Experimental and Theoretical Investigation of Stationary Stochastic EM Fields

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Outline

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Introduction

- In the context of modern computer-aided manufacturing accurate EMI modeling is required to design systems in a way that they comply with EMC standards.
- The modeling of stochastic fields differs from the modeling of deterministic fields since we have to consider the correlation between any pair of field samples.
- We present a methodology for the numerical computation of noisy electromagnetic fields excited by spatially distributed noise sources with arbitrary spatial correlation.

Introduction

- Near field characterization of the EMI radiated by a component is aimed to provide the information for modeling of EMI field distribution produced by this component when embedded into some system unit.
- Due to the equivalence principle an equivalent source distribution determined by amplitude and phase scanning of the tangential electric or magnetic field on a surface enclosing the radiating structure is equivalent to the internal sources and allows to model the environmental field.
- State of the art comprises electromagnetic interference (EMI) near-field scanners, scanning the electric or magnetic near-field amplitude distribution.
- Literature:
• J. A. Russer and P. Russer, "Network methods applied to the computation of stochastic electromagnetic fields," in *2011 Int. Conf. on Electromagnetics in Advanced Applications (ICEAA)*, Sep. 2011, pp. 1152–1155
• J. A. Russer and P. Russer, "Modeling of noisy EM field propagation using correlation information," *IEEE Transactions on Microwave Theory and Techniques*, vol. 63, no. 1, pp. 76–89, Jan. 2015

Outline

Network Oriented Noise Modeling

- Numerical values of noise amplitudes cannot be specified for stochastic signals.
- For numerical modeling of noisy circuits one has to deal with energy and power spectra.
- Stationary stochastic signals with Gaussian amplitude probability distribution can be completely described by their *auto-* and *cross correlation spectra*.

- Literature:
• W. B. Davenport and W. L. Root, *An Introduction to the Theory of Random Signals and Noise*. New York: McGraw-Hill, 1958
• H. A. Haus and R. W. Adler, *Circuit Theory of Linear Noisy Networks*, New York. John Wiley, 1959
• H. Hillbrand and P. Russer, "An efficient method for computer aided noise analysis of linear amplifier networks," , vol. 23, no. 4, pp. 235–238, Apr. 1976
• P. Russer and S. Muller, "Noise analysis of linear microwave circuits," *International Journal of Numerical Modelling, Electronic Networks, Devices and Fields (IJNM)*, vol. 3, pp. 287–316, 1990

Network Oriented Noise Modeling

- The spectrum of a stochastic signal does not exist.
- We can, however take a time-windowed sample $s_T(t)$ of a signal $s(t)$ defined by

$$s_T(t) = \begin{cases} s(t) & \text{for} \quad |t| \leqslant T \\ 0 & \text{for} \quad |t| > T \end{cases} \quad . \tag{1}$$

- From this time-windowed signal we can compute the spectrum $S_T(\omega)$

$$S_T(\omega) = \int_{-\infty}^{\infty} s_T(t) \mathrm{e}^{-\jmath\omega t}\, dt \,, \tag{2a}$$

$$s_T(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_T(\omega) \mathrm{e}^{\jmath\omega t}\, d\omega \,. \tag{2b}$$

Correlation Functions and Correlation Spectra

- For a stationary stochastic signal we define the *correlation function*

$$c_{ij}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} s_{iT}(t) s_{jT}(t-\tau)\, dt\,. \tag{3}$$

- For $i = j$, the function $c_{ii}(\tau)$ is called the *autocorrelation function*; $c_{ij}(\tau)$ with $i \neq j$ is called the *cross correlation function*.
- The Fourier transform $C_{ij}(\omega)$ of $c_{ij}(\tau)$ is the *correlation spectrum*:

$$C_{ij}(\omega) = \int\limits_{-\infty}^{+\infty} c_{ij}(\tau) e^{-\jmath\omega\tau}\, d\tau\,, \tag{4a}$$

$$c_{ij}(\tau) = \int\limits_{-\infty}^{+\infty} C_{ij}(\omega) e^{\jmath\omega\tau}\, d\omega\,, \tag{4b}$$

where $C_{ii}(\omega)$ is an *autocorrelation spectrum* and $C_{ij}(\omega)$ with $i \neq j$ is a *cross correlation spectrum*.

Network Oriented Noise Modeling

- We can also write the correlation spectra as

$$C_{ij}(\omega) = \lim_{T\to\infty} \frac{1}{2T}\langle S_{iT}(\omega) S_{jT}^{*}(\omega)\rangle\,, \tag{5}$$

  where the brackets $\langle \dots \rangle$ denote the forming of the *ensemble average*. The ensemble average has to be formed before the limiting process $T \to \infty$ since the amplitude spectrum does not exist for $T \to \infty$.
- The autocorrelation spectrum $C_{ii}(\omega)$ describes the *spectral energy density* of the signal $s_i(t)$.
- To compute the spectral energy densities of linear superpositions of signals we need also their cross correlation spectra.

Correlation Matrices

- We can summarize the correlation spectra of a number of n signals in the *correlation matrix*

$$C(\omega) = \begin{pmatrix} C_{11}(\omega) & C_{12}(\omega) & \cdots & C_{1n}(\omega) \\ C_{21}(\omega) & C_{22}(\omega) & \cdots & C_{2n}(\omega) \\ \vdots & \vdots & & \\ C_{n1}(\omega) & C_{n2}(\omega) & \cdots & C_{nn}(\omega) \end{pmatrix}. \tag{6}$$

- The correlation matrix is *Hermitian*.
- Summarizing the spectra of the time windowed signals $S_{1T}(\omega) \dots S_{nT}(\omega)$ in the vector $\boldsymbol{S}_T(\omega)$ we can represent the correlation matrix in the compact form

$$\boldsymbol{C}(\omega) = \lim_{T\to\infty} \frac{1}{2T} \langle \boldsymbol{S}_T(\omega)\ \boldsymbol{S}_T^\dagger(\omega) \rangle. \tag{7}$$

A General Rule for the Derivation of Network Equations for Correlation Matrices

- The equations describing linear networks have the form

$$\boldsymbol{S}_T^{'}(\omega) = \boldsymbol{M}(\omega)\, \boldsymbol{S}_T(\omega)\,. \tag{8}$$

- This yields to the relation between the correlation matrices

$$\boldsymbol{C}'(\omega) = \boldsymbol{M}(\omega)\, \boldsymbol{C}(\omega)\, \boldsymbol{M}^{\dagger}(\omega)\,. \tag{9}$$

