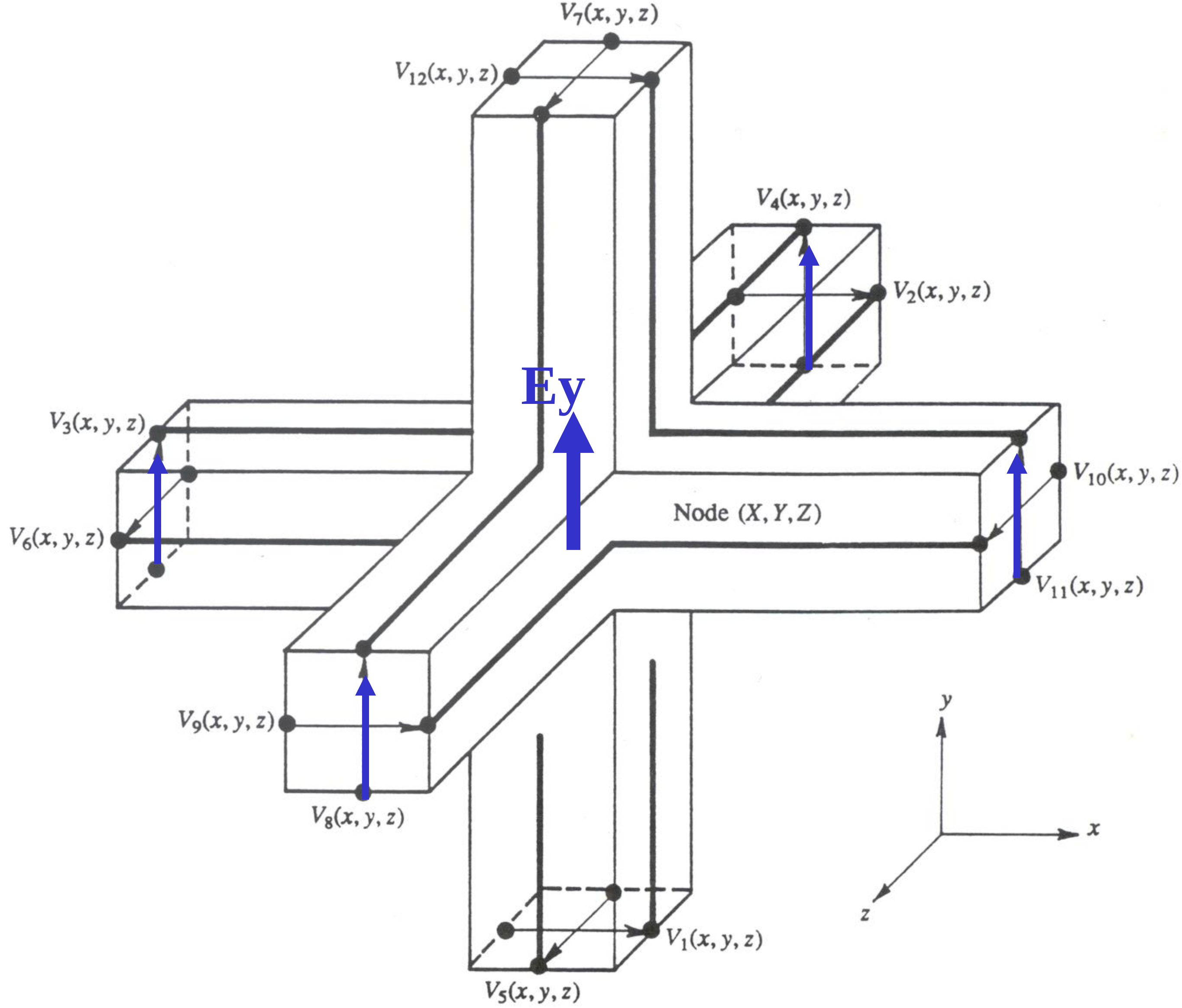
It can be confirmed directly using the scattering matrix, that for charge conservation, the sum of incident voltages to these ports is equal to the sum of reflected voltages.