- • H. Hillbrand and P. Russer, “An efficient method for computer aided noise analysis of linear amplifier networks,” , vol. 23, no. 4, pp. 235–238, Apr. 1976
  • P. Russer and S. Muller, “Noise analysis of linear microwave circuits,” *International Journal of Numerical Modelling, Electronic Networks, Devices and Fields (IJNM)*, vol. 3, pp. 287–316, 1990

Outline

Scalar Stochastic Fields

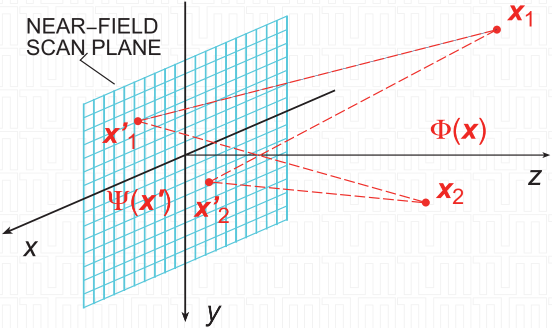
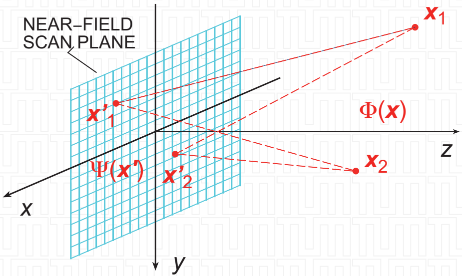


Figure: *Near and far field.*

The aperture field $\Psi(x')$ is the source of a far-field $\Phi(x)$.

Near and Far Field



The far-field $\Phi(\boldsymbol{x}, \omega)$ is related to the near-field $\Psi(\boldsymbol{x}', \omega)$ via

$$\Phi(\boldsymbol{x}, \omega) = \int_A G_0(\boldsymbol{x}, \boldsymbol{x}', \omega) \Psi(\boldsymbol{x}' \omega) d^3x' \,, \tag{10}$$

where the *scalar Green's function* $G_0(\boldsymbol{x}, \boldsymbol{x}')$ is given by

$$G_0(\boldsymbol{x}, \boldsymbol{x}', \omega) = \frac{\exp\left[-\jmath k(\boldsymbol{x} - \boldsymbol{x}')\right]}{|\boldsymbol{x} - \boldsymbol{x}'|} \,. \tag{11}$$

Stochastic Scalar Fields

$$c_\phi(\boldsymbol{x}_1,\boldsymbol{x}_2,\tau) = \lim_{T\to\infty}\frac{1}{2T}\int_{-\infty}^{\infty}\phi_T(\boldsymbol{x}_1,t)\phi_T(\boldsymbol{x}_2,t-\tau)dt\,. \qquad (12)$$

- s_T denotes the *time-windowed field*, defined by

$$\phi_T(\boldsymbol{x}_1,t) = \begin{cases}\phi(\boldsymbol{x}_1,t) & \text{for} \quad -T<t<T \\ 0 & \text{for} \quad |t|\geqslant T\end{cases}\,. \qquad (13)$$

- The Fourier transform of $c_\phi(\boldsymbol{x}_1,\boldsymbol{x}_2,\tau)$ is the *correlation spectrum*

$$\Gamma_\phi(\boldsymbol{x}_1,\boldsymbol{x}_2,\omega) = \int_{-\infty}^{\infty}c_\phi(\boldsymbol{x}_1,\boldsymbol{x}_2,\tau)\exp(-\jmath\omega\tau)d\tau\,. \qquad (14)$$

Stochastic Scalar Fields

- We can also obtain the correlation spectra directly from the spectra $\Phi_T(\boldsymbol{x}, \omega)$ of the time-windowed fields by

$$\Gamma_\phi(\boldsymbol{x}_1, \boldsymbol{x}_2, \omega) = \lim_{T \to \infty} \frac{1}{2T} \langle \Phi_T(\boldsymbol{x}_1, \omega) \Phi_T^*(\boldsymbol{x}_2, \omega) \rangle \,, \quad (15)$$

where the brackets denote the forming of the *ensemble average*.

Stochastic Scalar Fields

- To express the correlation spectrum of the far-field $\Gamma_\phi(\boldsymbol{x}_1, \boldsymbol{x}_2, \omega)$ as a function of the *correlation spectrum of the near-field*, given by

$$\Gamma_\psi(\boldsymbol{x}_1, \boldsymbol{x}_2, \omega) = \lim_{T\to\infty} \frac{1}{2T} \langle \Psi_T(\boldsymbol{x}_1, \omega) \Psi_T^*(\boldsymbol{x}_2, \omega) \rangle, \tag{16}$$

  we insert (10) into (15) and obtain

$$\begin{aligned} \Gamma_\phi(\boldsymbol{x}_1, \boldsymbol{x}_2) = \lim_{T\to\infty} \frac{1}{2T} \iint_A G_0(\boldsymbol{x}_1, \boldsymbol{x}_1') \\ \times \left\langle \Psi_T(\boldsymbol{x}_1') \Psi_T^*(\boldsymbol{x}_2') \right\rangle G_0^*(\boldsymbol{x}_2, \boldsymbol{x}_2') d^3x_1' d^3x_2' \\ = \iint_A G_0(\boldsymbol{x}_1, \boldsymbol{x}_1') \Gamma_\psi(\boldsymbol{x}_1', \boldsymbol{x}_2') G_0^*(\boldsymbol{x}_2, \boldsymbol{x}_2') d^3x_1' d^3x_2' \,. \end{aligned} \tag{17}$$

Stochastic Scalar Felds

- This allows to compute the *field correlation spectrum* $\Gamma_\phi(\boldsymbol{x}_1, \boldsymbol{x}_2)$ for the field amplitudes at the points of observation $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ from the correlation spectrum of the source field $\Gamma_\psi(\boldsymbol{x}'_1, \boldsymbol{x}'_2)$.
- *To compute the field excited by a distribution of stochastic sources requires not only the knowledge of the spatial distribution of the spectral energy density of the source but also the full information about the cross correlation of the source field amplitudes at any pair of points x'_1 and x'_2.*

Outline

Vectorial Stochastic Fields

- Consider a current density vector $\boldsymbol{J}(\boldsymbol{x},\omega)$ describing the source of the electromagnetic field. The electric field excited from $\boldsymbol{J}(\boldsymbol{x},\omega)$ is given by

$$\boldsymbol{E}(\boldsymbol{x},\omega) = \int_V \boldsymbol{G}(\boldsymbol{x},\boldsymbol{x}',\omega)\boldsymbol{J}(\boldsymbol{x}',\omega)d^3x'\,, \tag{18}$$

  where $\boldsymbol{G}(\boldsymbol{x},\boldsymbol{x}',\omega)$ is the total *Green's dyadic* .
- The integration is extended over the whole volume V where $\boldsymbol{J}(\boldsymbol{x},\omega)$ is nonvanishing.
  - J. V. Bladel, *Electromagnetic Fields*, 2nd. New York: J. Wiley, 2007

Vectorial Stochastic Fields

- *Stochastic source currents* can be described by the dyadic

$$\Gamma_J(\boldsymbol{x}_1, \boldsymbol{x}_2, \omega) = \lim_{T\to\infty} \frac{1}{2T} \langle \boldsymbol{J}_T(\boldsymbol{x}_1, \omega) \boldsymbol{J}_T^\dagger(\boldsymbol{x}_2, \omega) \rangle, \tag{19}$$

  where $J_T(\boldsymbol{x}, \omega)$ is the time-windowed current density, and $J_T^\dagger(\boldsymbol{x}, \omega)$ is its Hermitian conjugate.
- The *stochastic electric field* can be described by the dyadic

$$\Gamma_E(\boldsymbol{x}_1, \boldsymbol{x}_2, \omega) = \lim_{T\to\infty} \frac{1}{2T} \langle \boldsymbol{E}_T(\boldsymbol{x}_1, \omega) \boldsymbol{E}_T^\dagger(\boldsymbol{x}_2, \omega) \rangle, \tag{20}$$

  where $E_T(\boldsymbol{x}, \omega)$ is the spectrum of the *time-windowed electric field*.

Vectorial Stochastic Fields

- We obtain

$$\Gamma_E(\boldsymbol{x}_1, \boldsymbol{x}_2, \omega) = \iint_V \boldsymbol{G}(\boldsymbol{x}_1, \boldsymbol{x}_1') \times \Gamma_J(\boldsymbol{x}_1', \boldsymbol{x}_2', \omega) \boldsymbol{G}^\dagger(\boldsymbol{x}_2, \boldsymbol{x}_2') d^3x_1' d^3x_2' \,. \tag{21}$$

- With this we obtain from the *correlation dyadic* $\Gamma_J(\boldsymbol{x}_1, \boldsymbol{x}_2, \omega)$ *of the source currents* the *correlation dyadic of the electric field* $\Gamma_E(\boldsymbol{x}_1, \boldsymbol{x}_2, \omega)$.
- The *spectral electric energy density* $W_E(\boldsymbol{x}, \omega)$ is given by

$$W_E(\boldsymbol{x}, \omega) = \frac{\varepsilon}{2} |\Gamma_E(\boldsymbol{x}, \boldsymbol{x}, \omega)| \,, \tag{22}$$

where ε is the permittivity of the medium.

Environmental Field Computation

$$\boldsymbol{E}(\boldsymbol{x},\omega) = \int_V \boldsymbol{G}(\boldsymbol{x}-\boldsymbol{x}',\omega)\boldsymbol{J}(\boldsymbol{x}',\omega)d^3x'$$

$$\Gamma_J(\boldsymbol{x}_a,\boldsymbol{x}_b,\omega) = \lim_{T\to\infty}\frac{1}{2T}\langle \boldsymbol{J}_T(\boldsymbol{x}_a,\omega)\boldsymbol{J}_T^\dagger(\boldsymbol{x}_b,\omega)\rangle$$

$$\Gamma_E(\boldsymbol{x}_a,\boldsymbol{x}_b,\omega) = \lim_{T\to\infty}\frac{1}{2T}\langle \boldsymbol{E}_T(\boldsymbol{x}_a,\omega)\boldsymbol{E}_T^\dagger(\boldsymbol{x}_b,\omega)\rangle$$

$$\Gamma_E(\boldsymbol{x}_a,\boldsymbol{x}_b,\omega) = \iint\limits_V \boldsymbol{G}(\boldsymbol{x}_a-\boldsymbol{x}'_a)\,\Gamma_J(\boldsymbol{x}'_a,\boldsymbol{x}'_b,\omega)\,\boldsymbol{G}^\dagger(\boldsymbol{x}_b-\boldsymbol{x}'_b)d^3x'_a d^3x'_b\,.$$

With this we obtain from the correlation dyadic $\Gamma_J(\boldsymbol{x}_a,\boldsymbol{x}_b,\omega)$ of the source currents the correlation dyadic of the electric field $\Gamma_E(\boldsymbol{x}_a,\boldsymbol{x}_b,\omega)$.
The spectral electric energy density $W_E(\boldsymbol{x},\omega)$ is given by

$$W_E(\boldsymbol{x},\omega) = \frac{\varepsilon}{2}|\Gamma_E(\boldsymbol{x},\boldsymbol{x},\omega)|\,. \tag{23}$$

Outline

Numerical Computation of Stochastic Fields

- The numerical computation of stochastic electromagnetic fields can be performed in an efficient way by transforming the field problem to a network problem.
- Like in the case of deterministic electromagnetic fields also in the case of stochastic electromagnetic fields network methods can reduce the computational effort considerably and beyond this can contribute to *compact model* generation.
- Network methods for deterministic fields already have been described in

• P. Russer, *Electromagnetics, Microwave Circuit and Antenna Design for Communications Engineering*, Second. Boston: Artech House, 2006
• L. B. Felsen, M. Mongiardo, and P. Russer, *Electromagnetic Field Computation by Network Methods*. Springer-Verlag, Mar. 2009

Numerical Computation of Stochastic Fields

- In the following we describe the computation of stochastic electromagnetic fields by the *Method of Moments* (MoM).
- The MoM allows to transform a field problem into a network-like problem described by algebraic equations.
  - R. F. Harrington, *Time Harmonic Electromagnetic Fields*. New York: McGraw-Hill, 1961

Numerical Computation of Stochastic Fields

- Let us first apply the MoM to compute the integral expression for deterministic fields. We expand the field functions $\boldsymbol{J}(\boldsymbol{x}, \omega)$ and $\boldsymbol{E}(\boldsymbol{x}, \omega)$ into basis functions

$$\boldsymbol{J}(\boldsymbol{x}, \omega) = \sum_n I_n(\omega) \boldsymbol{u}_n(\boldsymbol{x}) \,, \tag{24a}$$

$$\boldsymbol{E}(\boldsymbol{x}, \omega) = \sum_n V_n(\omega) \boldsymbol{u}_n(\boldsymbol{x}) \,, \tag{24b}$$

  where the $\boldsymbol{u}_n(\boldsymbol{x})$ are *vectorial basis functions* and $I_n(\omega)$ and $V_n(\omega)$ are the expansion coefficients.
- We can consider $I_n(\omega)$ and $V_n(\omega)$ as *generalized voltages* and *currents*, respectively. If use a complete set of basis functions, the series expansions will converge to the exact value.
- However, to facilitate a numerical treatment of the problem we have to truncate the series expansion after a finite number of elements.