Hence:

$$V_x = \frac{1}{2}\left(V_1^i + V_2^i + V_9^i + V_{12}^i\right)$$

and

$$E_x = -\frac{V_1^i + V_2^i + V_9^i + V_{12}^i}{2\Delta l}$$



$V_7(x, y, z)$
$V_{12}(x, y, z)$
$V_4(x, y, z)$
$V_2(x, y, z)$
Ey
$V_3(x, y, z)$
$V_{10}(x, y, z)$
Node (X, Y, Z)
$V_6(x, y, z)$
$V_{11}(x, y, z)$
$V_9(x, y, z)$
$V_8(x, y, z)$
y
x
z
$V_1(x, y, z)$
$V_5(x, y, z)$

The magnetic field component H_x may be obtained from

$$H_x = \frac{I_x}{\Delta_x}$$

The current I_x is in turn calculated from

$$H_x = \frac{V_4^i + V_7^i - V_5^i - V_8^i}{2Z\Delta I}$$

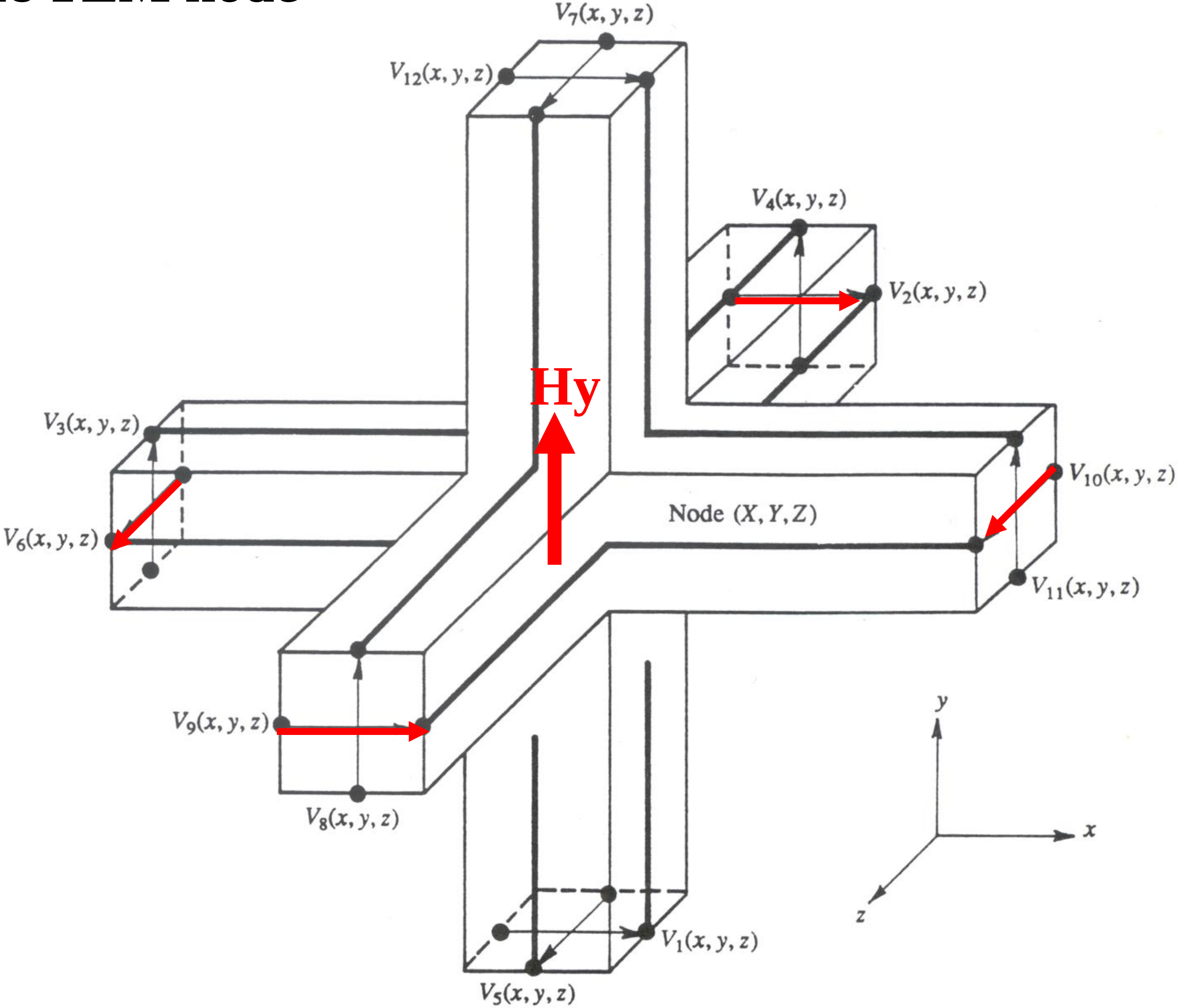
Ports on the z-y plane!