Numerical Computation of Stochastic Fields

- With these series expansions we obtain

$$\sum_n V_n(\omega)\boldsymbol{u}_n(\boldsymbol{x}) = \sum_n I_n(\omega)\int_V \boldsymbol{G}(\boldsymbol{x},\boldsymbol{x}',\omega)\boldsymbol{u}_n(\boldsymbol{x}')d^3x' \,. \tag{25}$$

- Using expansion functions $\boldsymbol{u}_n(\boldsymbol{x})$ with the property

$$\int_V \boldsymbol{u}_m^\dagger(\boldsymbol{x})\boldsymbol{u}_n(\boldsymbol{x})d^3x = \delta_{mn} \,, \tag{26}$$

where δ_{mn} is the Kronecker delta, and multiplying (25) from the left with $\boldsymbol{u}_m^\dagger(\boldsymbol{x})$ and integrating over V yields

$$V_m(\omega) = \sum_m Z_{mn}(\omega)I_n(\omega) \,. \tag{27}$$

Numerical Computation of Stochastic Fields

We expand the correlation dyadics $\Gamma_J(\boldsymbol{x}_a, \boldsymbol{x}_b, \omega)$ and $\Gamma_E(\boldsymbol{x}_a, \boldsymbol{x}_b, \omega)$ into basis functions

$$C_{I,mn}(\omega) = \iint\limits_V \boldsymbol{u}_m^\dagger(\boldsymbol{x})\Gamma_J(\boldsymbol{x}, \boldsymbol{x}', \omega)\boldsymbol{u}_n(\boldsymbol{x}')d^3x\, d^3x' \,, \tag{28a}$$

$$C_{V,mn}(\omega) = \iint\limits_V \boldsymbol{u}_m^\dagger(\boldsymbol{x})\Gamma_E(\boldsymbol{x}, \boldsymbol{x}', \omega)\boldsymbol{u}_n(\boldsymbol{x}')d^3x\, d^3x' \,. \tag{28b}$$

These matrix elements can be summarized in the matrices

$$\boldsymbol{C}_I(\omega) = \begin{bmatrix} C_{I,11}(\omega) & \dots & C_{I,1N}(\omega) \\ \vdots & \ddots & \vdots \\ C_{I,N1}(\omega) & \dots & C_{I,NN}(\omega) \end{bmatrix} , \tag{29a}$$

$$\boldsymbol{C}_V\omega) = \begin{bmatrix} C_{V,11}(\omega) & \dots & C_{V,1N}(\omega) \\ \vdots & \ddots & \vdots \\ C_{V,N1}(\omega) & \dots & C_{V,NN}(\omega) \end{bmatrix} . \tag{29b}$$

Numerical Computation of Stochastic Fields

The matrix elements $Z_{mn}(\omega)$ are given by

$$Z_{mn}(\omega) = \iint\limits_V \boldsymbol{u}_m^\dagger(\boldsymbol{x}) \boldsymbol{G}(\boldsymbol{x}, \boldsymbol{x}', \omega) \boldsymbol{u}_n(\boldsymbol{x}') \, d^3x \, d^3x' \, . \tag{30}$$

For a chosen dimension N of the series expansions (24a) and (24b) we introduce the *generalized current and voltage vectors*

$$\boldsymbol{I}(\omega) = [I_1(\omega) \ldots I_N(\omega)]^T \, , \tag{31}$$

$$\boldsymbol{V}(\omega) = [V_1(\omega) \ldots V_N(\omega)]^T \, , \tag{32}$$

and the *impedance matrix*

$$\boldsymbol{Z}(\omega) = \begin{bmatrix} Z_{11}(\omega) & \ldots & Z_{1N}(\omega) \\ \vdots & \ddots & \vdots \\ Z_{N1}(\omega) & \ldots & Z_{NN}(\omega) \end{bmatrix} . \tag{33}$$

Numerical Computation of Stochastic Fields

We can write

$$V_m(\omega) = \sum_m Z_{mn}(\omega) I_n(\omega)\,,$$

in matrix form as

$$\boldsymbol{V}(\omega) = \boldsymbol{Z}(\omega)\boldsymbol{I}(\omega)\,. \tag{34}$$

as

$$\boldsymbol{C}_V(\omega) = \boldsymbol{Z}(\omega)\ \boldsymbol{C}_I(\omega)\ \boldsymbol{Z}^\dagger(\omega) \tag{35}$$

with

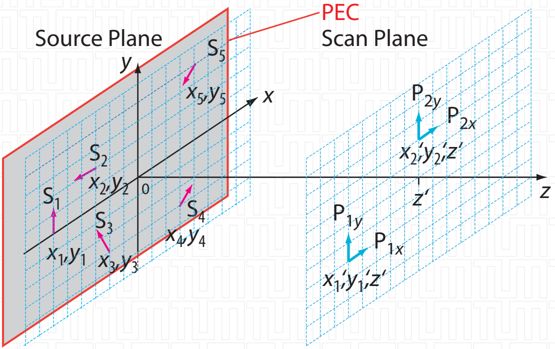
$$\boldsymbol{C}_I(\omega) = \lim_{T\to\infty} \frac{1}{2T} \langle \boldsymbol{I}_T(\omega) \boldsymbol{I}_T(\omega)^\dagger \rangle\,, \tag{36}$$

$$\boldsymbol{C}_V(\omega) = \lim_{T\to\infty} \frac{1}{2T} \langle \boldsymbol{V}_T(\omega) \boldsymbol{V}_T(\omega)^\dagger \rangle\,. \tag{37}$$

Using the MoM we have reduced the field problem to a network problem.

Outline

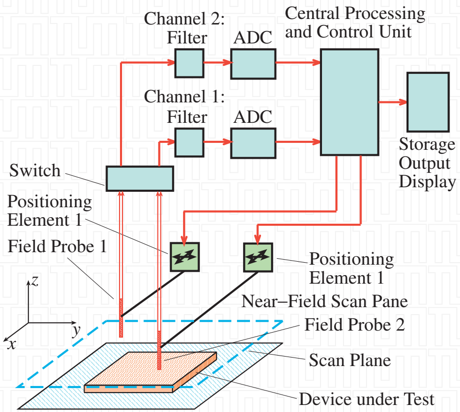
Near-Field Distribution of Sources in a Plane



Planar array of 5 sources sampled at a plane parallel to the source plane.

- J. A. Russer and P. Russer, “Modeling of noisy EM field propagation using correlation information,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 63, no. 1, pp. 76–89, Jan. 2015
- J. A. Russer, F. Mukhtar, O. Filonik, G. Scarpa, and P. Russer, “Modelling of noisy EM field propagation using correlation information of sampled data,” in *IEEE Int. Conf. on Numerical Electromagnetical Modeling and Optimization NEMO2014*, Pavia, Italia, May 2014

Near-Field Distribution of Sources in a Plane



Schematic drawing of the near-field scanning system.

Computation of the Near-Field

Accounting also for the near-field contributions the Green's dyadic is given by

$$\boldsymbol{G}(\boldsymbol{x},\omega) = \left[g_1(\boldsymbol{x},\omega)\mathbf{1} + g_2(\boldsymbol{x},\omega)\boldsymbol{x}\boldsymbol{x}^T\right]\mathrm{e}^{-\jmath\beta(\omega)|\boldsymbol{x}|}\,. \tag{38}$$

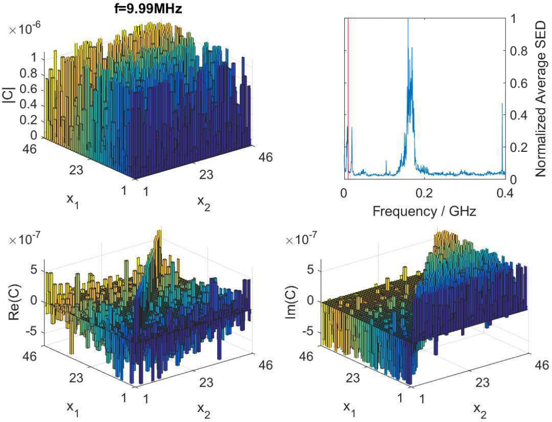
- $\mathbf{1}$: identity matrix
- $|\boldsymbol{x}| = \sqrt{x^2+y^2+z^2}$
- $\beta = \omega/c_0$: phase coefficient
- c_0: free-space light velocity

$$g_1(\boldsymbol{x},\beta) = -\frac{\jmath Z_{F0}\beta^2}{4\pi}\left[\frac{1}{\beta|\boldsymbol{x}|} + \frac{\jmath}{\beta^2|\boldsymbol{x}|^2} - \frac{1}{\beta^3|\boldsymbol{x}|^3}\right], \tag{39a}$$

$$g_2(\boldsymbol{x},\beta) = \frac{\jmath Z_{F0}\beta^2}{4\pi}\left[\frac{1}{\beta|\boldsymbol{x}|^3} + \frac{3\jmath}{\beta^2|\boldsymbol{x}|^4} - \frac{3}{\beta^3|\boldsymbol{x}|^5}\right]. \tag{39b}$$

$Z_{F0} = \sqrt{\varepsilon_0/\mu_0}$ is the free space wave impedance, and the ^{T} denotes the transpose of the vector.

Frequency Dependence of the Correlation Matrix



- J. A. Russer, M. Haider, M. H. Baharuddin, C. Smartt, A. Baev, S. Wane, D. Bajon, Y. Kuznetsov, D. Thomas, and P. Russer, "Correlation measurement and evaluation of stochastic electromagnetic fields," in *To be published at the EMC Europe 2016, Wroclaw*, 2016

Data Analysis - PCA

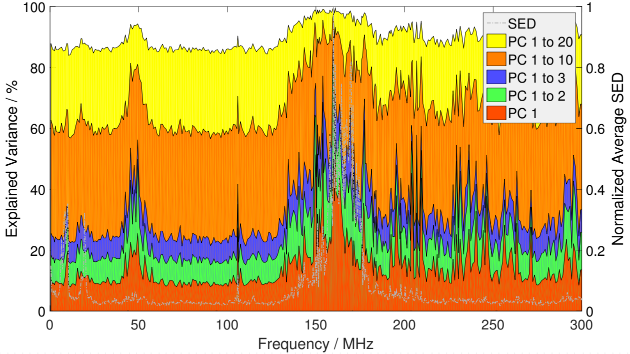
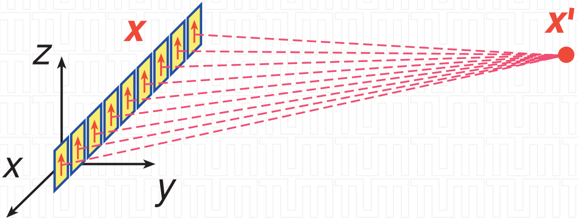


Figure: *Cumulative explained variance of the principal component vs. frequency and spectral energy density.*

• J. A. Russer, M. Haider, M. H. Baharuddin, C. Smartt, A. Baev, S. Wane, D. Bajon, Y. Kuznetsov, D. Thomas, and P. Russer, "Correlation measurement and evaluation of stochastic electromagnetic fields," in *To be published at the EMC Europe 2016, Wroclaw*, 2016

Outline

Far-Field for Partially Coherent Excitation



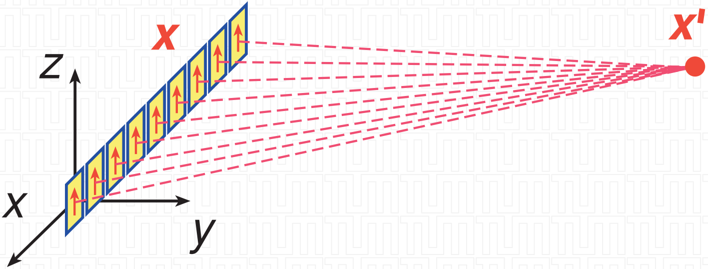
- In spherical coordinates the far-field exhibits only a ϑ-component of the electrical field and a φ-component of the magnetic field, given by

$$E_\vartheta = Z_{F0} H_\varphi = j Z_{F0} F(\vartheta, \varphi) \sum_{\nu=0}^{N-1} I_\nu \frac{\mathrm{e}^{-jkr_\nu}}{2\pi r_\nu}\,, \tag{40}$$

  where I_ν is the excitation current of the ν-th dipole, Z_0 is the wave impedance of the free space, $F(\vartheta, \varphi)$ is the single dipole characteristics.