Similarly,

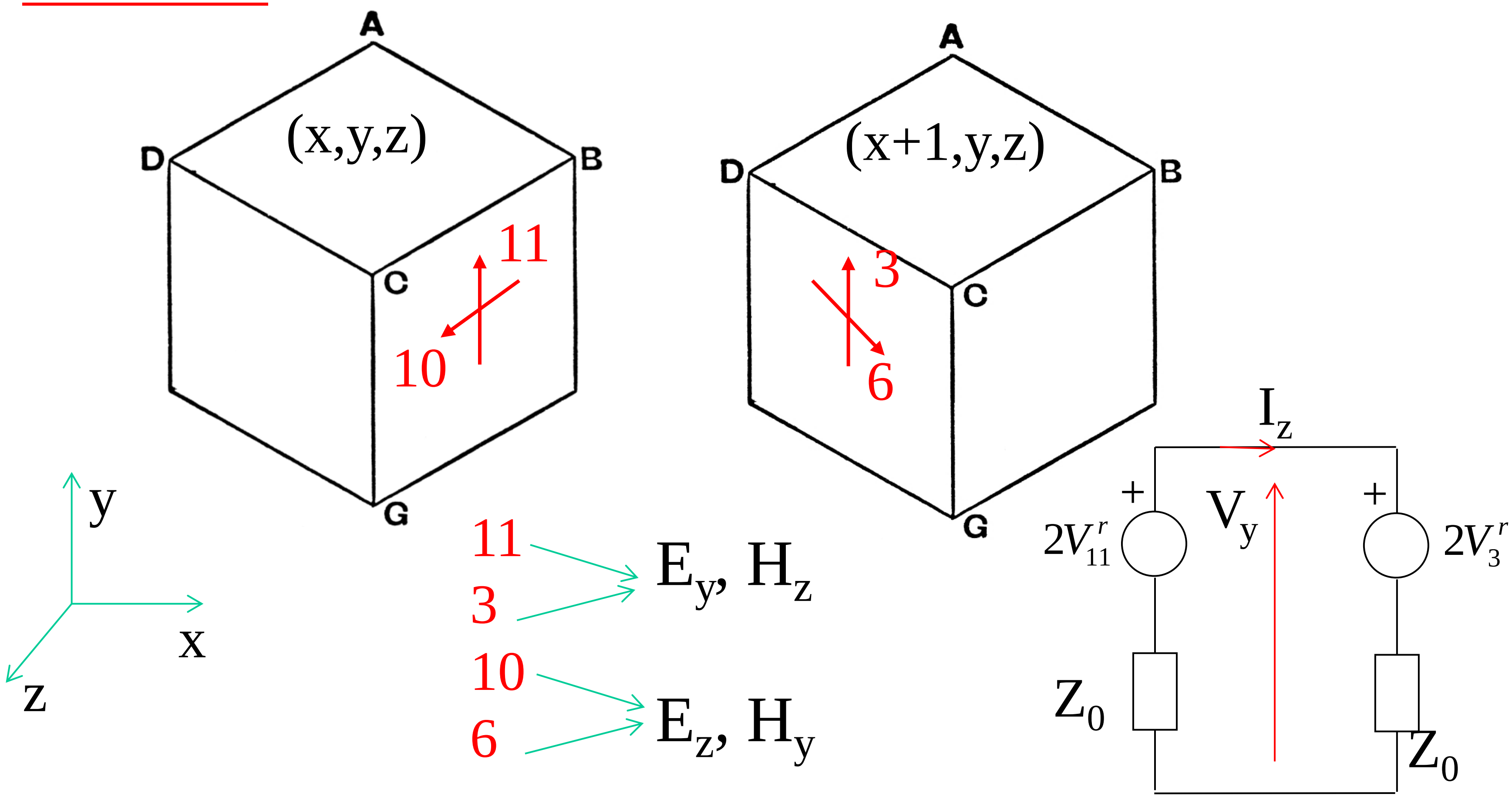
$$H_z = \frac{V_1^i - V_3^i + V_{11}^i - V_{12}^i}{2Z\Delta I}$$