• J. A. Russer and P. Russer, "An efficient method for computer aided analysis of noisy electromagnetic fields," in *Microwave Symposium Digest (MTT), 2011 IEEE MTT-S International*, IEEE, Jun. 2011, pp. 1–4

Far-Field for Partially Coherent Excitation



- The distances between the far-field point of observation and the center of the νth dipole is given by

$$r_\nu = r_0 - x_\nu \sin\vartheta \cos\varphi - y_\nu \sin\vartheta \sin\varphi - z_\nu \cos\vartheta \,. \tag{41}$$

Far-Field for Partially Coherent Excitation

- On the right-hand side of (40) we can approximate in the far field the r_ν in the denominators by r_0.
- We summarize the antenna element feed currents in the vector

$$\boldsymbol{I} = [I_1 \ldots I_N]^T \ . \tag{42}$$

- Furthermore, we introduce the vector

$$\boldsymbol{M}(\vartheta, \varphi) = jZ_{F0} \frac{F(\vartheta, \varphi)}{2\pi r_0} \left[\mathrm{e}^{-jkr_1} \ldots \mathrm{e}^{-jkr_{N-1}} \right] \ . \tag{43}$$

- With (40) this yields

$$E_\vartheta(\vartheta, \varphi) = \boldsymbol{M}(\vartheta, \varphi) \boldsymbol{I} \ . \tag{44}$$

Far-Field for Partially Coherent Excitation

- To describe a stochastic excitation of these near-field cells we introduce the correlation matrix of the antenna feed currents

$$\boldsymbol{C}_I(\omega) = \lim_{T\to\infty} \frac{1}{2T} \langle \boldsymbol{I}_T(\omega)\, \boldsymbol{I}_T^\dagger(\omega)\rangle \,. \tag{45}$$

- We obtain

$$\Gamma_{E\vartheta}(\vartheta_1, \varphi_1; \vartheta_2, \varphi_2) = \boldsymbol{M}(\vartheta_1, \varphi_1)\boldsymbol{C}_I \boldsymbol{M}^\dagger(\vartheta_2, \varphi_2) \,. \tag{46}$$

- This yields the spectral electric energy density of the far-field

$$W_E(\vartheta, \varphi) = \frac{\varepsilon}{2}\boldsymbol{M}(\vartheta, \varphi)\boldsymbol{C}_I \boldsymbol{M}^\dagger(\vartheta, \varphi) \,. \tag{47}$$

Far-Field for Partially Coherent Excitation

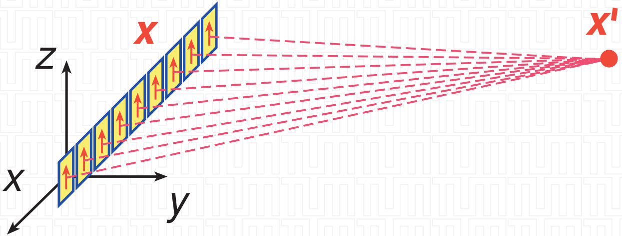


Figure: *Linear array of $N = 9$ near-field cells.*

Let us make the following assumptions:

- All currents in $\boldsymbol{I}_1$ are mutually correlated and in phase.
- The currents in $\boldsymbol{I}_2$ are assumed to be mutually correlated and to exhibit a phase delay of $n\pi/6$ with respect to I_{21}.
- The currents of $\boldsymbol{I}_3$ all may have equal amplitude but are mutually uncorrelated.
- The currents $\boldsymbol{I}_i$ and $\boldsymbol{I}_j$ are mutually uncorrelated for $i \neq j$.

Far-Field for Partially Coherent Excitation

- The correlation matrices of $\boldsymbol{I}_1$, $\boldsymbol{I}_2$ and $\boldsymbol{I}_3$ are given by

$$\boldsymbol{C}_{ij}^{I} = \langle \boldsymbol{I}_i \boldsymbol{I}_j \rangle \,. \tag{48}$$

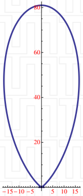
- The mutually uncorrelated currents $\boldsymbol{I}_i$ and $\boldsymbol{I}_j$ yield

$$\boldsymbol{C}_{ij}^{I} = \boldsymbol{0} \quad \text{for} \quad i \neq j \,. \tag{49}$$

- The mutually correlated in-phase currents $\boldsymbol{I}_1$ of equal amplitude I_1 are described by

$$\boldsymbol{C}_{11}^{I} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & 1 \end{bmatrix} . \tag{50}$$

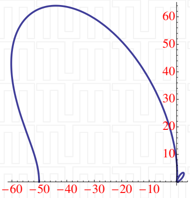
Far-Field for Partially Coherent Excitation



- Excitation with C^I_{11}
- The currents are correlated and all in phase.
- Angular far-field spectral energy density distribution for $\vartheta = \pi/2$.

$$C^I_{11} = |I_1|^2 \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{51}$$

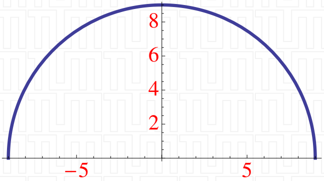
Far-Field for Partially Coherent Excitation



- Excitation with C^{I}_{11}
- The currents are correlated and mutually delayed by $\pi/6$.
- Angular far-field spectral energy density distribution for $\vartheta = \pi/2$.

$$\begin{bmatrix}
1 & e^{-\frac{i\pi}{6}} & e^{-\frac{i\pi}{3}} & -i & e^{-\frac{2i\pi}{3}} & e^{-\frac{5i\pi}{6}} & -1 & e^{\frac{5i\pi}{6}} & e^{\frac{2i\pi}{3}} \\
e^{\frac{i\pi}{6}} & 1 & e^{-\frac{i\pi}{6}} & -ie^{\frac{i\pi}{6}} & -i & e^{-\frac{2i\pi}{3}} & -e^{\frac{i\pi}{6}} & -1 & e^{\frac{5i\pi}{6}} \\
e^{\frac{i\pi}{3}} & e^{\frac{i\pi}{6}} & 1 & -ie^{\frac{i\pi}{3}} & e^{-\frac{i\pi}{3}} & -i & -e^{\frac{i\pi}{3}} & e^{-\frac{5i\pi}{6}} & -1 \\
i & ie^{-\frac{i\pi}{6}} & ie^{-\frac{i\pi}{3}} & 1 & ie^{-\frac{2i\pi}{3}} & ie^{-\frac{5i\pi}{6}} & -i & ie^{\frac{5i\pi}{6}} & ie^{\frac{2i\pi}{3}} \\
e^{\frac{2i\pi}{3}} & i & e^{\frac{i\pi}{3}} & -ie^{\frac{2i\pi}{3}} & 1 & e^{-\frac{i\pi}{6}} & -e^{\frac{2i\pi}{3}} & -i & e^{-\frac{2i\pi}{3}} \\
e^{\frac{5i\pi}{6}} & e^{\frac{2i\pi}{3}} & i & -ie^{\frac{5i\pi}{6}} & e^{\frac{i\pi}{6}} & 1 & -e^{\frac{5i\pi}{6}} & e^{-\frac{i\pi}{3}} & -i \\
-1 & -e^{-\frac{i\pi}{6}} & -e^{-\frac{i\pi}{3}} & i & -e^{-\frac{2i\pi}{3}} & -e^{-\frac{5i\pi}{6}} & 1 & -e^{\frac{5i\pi}{6}} & -e^{\frac{2i\pi}{3}} \\
e^{-\frac{5i\pi}{6}} & -1 & e^{\frac{5i\pi}{6}} & -ie^{-\frac{5i\pi}{6}} & i & e^{\frac{i\pi}{3}} & -e^{-\frac{5i\pi}{6}} & 1 & e^{-\frac{i\pi}{6}} \\
e^{-\frac{2i\pi}{3}} & e^{-\frac{5i\pi}{6}} & -1 & -ie^{-\frac{2i\pi}{3}} & e^{\frac{2i\pi}{3}} & i & -e^{-\frac{2i\pi}{3}} & e^{\frac{i\pi}{6}} & 1
\end{bmatrix} \tag{52}$$