etc.

Basic TLM node



One-to-one correspondence between fields on cell surface and voltage pulses…fields at same point and same time!



Simple mapping…

$$E_y = -\frac{V_y}{\Delta\ell}, \quad H_z = -\frac{I_z}{\Delta\ell}$$

Input:
Excitation in a SCN Mesh

To excite E_x :

$$V_1^i = V_2^i = V_9^i = V_{12}^i = -E_0 \Delta 1/2$$

$$\Rightarrow \quad E_x = E_0$$

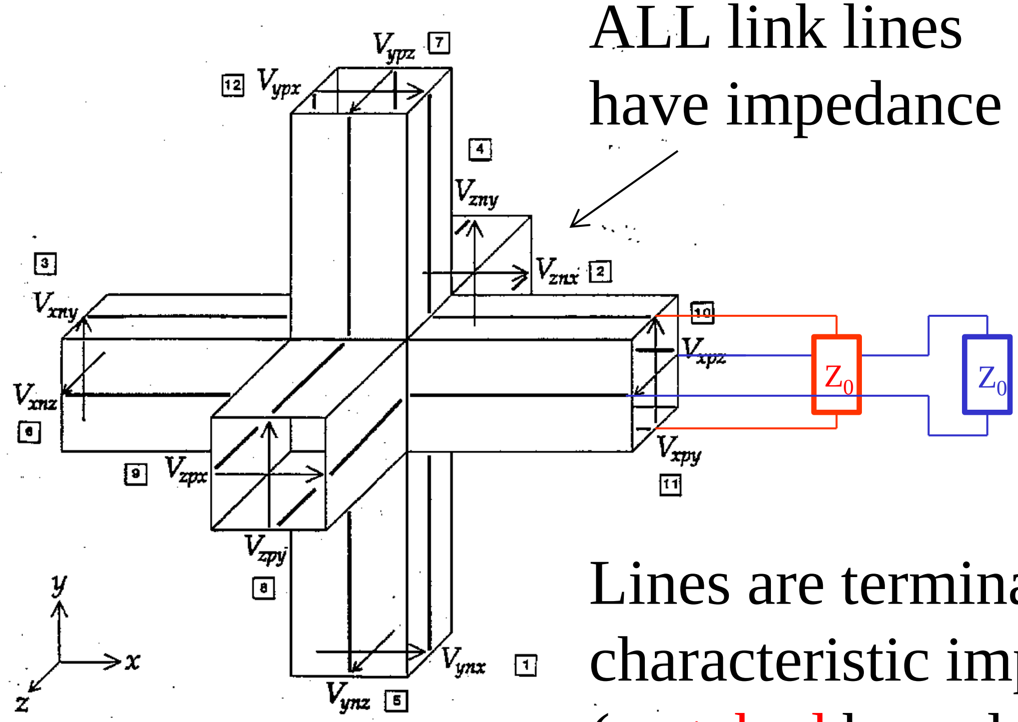
To excite H_x :

$$V_4^i = V_7^i = H_0 Z \Delta 1/2$$

$$V_5^i = V_8^i = -H_0 Z \Delta 1/2$$

$$\Rightarrow \quad H_x = H_0$$

Boundaries…



ALL link lines have impedance Z_0

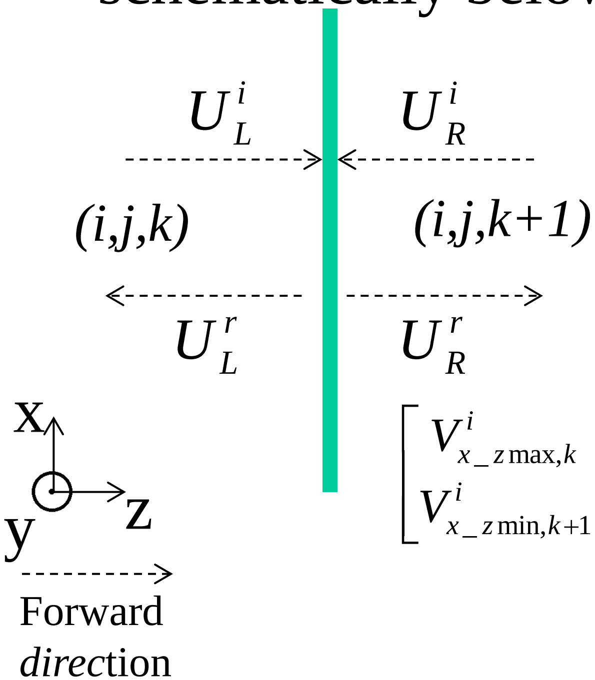
Can we alter BCs dynamically to develop random environments ?

Lines are terminated by their own characteristic impedance-no reflections (matched boundary condition)…simplest absorbing boundary condition.

If the termination impedance is equal to zero (s/c) then we have a conducting boundary, an o/c is a symmetry boundary etc.

Thin layers are normally modelled by *impedance boundary conditions* (rather than attempting to discretize the layers). They are placed between cells (i.e. between link lines forming nodes) and hence are treated during the connection process as shown schematically below. The electric and magnetic field components are related through an impedance matrix,

$$\begin{bmatrix} E_{x,L} \\ E_{x,\mathrm{R}} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} H_{\mathrm{y},L} \\ H_{\mathrm{y,R}} \end{bmatrix}$$



$$\begin{bmatrix} V^i_{x\_z\max,k} \\ V^i_{x\_z\min,k+1} \end{bmatrix} = \left[\frac{1}{Z_0} \begin{bmatrix} Z_{11} & -Z_{12} \\ Z_{21} & -Z_{22} \end{bmatrix} + [1] \right]^{-1} \left[\frac{1}{Z_0} \begin{bmatrix} Z_{11} & -Z_{12} \\ Z_{21} & -Z_{22} \end{bmatrix} - [1] \right]^{-1} \begin{bmatrix} V^r_{x\_z\max,k} \\ V^r_{x\_z\max,k} \end{bmatrix}$$

In the case of <u>frequency-dependent thin boundaries</u> similar techniques apply-the impedance matrix [Z] is separated into the sum of two matrices , the first part corresponding to a fast and the second to a delayed (slow) response. Digital Filters are then devised to deal with the slow part of the response.