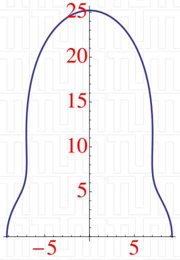
Far-Field for Partially Coherent Excitation



- Excitation with $\boldsymbol{C}^I_{33}$
- All currents are mutually uncorrelated.
- Angular far-field spectral energy density distribution for $\vartheta = \pi/2$.

$$\boldsymbol{C}^I_{33} = |I_3|^2 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{53}$$

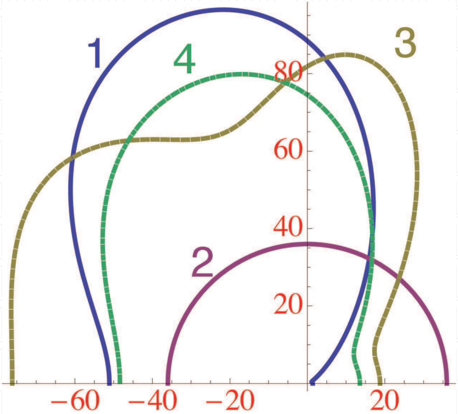
Far-Field for Partially Coherent Excitation



- Excitation with superposition of correlated currents C_{11}^{I} and uncorrelated currents C_{33}^{I}
- Angular far-field spectral energy density distribution for $\vartheta = \pi/2$.

$$C_{11}^{I} + 0.2C_{33}^{I} = |I_1|^2 \begin{bmatrix} 1.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 1.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 1.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 1.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 & 1.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 1.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 1.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 1.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 1.2 \end{bmatrix} \tag{54}$$

Far-Field for Partially Coherent Excitation



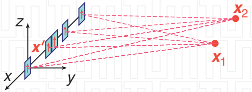
1. $\boldsymbol{C}^I_{11} + \boldsymbol{C}^I_{22}$
2. $4.0\boldsymbol{C}^I_{33}$
3. $\boldsymbol{C}^I_{44} + 0.2\boldsymbol{C}^I_{22}$
4. $0.7\boldsymbol{C}^I_{11} + 0.7\boldsymbol{C}^I_{22} + 1.4\boldsymbol{C}^I_{33}$

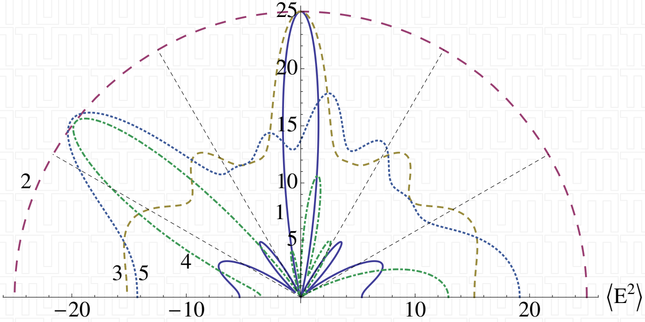
Electric far-field distribution over φ for $\vartheta = \pi/2$.

$$\boldsymbol{C}^I_{44}(\omega) = \lim_{T\to\infty} \frac{1}{2T}\langle[\boldsymbol{I}_{1T}(\omega) + \boldsymbol{I}_{2T}(\omega)][\boldsymbol{I}^\dagger_{1T}(\omega)\boldsymbol{I}^\dagger_{2T}(\omega)]\rangle\,.$$

Far-Field for Partially Coherent Excitation

Non-uniform linear array of Hertzian dipoles.





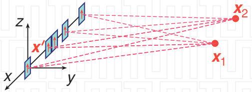
Polar plot of $\langle |E_z(\varphi)|^2 \rangle$ in the far-field.

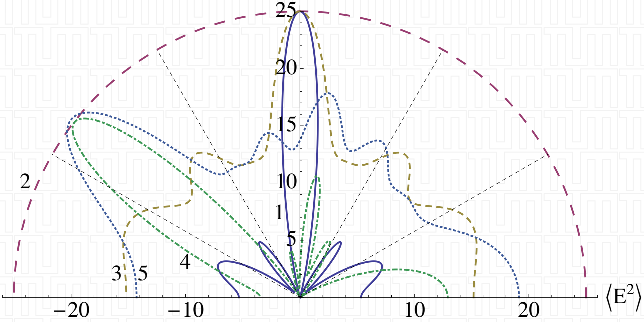
$$\boldsymbol{C_1^I} = i_0^2 \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}, \quad \boldsymbol{C_2^I} = 5 i_0^2 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\boldsymbol{C_3^I} = 0.5\boldsymbol{C_1^I} + 0.5\boldsymbol{C_2^I}\,.$$

Far-Field for Partially Coherent Excitation

Non-uniform linear array of Hertzian dipoles.



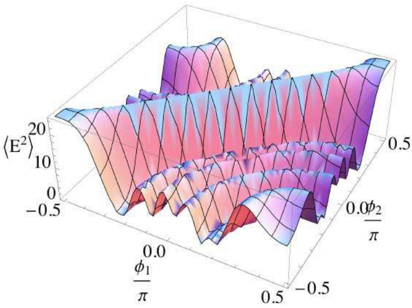


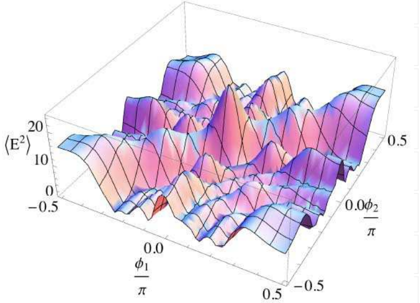
Polar plot of $\langle |E_z(\varphi)|^2 \rangle$ in the far-field.