Modelling of frequency-dependent impedances…

$$Z_C\left(z^{-1}\right)=\frac{V\left(z^{-1}\right)}{I\left(z^{-1}\right)}=\alpha_0'+\frac{\alpha_1'z^{-1}+\alpha_2'z^{-2}+\cdots}{\beta_0'+\beta_1'z^{-1}+\beta_2'z^{-2}+\cdots}=Z_{Cf}+Z_{Cs}\left(z^{-1}\right)$$

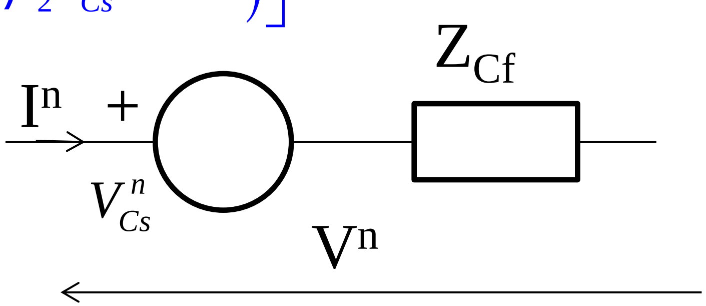
$$\frac{V_{Cs}\left(z^{-1}\right)}{I\left(z^{-1}\right)}$$

This means that we have an impedance Z_{Cf} representing the conductivity term in series with an impedance represented by a digital filter the voltage across this impedance $V_{Cs}(z^{-1})$ is related to the current by,

$$\frac{V_{Cs}\left(z^{-1}\right)}{I\left(z^{-1}\right)}=\frac{\alpha_1'z^{-1}+\alpha_2'z^{-2}+\cdots}{\beta_0'+\beta_1'z^{-1}+\beta_2'z^{-2}+\cdots}\Rightarrow\beta_0'V_{Cs}^{n}+\beta_1'V_{Cs}^{n-1}+\beta_2'V_{Cs}^{n-2}+\cdots=\alpha_1'I^{n-1}+\alpha'I^{n-2}+\cdots$$

$$\Rightarrow V_{Cs}^{n}=\frac{1}{\beta_0'}\left[\alpha_1'I^{n-1}+\alpha'I^{n-2}+\cdots-\left(\beta_1'V_{Cs}^{n-1}+\beta_2'V_{Cs}^{n-2}+\cdots\right)\right]$$

The circuit equivalent is thus,

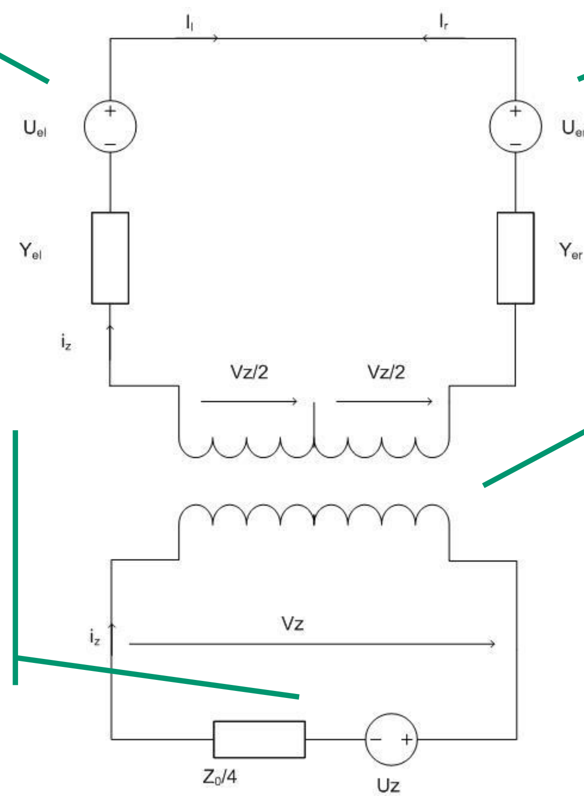


Wire-to-field interface:

The simplified model for a segment of wire Δℓ long is shown below, where the top network representing the wire is shown by the Thevenin equivalents looking right and left. Scattering and connections proceed in synchronism with the field solution. The field and wire models are coupled and self-consistent.

Wire segment end on the Left

Wire segment end on the Right



Wire-Field coupling

Field components affecting wire current

We can embed wires of smaller diameter than Δℓ…

Contact

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