$$\boldsymbol{C}_{\boldsymbol{4}}^{\boldsymbol{I}} = i_0^2 \begin{bmatrix} 1 & e^{0.2\pi j} & e^{-0.4\pi j} & e^{-1.5\pi j} & e^{-1.1\pi j} \\ e^{-0.2\pi j} & 1 & e^{-0.6\pi j} & e^{-1.7\pi j} & e^{-1.3\pi j} \\ e^{0.4\pi j} & e^{0.6\pi j} & 1 & e^{-1.1\pi j} & e^{-0.7\pi j} \\ e^{1.5\pi j} & e^{1.7\pi j} & e^{1.1\pi j} & 1 & e^{0.4\pi j} \\ e^{1.1\pi j} & e^{1.3\pi j} & e^{0.7\pi j} & e^{-0.4\pi j} & 1 \end{bmatrix}, \quad \boldsymbol{C}_{\boldsymbol{5}}^{\boldsymbol{I}} = 2.5\boldsymbol{C}_{\boldsymbol{2}}^{\boldsymbol{I}} + 0.54\boldsymbol{C}_{\boldsymbol{3}}^{\boldsymbol{I}} \,.$$

Far-Field for Partially Coherent Excitation

Excitation Dependence of Far-Field Correlation





Uncorrelated

Excitation Currents

Partially Correlated

The magnitude $|\underline{\underline{\Gamma}}_{Ezz}(\varphi_1, \varphi_2)|$ of the correlation spectrum of the electric field amplitudes in the far-field at angular positions φ_1 and φ_2 for uncorrelated and partially correlated excitation currents.

Outline

TLM - A Discrete Scheme of Electromagnetism

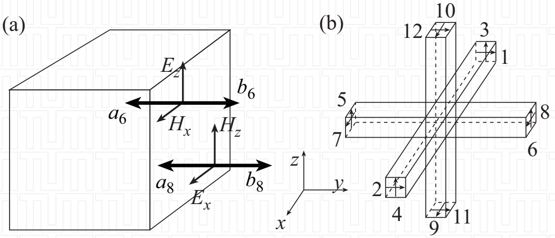


Figure: *Schematic of the TLM cell: a) Space cell with samples of the tangential electric and magnetic field values and wave pulse amplitudes, b) TLM node.*

In the TLM–method, the electromagnetic field is modeled by wave pulses propagating on a Cartesian mesh of transmission lines.

- W. Hoefer, "The transmission line matrix (TLM) method," in *Numerical Techniques for Microwave and Millimeter Wave Passive Structures*, T. Itoh, Ed., New York: J. Wiley, 1989, pp. 496–591

TLM - A Discrete Scheme of Electromagnetism

- In a compact formulation of the TLM scheme we summarize all $12N$ incident wave pulses in the vector $a[k]$ and all $12N$ scattered wave pulses in the vector $b[k]$.
- The argument k enumerates the discrete time step. We can formulate the TLM scheme in the compact Hilbert space notation where the scattering matrix $\boldsymbol{S}$ describes the instantaneous scattering of the wave pulses in the TLM node and $\mathbf{\Gamma}$ describes the connection of the TLM nodes with the adjacent TLM nodes.

Correlation Green's Function

We introduce the *Correlation Green's Function (CGF)* $K_{ij;pq}[k]$ for the TLM wave amplitude correlation functions

$$K_{ij;pq}[k] = \sum_{l=-\infty}^{\infty} G_{i,p}[l] G_{j,q}[l+k]\,. \tag{55}$$

We obtain

$$\boxed{c^b_{ij}[m] = \sum_{n_r,n_s\in B} \sum_{l=-\infty}^{\infty} K_{ij;rs}[l] c^a_{rs}[m-l]\,,} \tag{56}$$

relating the auto- and cross correlation functions $c^b_{ij}[m]$ of the wave amplitudes scattered from the boundary to the auto- and cross correlation functions $c^a_{rs}[m]$ incident to the boundary.

• J. A. Russer, A. Cangellaris, and P. Russer, "Correlation transmission line matrix (CTLM) modeling of stochastic electromagnetic fields," in *Microwave Symposium (IMS), 2015 IEEE MTT-S International*, May 2016, to be published

Numerical Example

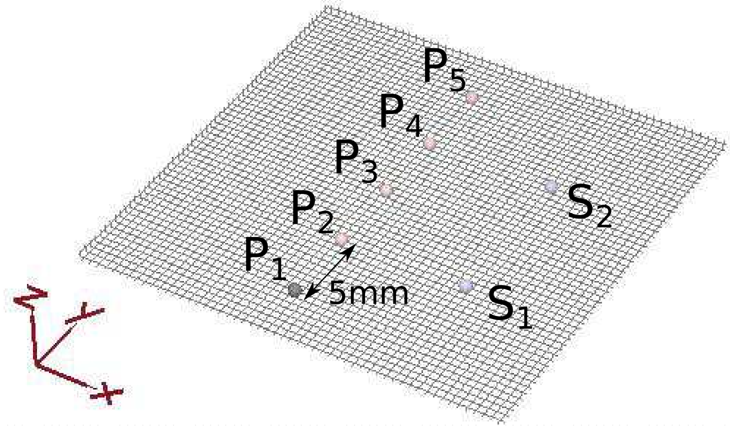


Figure: *Arrangement of source and observation points.*

Numerical Example

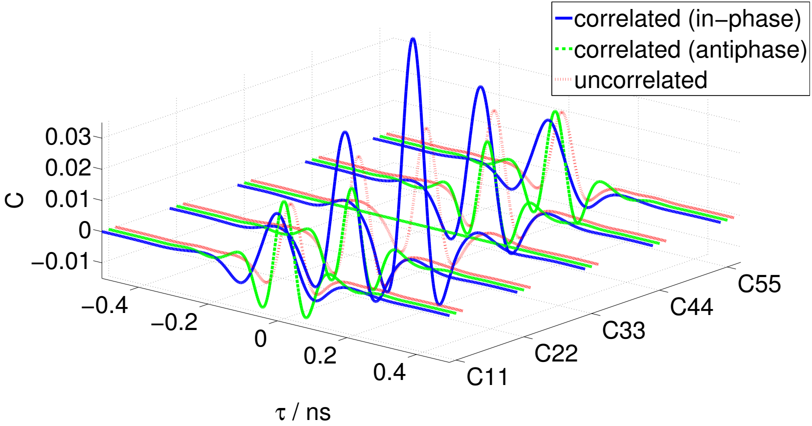


Figure: *Time domain autocorrelations of the observation points for a two-source excitation with correlated in-phase, correlated antiphase, and uncorrelated sources.*

Outline

Conclusion

- For computation of the response to stochastic field excitation the response functions computed for deterministic fields can be used.
- For the correct modeling of stochastic electromagnetic fields the spatial correlations of the source distributions have to be considered.
- The measurement effort is feasible if modern time-domain EMI measurement systems are applied. The intensity and mutual correlation of noise sources can be analyzed.

THANK YOU FOR YOUR KIND ATTENTION!